

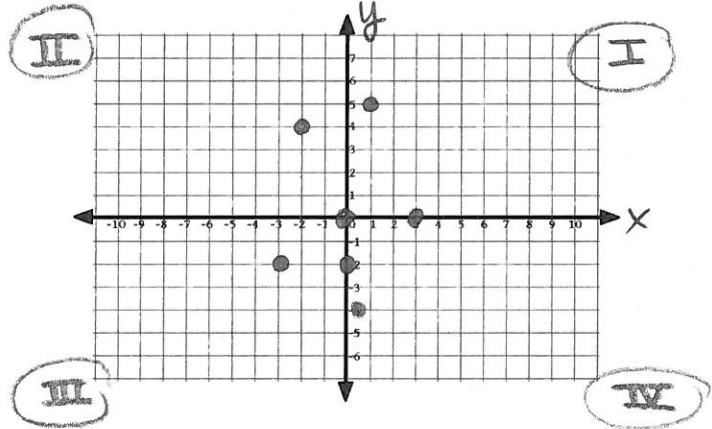
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# 9A1 Angles in Standard Position

Notes / Classwork

First, we need to establish that you are comfortable with the basics of the rectangular coordinate system:

1. Label the x and y axes.
2. Put a point at the origin.
3. Label the quadrants.
4. Point plot the following coordinates:  
(1,5) (-2,4) (-3,-2) (1/2, -4) (3,0) (0,-2)
5. For each point, name the quadrant it is in and calculate its distance from the origin.  
Give exact (= radical) AND approximate ( $\approx$  decimal)



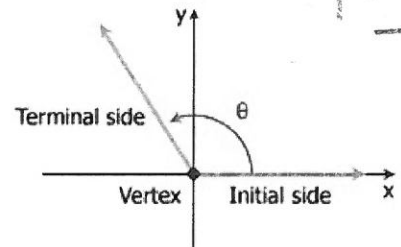
$(5, 12) \sqrt{5^2 + 12^2} = \sqrt{169} = 13 \text{ units}$   
**Q I**  
 $(8, -4) \sqrt{8^2 + 4^2} = \sqrt{80} = 4\sqrt{5} \text{ units}$   
 $\approx 8.944$   
**Q IV**

$(-2, 8) \sqrt{2^2 + 8^2} = \sqrt{68} = 2\sqrt{17} \text{ u}$   
 $\approx 8.246 \text{ u}$   
**Q II**  
 $(-3, -3) \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2} \text{ u}$   
 $\approx 4.243 \text{ u}$   
**Q III**

### Angles in Standard Position: $\theta$

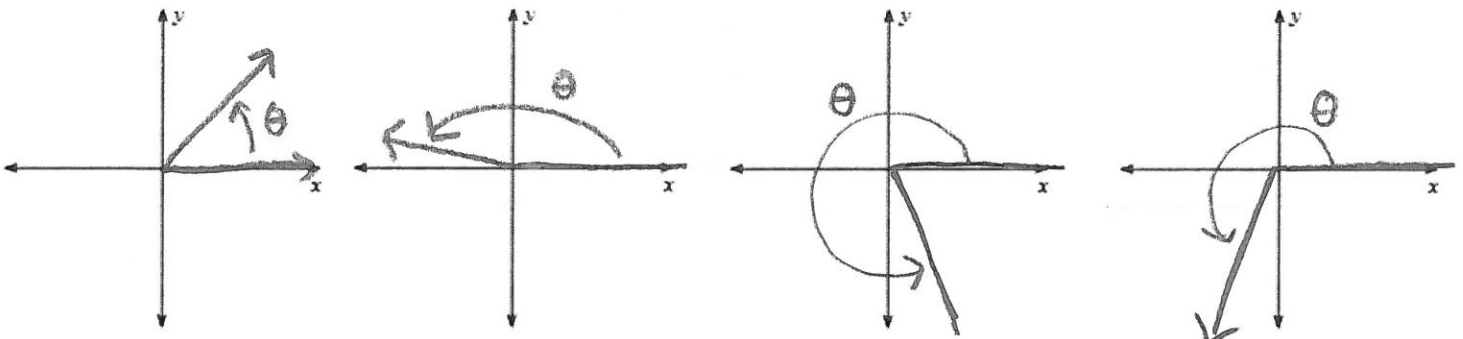
An angle in the coordinate plane is in standard position when its initial side is along the positive x-axis and its vertex is at the origin. The other side is called the terminal side.

The terminal side always opens up counterclockwise about the origin. The angle that is formed is referred to as  $\theta$  (theta).



