

Name: Answer Key
 Serafino · Algebra 2E

Per: _____ Date: _____

9A2 Trig Ratios in the Coordinate Plane

Notes / Classwork

Evaluating Trig Ratios:

Use your knowledge of ratios to fill in the blanks. Then check.

$\sin B \approx 0.471$ $\cos A \approx 0.471$

$\tan B \approx 0.533$

$\sin D \approx 0.667$ $\cos C \approx 0.667$

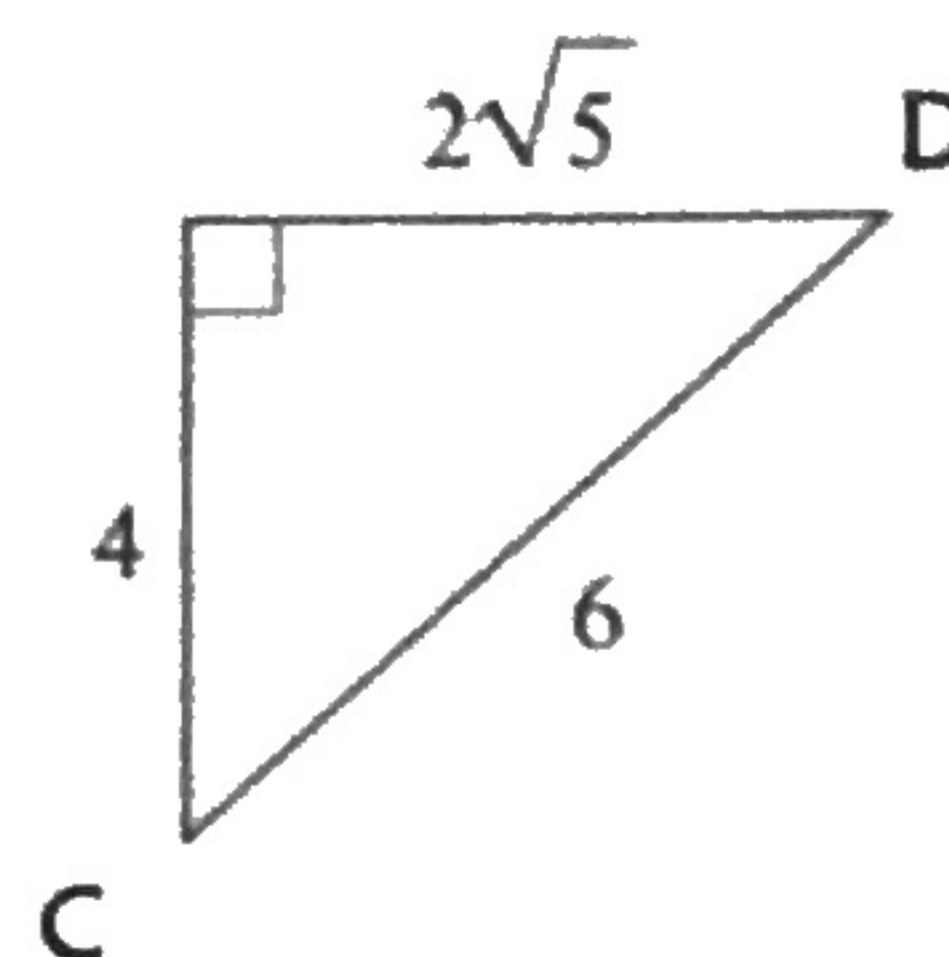
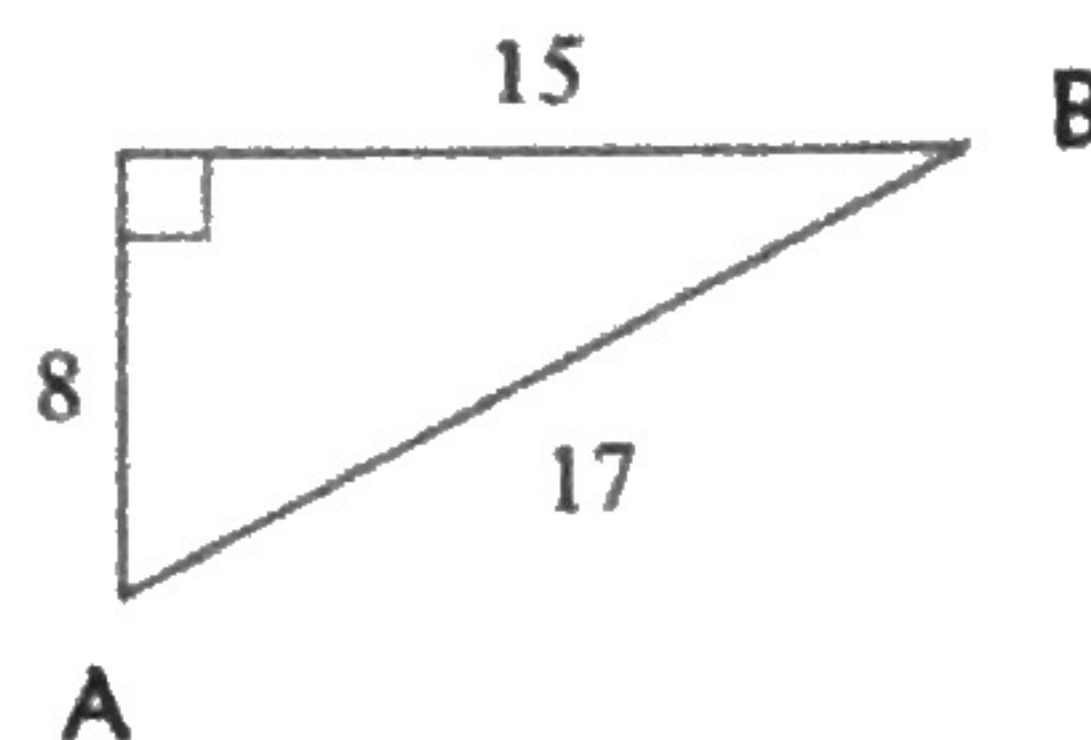
$\tan D \approx 0.894$

