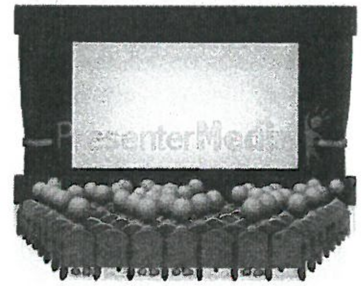
Name: Answer Key Per: ________ Date: ______________

Serafino ▪ Algebra 2E

8C Arithmetic Series

Notes/CW/HW



$a_n = 7 + 3(n-1)$

Ex 1) Theater Seating: Let's consider the seats in a theater.
The first row has 7 seats, and each row has 3 more seats than the row in front of it.

How many seats are there in the 2nd row? 6th? 30th ?

$a_2 = 10$ $a_6 = 22$ $a_{30} = 94$

Obviously, you can see the benefit of having an explicit arithmetic formula for sequences. You can find out any term in a sequence of numbers, which means you can know the number of seats in any row. But What if you wanted to know how many TOTAL seats are in the first 10 rows? Go ahead.... I'll wait....

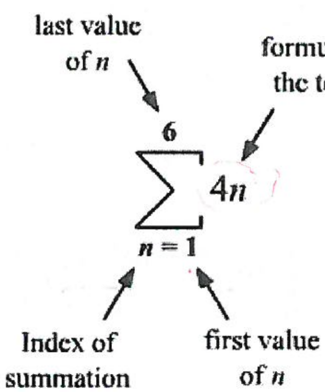
$7 + 10 + 13 + 16 + 19 + 22 + 25 + 28 + 31 + 34 = 205$

You probably did a lot of listing and adding. That's okay, but what if you wanted to know the total number of seats the first 75 rows? What about the total seats in rows 27 through 48? There's a notation for this exact sort of thing.

$$\sum_{n=1}^{10} 7 + 3(n-1) \qquad \sum_{n=1}^{75} 7 + 3(n-1) \qquad \sum_{n=27}^{48} 7 + 3(n-1)$$

Sigma: Summation Notation

You can use any variable but n, k, a, and i are the ones you'll see most often.
Rewrite each series as the sum of the terms indicated, then evaluate with the sigma function in your calculator:



$$\sum_{a=1}^{5} a^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$$
$1 + 4 + 9 + 16 + 25$
$= \boxed{55}$

$$\sum_{k=3}^{6} 2k = 6 + 8 + 10 + 12$$
$= \boxed{36}$

$$\sum_{i=15}^{17} 50 - i = 35 + 34 + 33$$
$= \boxed{102}$

$$\sum_{m=0}^{4} \frac{m}{m+1} = 0 + \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5}$$
$= \boxed{\frac{163}{60}}$

$$\sum_{2}^{5} m(m-1) = 2(2-1) + 3(3-1) + 4(4-1) + 5(5-1)$$
$1 \quad 2 \quad 3 \quad 4$
$2 + 6 + 12 + 20 = \boxed{40}$

$$\sum_{x=0}^{5} 2^x = 2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5$$
$1 + 2 + 4 + 8 + 16 + 32$
$= \boxed{63}$

$$\sum_{3}^{7} m^2 = 3^2 + 4^2 + 5^2 + 6^2 + 7^2 = \boxed{135}$$
$9 + 16 + 25 + 36 + 49$

$$\sum_{n=3}^{5} n(n+1) = 3(4) + 4(5) + 5(6)$$
$12 + 20 + 30$
$= \boxed{62}$

$$\sum_{4}^{9} \frac{1}{m} = \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} = \boxed{\frac{2509}{2520}}$$

Ex 2) Finding pencils: The first week of school, I found 3 pencils on the floor. The next week, I found 5, the next week I found 7, etc…

$a_n = 3+2(n-1)$

Use Sigma notation and evaluate with a calculator.

a) How many total pencils have I found after 4 weeks? 12 weeks? 40 weeks?