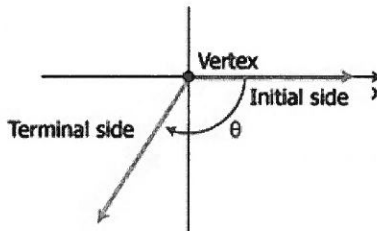
Draw the following angles in standard position. Indicate which Quadrant the angle terminates.

- $\theta = 45^\circ \in \text{Q } \underline{\text{I}}$      
  $\theta = 170^\circ \in \text{Q } \underline{\text{II}}$      
  $\theta = 300^\circ \in \text{Q } \underline{\text{IV}}$      
  $\theta = 250^\circ \in \text{Q } \underline{\text{III}}$



**Negative Angles in Standard Position:**

Now, angles can be negative.  
This means we go clockwise about the origin.



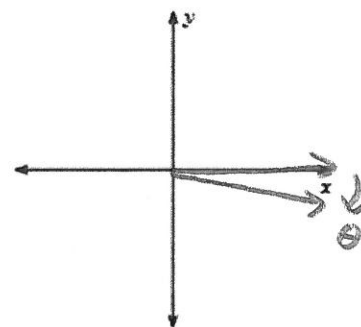
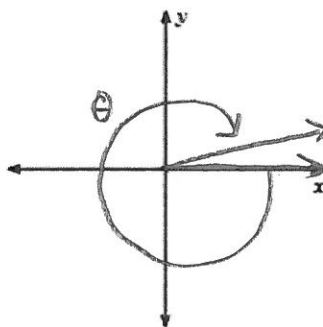
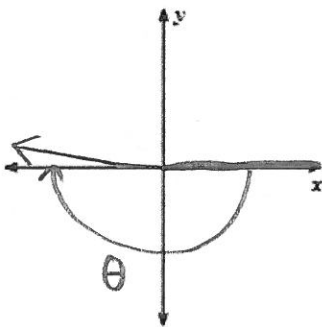
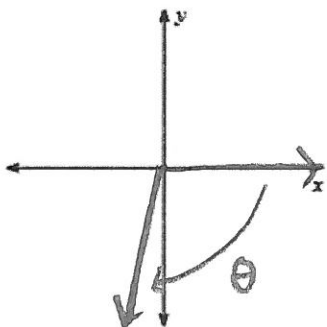
Draw the following angles in standard position:

$\theta = -100^\circ \in Q$  III

$\theta = -185^\circ \in Q$  II

$\theta = -350^\circ \in Q$  I

$\theta = -10^\circ \in Q$  IV

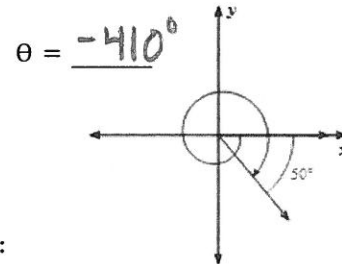


**Coterminal Angles: Go beyond 360°**

Since we're swinging angles around a center, we can go around more than one time.  
Angles that are one full rotation away from one another are coterminal.  
Visually, their terminal sides end up in the exact same position.

The way we denote the swinging around is to draw a swoopy arrow.

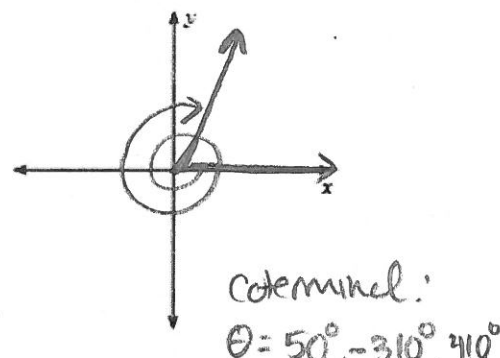
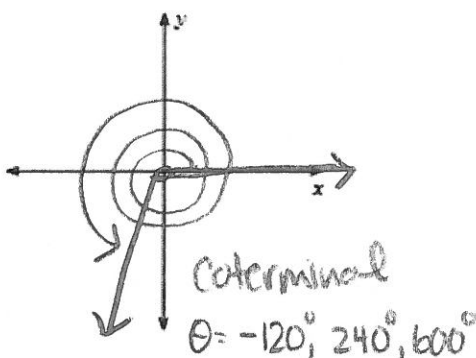
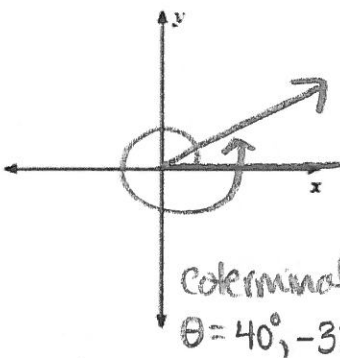
Indicate which quadrant the given angle is in. Draw it, then name three coterminal angles:



$\theta = 400^\circ \in Q$  I

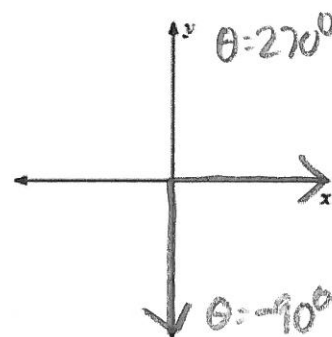
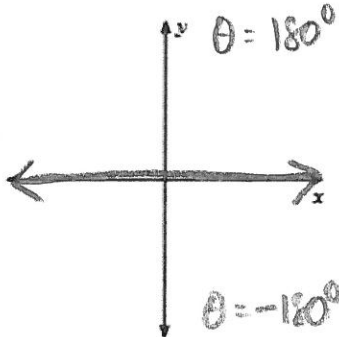
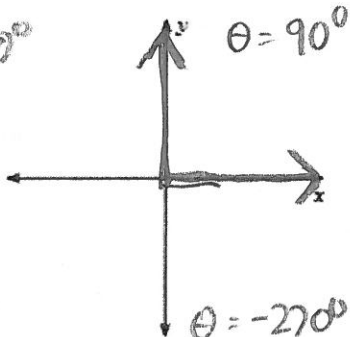
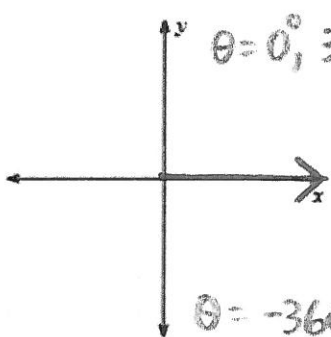
$\theta = 960^\circ \in Q$  III

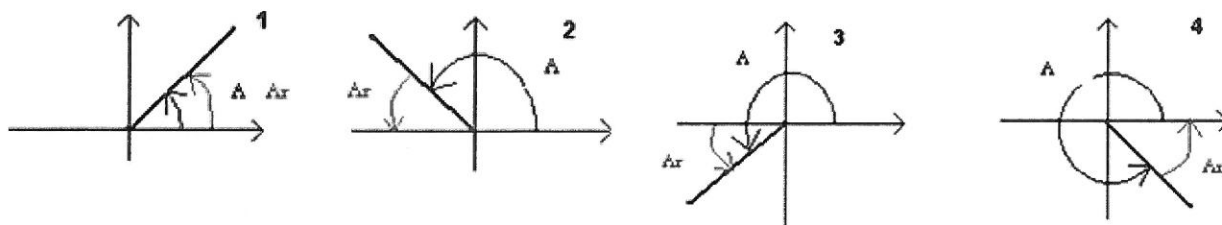
$\theta = -675^\circ \in Q$  I



**Quadrantal Angles:**

Angles whose terminal side lies on an axis. They are NOT in a quadrant.



**Reference Angles:  $\theta'$** An acute angle formed by the terminal side and the nearest x-axis

FINDING REFERENCE ANGLES					USING REFERENCE ANGLES				
Q	$\theta$	$\theta'$	Sketch	What math did you do?	Q	$\theta'$	$0^\circ \leq \theta < 360^\circ$	Sketch	What math did you do?
I	$6^\circ$	$6^\circ$		none ☺	I	$20^\circ$	$20^\circ$		none ☺
	$27^\circ$	$27^\circ$				$48^\circ$	$48^\circ$		
	$88^\circ$	$88^\circ$				$81^\circ$	$81^\circ$		
II	$100^\circ$	$80^\circ$		$180^\circ - \theta$	II	$70^\circ$	$110^\circ$		$180 - \theta'$
	$132^\circ$	$48^\circ$				$55^\circ$	$125^\circ$		
	$179^\circ$	$1^\circ$				$20^\circ$	$160^\circ$		
III	$196^\circ$	$16^\circ$		$\theta - 180^\circ$	III	$1^\circ$	$181^\circ$		$180 + \theta'$
	$201^\circ$	$21^\circ$				$15^\circ$	$195^\circ$		
	$268^\circ$	$88^\circ$				$40^\circ$	$220^\circ$		
IV	$305^\circ$	$55^\circ$		$360^\circ - \theta$	IV	$80^\circ$	$280^\circ$		$360 - \theta'$
	$352^\circ$	$8^\circ$				$50^\circ$	$310^\circ$		
	$359.5^\circ$	$0.5^\circ$				$10^\circ$	$350^\circ$		

Some "gentle" NEVERS about reference angles:



- Reference Angles are NEVER right or obtuse. They are ALWAYS acute.
- They are NEVER measured from the y-axis, and will NEVER be calculated with  $90^\circ$  nor  $270^\circ$ .