$\sin C \approx 0.745$ $\cos D \approx 0.745$

$\tan C \approx 1.118$

$\sin A \approx 0.882$ $\cos B \approx 0.882$

$\tan A \approx 1.875$



Cofunction Theorem: This demonstrates a convenient little shortcut about CO-FUNCTIONS (like sine and COsine) The trig ratio of an angle is exactly the same as the co-function of its complement.

Examples: $\cos 70 = \sin 20$; $\sin 40 = \cos 50$; $\sin 36.8 = \cos 53.2^\circ$; $\cos \theta = \sin (90 - \theta)$

Using Inverses to Solve for Angles:

See if you can use your Trig Ratios table to estimate the angle before you use inverses to solve for it. (see Trig Table)

$m \angle A \approx 61.93^\circ$

$m \angle B \approx 28.07^\circ$

$m \angle C \approx 48.19^\circ$

$m \angle D \approx 41.81^\circ$

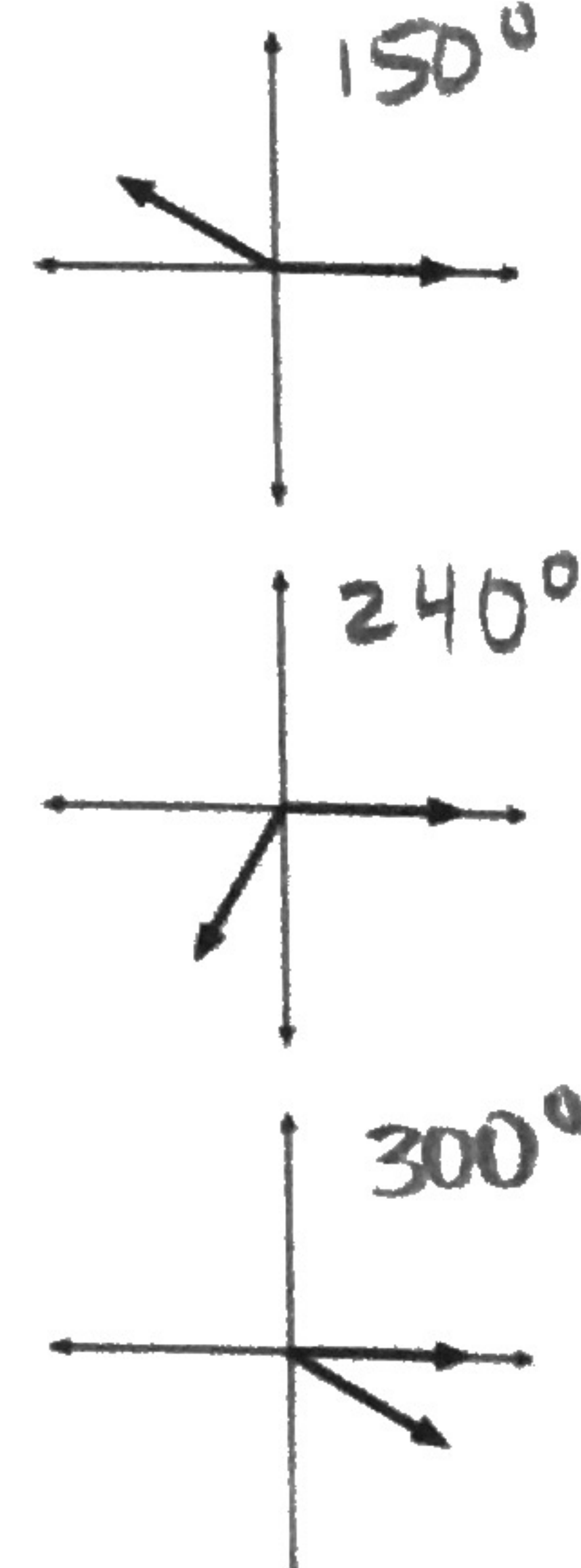
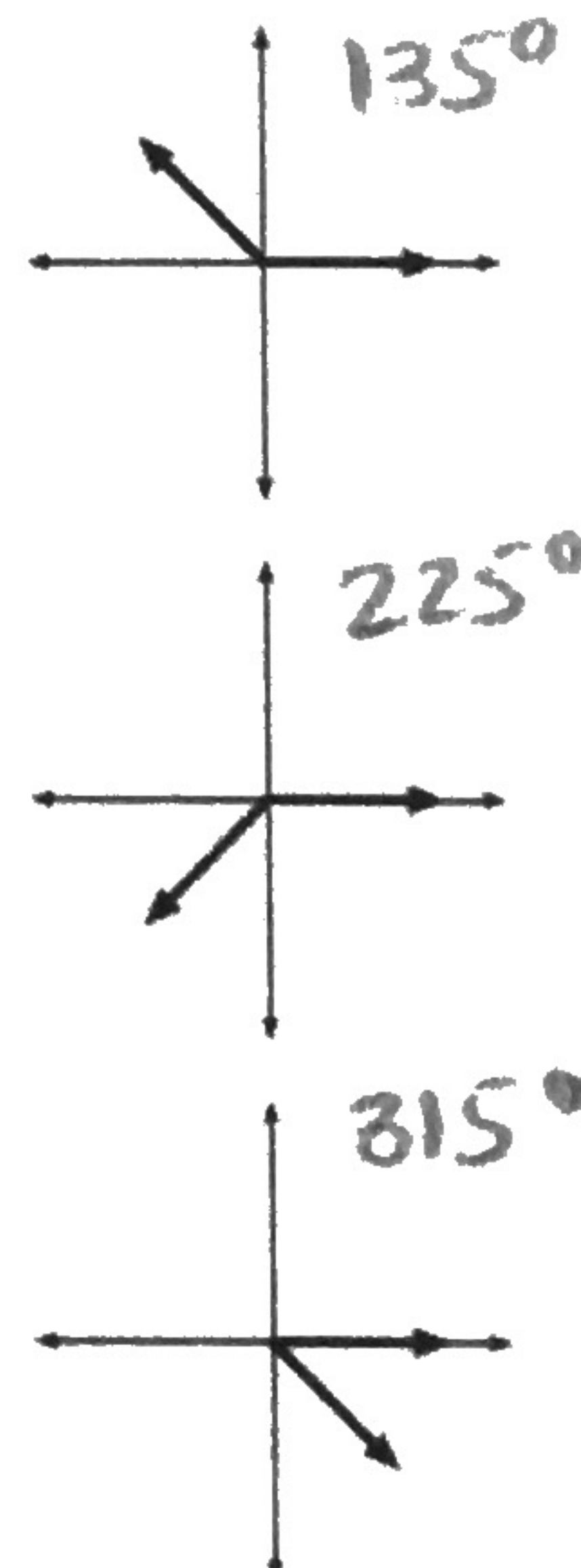
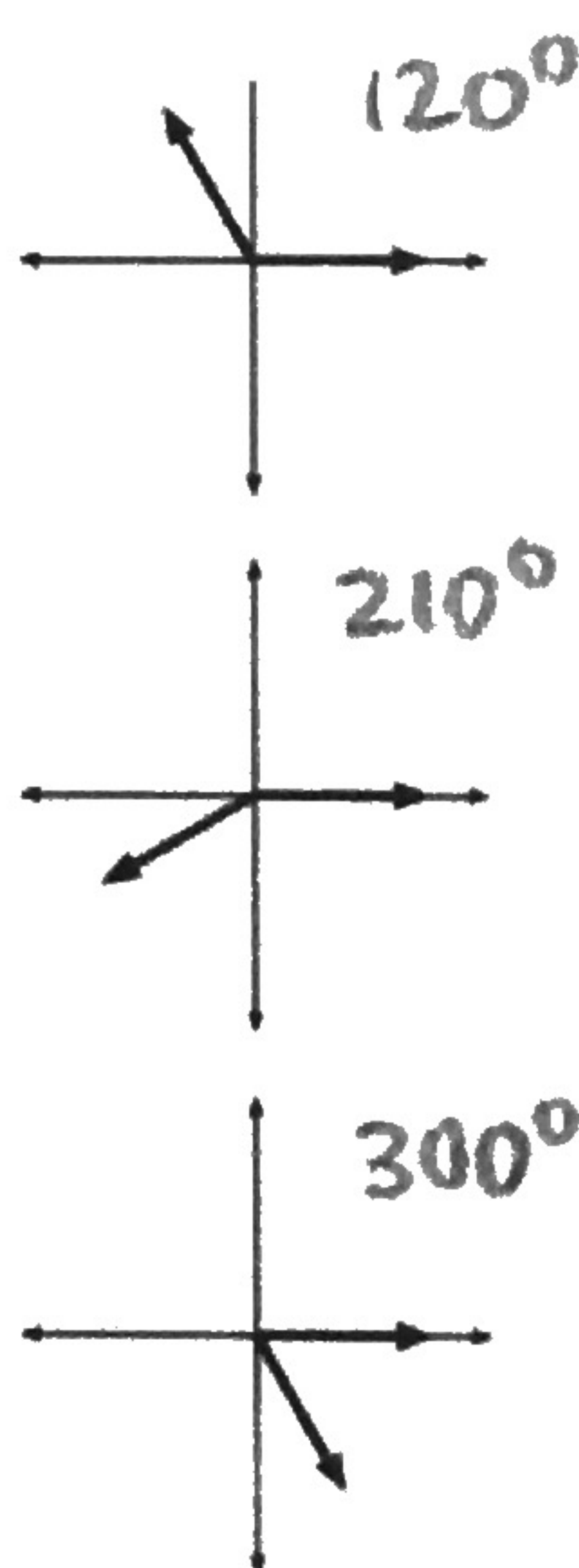
Coterminal and Reference Angles:

Fill in the chart

Match each with the Special Angle:

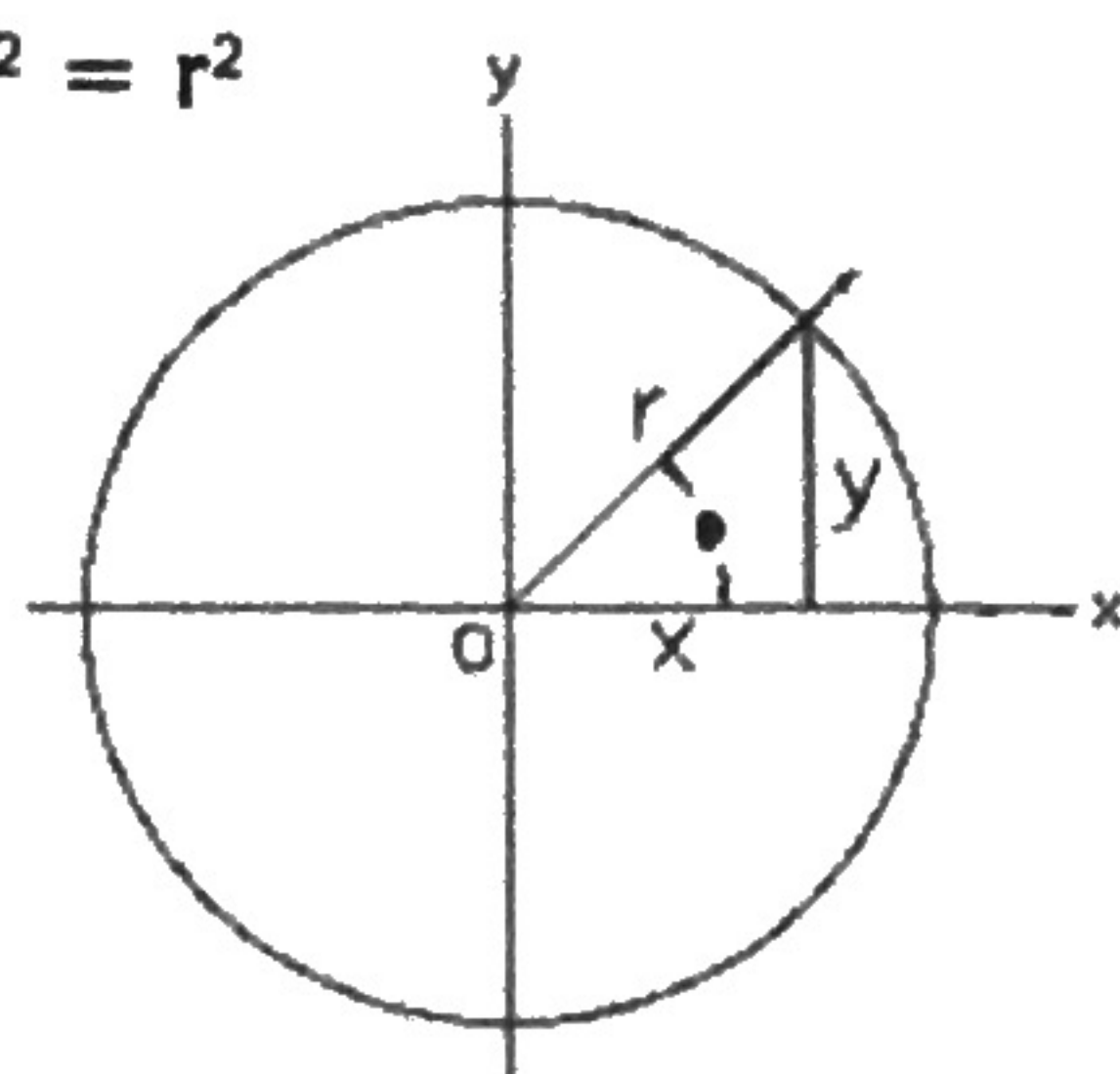
150° 210° 330° 120° 240° 300° 135° 225° 315°

Any θ Coterminal	Coterminal $0^\circ \leq \theta < 360^\circ$	Quad	θ'
-130°	230°	III	50°
1,070°	350°	IV	10°
560°	200°	III	20°
470°	110°	QII	70°
-85°	275°	QIV	85°



Noticing connections...

You may recall from Geometry that the Equation of a circle with a center at the origin is: $x^2 + y^2 = r^2$
 All radii of the circle are hypotenuses of all possible right triangles with legs x and y .
 The length of the hypotenuse is its distance from the origin.