$\sum_{n=1}^{4} 3+2(n-1)$ $\sum_{n=1}^{12} 3+2(n-1)$ $\sum_{n=1}^{40} 3+2(n-1)$

Without a calculator, there's a quick formula.
S_n = the sum of the first n terms

Add 1st last

$$\boxed{S_n = n\left(\frac{a_1+a_n}{2}\right)}$$

sum of terms up to a_n #terms

Why would you do this when you have a calculator with a sigma function? Well, you wouldn't. It's useful for something later in this packet. But If you don't have a calculator, here's how you do it. So, to make things easier and move things along, <u>I will let you use the formula for the arithmetic, but you must set up and use the formula</u>.

b) Use the partial sum formula to find how many pencils I have at the end of 4 weeks:

$S_4 = 4\left(\frac{a_1+a_4}{2}\right)$ $S_4 = 4\left(\frac{3+9}{2}\right) = 4(6) = \boxed{24}$

c) How many pencils will I have at the end of week 12? Week 40?

$S_{12} = 12\left(\frac{3+25}{2}\right) = 12(14) = \boxed{168}$ $S_{40} = 40\left(\frac{3+81}{2}\right) = \boxed{1,680}$

d) On what week will I have found a grand total of 3968 pencils?

$S_n = \boxed{n}\left(\frac{a_1+a_n}{2}\right)$

$3968 = n\left(\frac{3+(3+\underbrace{2(n-1)}_{2n-2})}{2}\right)$

$2 \cdot 3968 = n\left(\frac{4+2n}{\cancel{2}}\right)$

$7936 = 4n+2n^2$

$2n^2+4n-7936$

$2(n^2+2n-3968)$

Factor or do QF

$\frac{-2 \pm \sqrt{4-4(1)(-3968)}}{2(1)}$

$\frac{-2 \pm 126}{2}$ → 62 → -64

$\boxed{62^{nd} \text{ term}}$

Find the sum of the finite sequence with the partial sum formula; check by setting up Sigma notation and then check the sum with your calculator.

1) 13, 15, 17, 19, 21, 23 $\boxed{=108}$

$\sum_{n=1}^{6} 13+2(n-1)$

2) 6, 11, 16, 21, 26, 31, 36 $\boxed{=147}$

$\sum_{n=1}^{n=7} 6+5(n-1)$

3) 22, 28, 34, 40, 46 $\boxed{=170}$

$\sum_{n=1}^{5} 22+6(n-1)$

4) 39, 49, 59, 69 $\boxed{=216}$

$\sum_{n=1}^{4} 39+10(n-1)$

For each arithmetic series, rewrite how to calculate the sum using the Partial Sum Formula.

5) $\sum_{k=1}^{35}(5k-2) = 35\left(\frac{3+173}{2}\right)$

$\boxed{= 3080}$

6) $\sum_{i=1}^{35}(3i-13) = 35\left(\frac{-10+92}{2}\right)$

$= \boxed{1435}$

7) $\sum_{m=1}^{15} 4m = 15\left(\frac{4+60}{2}\right)$

$\boxed{= 480}$

8) $\sum_{m=1}^{10}(7m-2) = 10\left(\frac{5+68}{2}\right)$

$\boxed{= 365}$

9) $\sum_{i=1}^{6} 3i = 6\left(\frac{3+18}{2}\right)$

$\boxed{= 63}$

10) $\sum_{n=1}^{45}(3n-9) \quad 45\left(\frac{-6+126}{2}\right)$

$\boxed{= 2{,}700}$

For each, find the sum using the partial sum formula, then rewrite the expression in Sigma Notation.