**Why are Reference Angles a thing?**If  $\theta = 170^\circ$ ,  $\theta' = 10^\circ$  in Quadrant IIIf  $\theta = 190^\circ$ ,  $\theta' = 10^\circ$  in Quadrant IIIIf  $\theta = 350^\circ$ ,  $\theta' = 10^\circ$  in Quadrant IVIf  $\theta = 20^\circ$ ,  $\theta' = 10^\circ$  in Quadrant I~~2000~~  
2,170°

Cool.... You'll see more in the next section, but essentially, when we do trig in the coordinate plane, the ONLY two pieces of valuable information we need from any angle is what quadrant it's in and its reference angle. That's all you need to do trig.

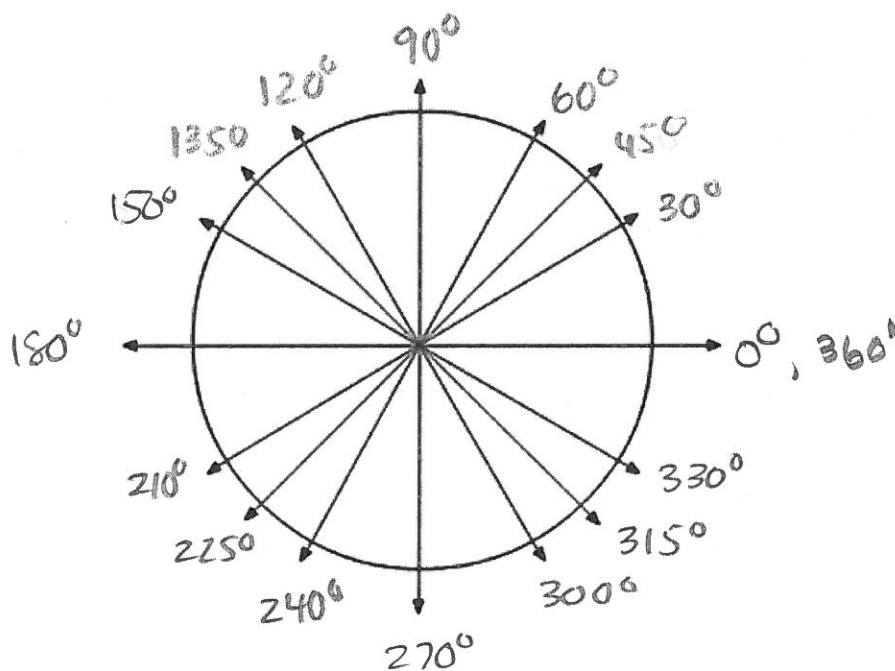
**Special Angles:**

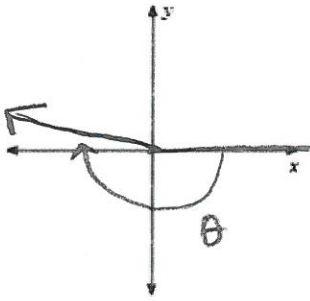
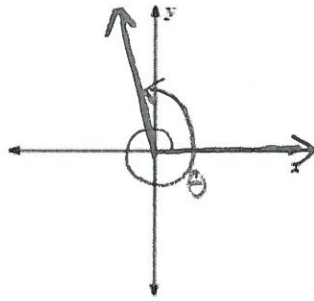
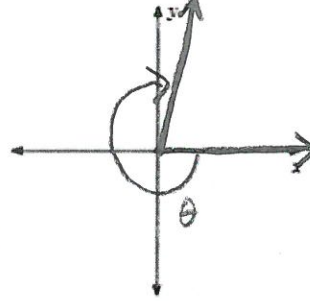
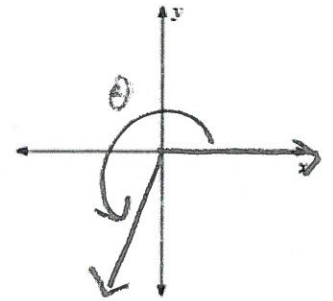
Just like there are special right triangles, there are special angles in the coordinate plane.