$\frac{104}{13} = 8$

11) $a_1 = 42,\ a_n = 146,\ n = 14$

$S_{14} = 14\left(\frac{42+146}{2}\right)$

$= \boxed{1316}$

$\sum_{n=1}^{14} 42 + 8(n-1)$

12) $a_1 = 4,\ a_n = 22,\ n = 10$

$S_{10} = 10\left(\frac{4+22}{2}\right)$

$= \boxed{130}$

$\sum_{n=1}^{10} 4 + 2(n-1)$

13) $a_1 = 2,\ a_n = 122,\ n = 13$

$S_{13} = 13\left(\frac{2+122}{2}\right)$

$= \boxed{806}$

$\sum_{n=1}^{13} 2 + 10(n-1)$

14) $a_1 = -18,\ a_n = -102,\ n = 13$

$S_{13} = 13\left(\frac{-18-102}{2}\right)$

$= \boxed{-780}$

$\sum_{n=1}^{13} -18 - 7(n-1)$

15) $20 + 27 + 34 + 41\ldots,\ n = 16$

$S_{16} = 16\left(\frac{20+125}{2}\right)$

$= \boxed{1160}$

$\sum_{n=1}^{16} 20 + 7(n-1)$

16) $20 + 30 + 40 + 50\ldots,\ n = 15$

$S_{15} = 15\left(\frac{20+160}{2}\right)$

$= \boxed{1350}$

$\sum_{n=1}^{15} 20 + 10(n-1)$

17) $7 + 9 + 11 + 13\ldots,\ n = 10$

$S_{10} = 10\left(\frac{7+25}{2}\right)$

$= \boxed{160}$

$\sum_{n=1}^{10} 7 + 2(n-1)$

18) $10 + 12 + 14 + 16\ldots,\ n = 11$

$S_{11} = 11\left(\frac{10+30}{2}\right)$

$= \boxed{220}$

$\sum_{n=1}^{11} 10 + 2(n-1)$

The sum of the finite arithmetic series is given. Determine the number of terms in the series.

19) $a_1 = 19,\ a_n = 96,\ S_n = 690$

$690 = n\left(\frac{19+96}{2}\right)$ $\boxed{n=12}$

20) $a_1 = 16,\ a_n = 163,\ S_n = 4475$

$4475 = n\left(\frac{16+163}{2}\right)$ $\boxed{n=50}$

21) $a_1 = 19,\ a_n = 118,\ S_n = 822$

$822 = n\left(\frac{19+118}{2}\right)$ $\boxed{n=12}$

22) $a_1 = 15,\ a_n = 79,\ S_n = 423$

$423 = n\left(\frac{15+79}{2}\right)$ $\boxed{n=9}$

23) $a_1 = -3,\ d = 2,\ S_n = 21$

$21 = n\left(\frac{-3+[-3+2(n-1)]}{2}\right)$ $\boxed{n=7}$

24) $a_1 = 4,\ d = 7,\ S_n = 228$

$228 = n\left(\frac{4+\ 4+7(n-1)}{2}\right)$ $\boxed{n=8}$

Skills Check Practice:

1. Evaluate the arithmetic series by using the partial sum formula.

$$\sum_{n=1}^{16} -12 + 3(n-1) \qquad 16\left(\frac{-12+33}{2}\right) \qquad \boxed{=168}$$

$$\sum_{k=1}^{22} -\frac{1}{2}k + 8 \qquad 22\left(\frac{7.5-3}{2}\right) \qquad = \boxed{49.5}$$

2. The finite series: 7 + 11 + 15 + … + 207 $\quad a_n = 7+4(n-1)$

a. Find n and write the expression in Sigma Notation.

$207 = 7+4(n-1)$

$\boxed{n=51}$

$\boxed{\sum_{n=1}^{51} 7+4(n-1)}$

b. Evaluate

$S_{51} = \boxed{5457}$

3. Kwanzaa is an African American harvest festival that celebrates the New Year. It involves a candle lighting ceremony. On the first night, a candle is lit and blown out. On the second night, two candles are lit, and blown out, and so on. By the 7th night, how many candle–lightings have taken place? 1, 2, 3, …

$\sum_{n=1}^{7} 1+1(n-1) = 7\left(\frac{1+7}{2}\right) = \boxed{28 \text{ candles}}$

4. A skydiver jumps from an airplane and falls. She falls 16 feet in the first second, 48 during the second second, 80 feet in the third second, etc. How many feet did she fall after 20 seconds?

$a_n = 16+32(n-1)$ $\quad S_{20} = (20)\left(\frac{16+624}{2}\right) = \boxed{6400 \text{ total feet}}$

5. Dean is obsessed with Chuck Norris. He decides to start tweeting at him every day, and to increase the number of daily tweets until he hears back. On the 2nd day, he wrote 7 tweets. On the 5th day, he wrote 22 tweets. Chuck Norris finally gives up and responds. By then, Chuck had received a TOTAL of 3,045 tweets from Dean. How many days did it take for Dean's plan to work?

$d = 5 \quad a_1 = 2$

$3045 = n\left(\frac{2+\ \overbrace{2+5(n-1)}^{5n-5}}{2}\right)$

$3045 = n\left(\frac{5n-1}{2}\right)$

$5n^2 - n - 6090$

$\frac{1 \pm \sqrt{1-4(5)(-6090)}}{2(5)}$

$\frac{1 \pm 349}{10}$ $\quad n = \boxed{35 \text{ days}}$