These angles are the angles whose reference angles are special:  $30^\circ$ ,  $45^\circ$  or  $60^\circ$

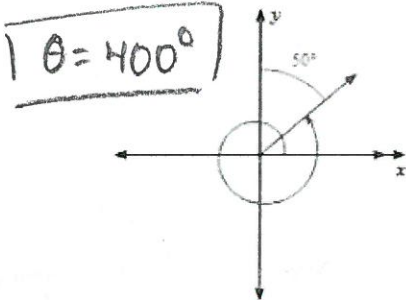
<b>"SPECIAL" REFERENCE ANGLES</b>				
$45^\circ$ reference angles cut the quadrants in half. $30^\circ$ are closer to the x axis, $60^\circ$ are further away.				
Quadrant	$\theta =$	$\theta' =$	Sketch	What math did you do?
I	$30^\circ$	$30^\circ$		none ☺
	$45^\circ$	$45^\circ$		
	$60^\circ$	$60^\circ$		
II	$120^\circ$	$60^\circ$		$180 - \theta$
	$135^\circ$	$45^\circ$		
	$150^\circ$	$30^\circ$		
III	$210^\circ$	$30^\circ$		$\theta - 180$
	$225^\circ$	$45^\circ$		
	$240^\circ$	$60^\circ$		
IV	$300^\circ$	$60^\circ$		$360 - \theta$
	$315^\circ$	$45^\circ$		
	$330^\circ$	$30^\circ$		

Label all special angles & all quadrantal angles: Look at them. Get familiar with their numbers and where they are. These 16 angles will be old friends by the time we're done ☺

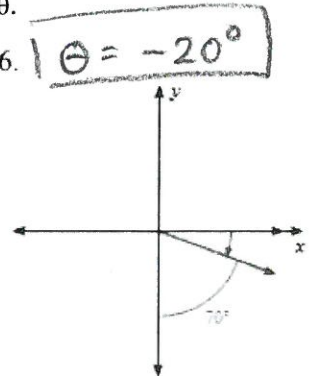


**Practice:**Draw each angle,  $\theta$ , in standard position1.  $-190^\circ$ 2.  $460^\circ$ 3.  $-280^\circ$ 4.  $250^\circ$ Find the measure of the angle,  $\theta$ .

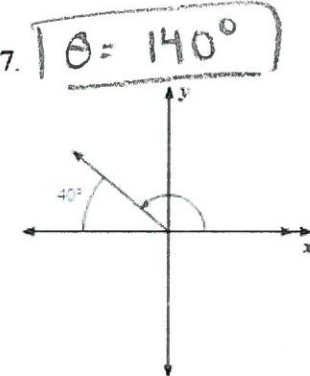
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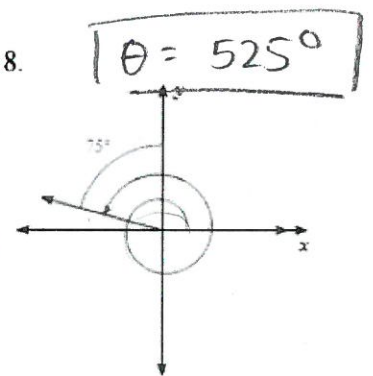
6.



7.



8.

For each angle, find a) a coterminal angle  $0^\circ \leq \theta < 360^\circ$  b) the Quadrant in which  $\theta$  terminates c)  $\theta'$ 9.  $-633^\circ$ 

- a)  $87^\circ$   
 b) Q I  
 c)  $\theta' = 87^\circ$

10.  $684^\circ$ 

- a)  $324^\circ$   
 b) Q IV  
 c)  $\theta' = 36^\circ$

11.  $-102^\circ$ 

- a)  $258^\circ$   
 b) Q III  
 c)  $\theta' = 78^\circ$

12.  $305^\circ$ 

- a)  $305^\circ$   
 b) Q IV  
 c)  $55^\circ$

13.  $324^\circ$ 

- a)  $324^\circ$   
 b) Q IV  
 c)  $\theta' = 36^\circ$

14.  $-225^\circ$ 

- a)  $135^\circ$   
 b) Q II  
 c)  $45^\circ$

15.  $270^\circ$ 

- a)  $270^\circ$   
 b) Not in a Quad.  
 c) No  $\theta'$

16.  $-150^\circ$ 

- a)  $210^\circ$   
 b) Q III  
 c)  $30^\circ$

17.  $-245^\circ$ 

- a)  $115^\circ$   
 b) Q II  
 c)  $65^\circ$

18.  $755^\circ$ 

- a)  $35^\circ$   
 b) Q I  
 c)  $35^\circ$

19.  $-600^\circ$ 

- a)  $120^\circ$   
 b) Q II  
 c)  $60^\circ$

20.  $555^\circ$ 

- a)  $195^\circ$   
 b) Q III  
 c)  $15^\circ$