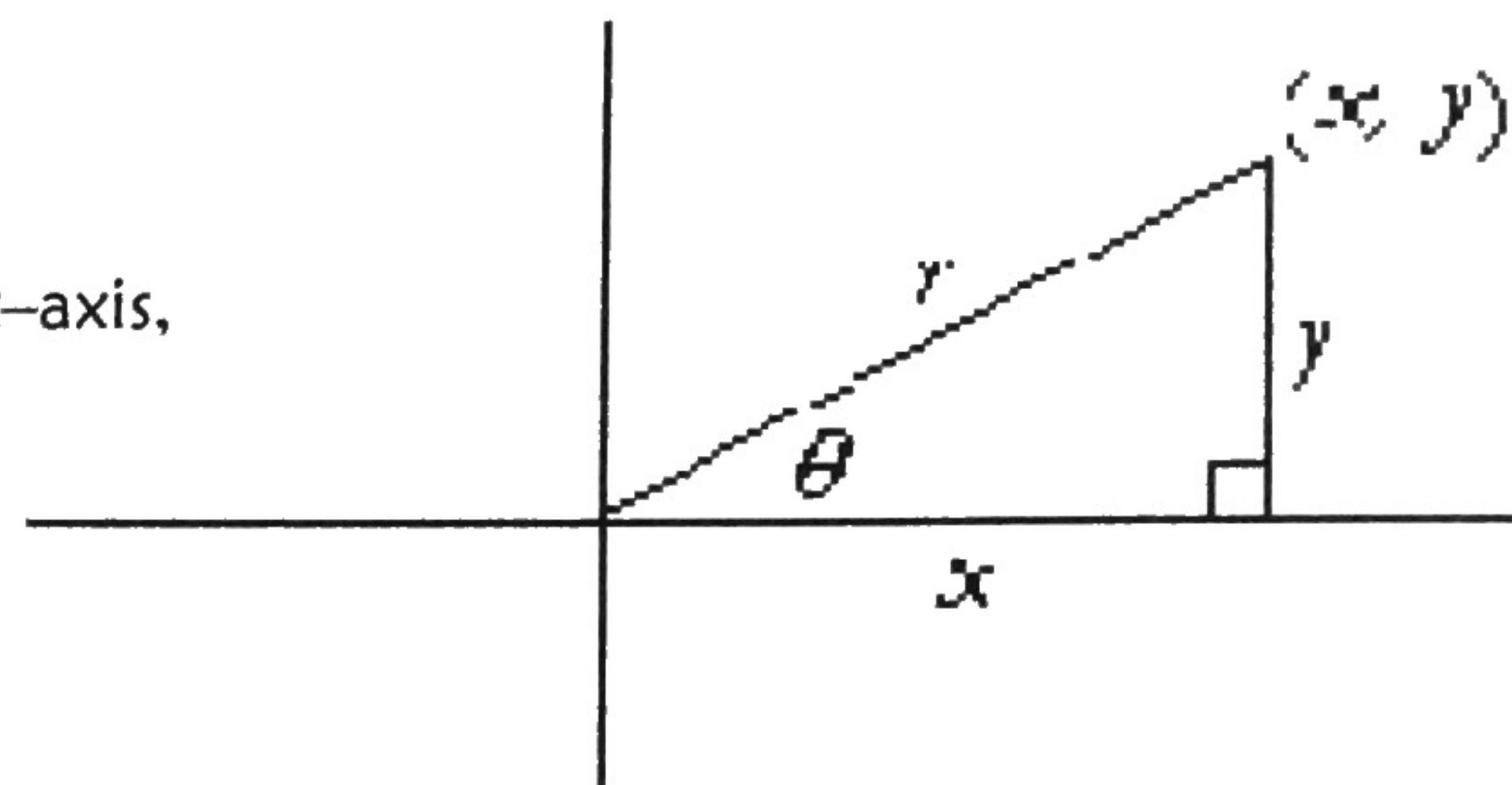
This should make sense because the distance formula is: $(\Delta x)^2 + (\Delta y)^2 = d^2$

This should also fit in with what we know about the Pythagorean Theorem is: $a^2 + b^2 = c^2$

Drawing Reference Triangle:

When you have an angle, θ , in standard position, the terminal side is going to intersect a bunch of points: (x, y)

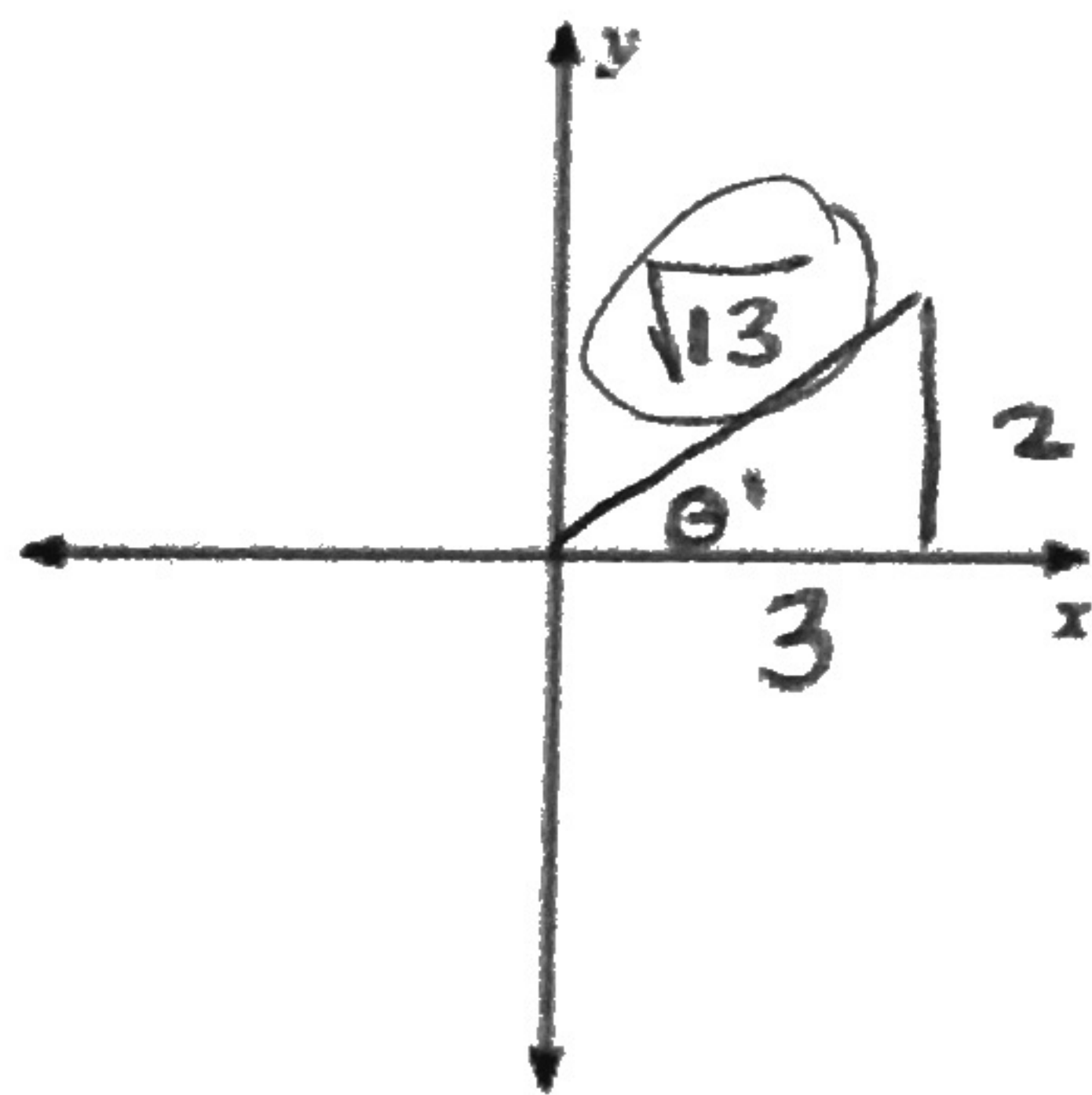
If you pick a point and drop down a line perpendicular to the nearest x -axis, you will make a right triangle.



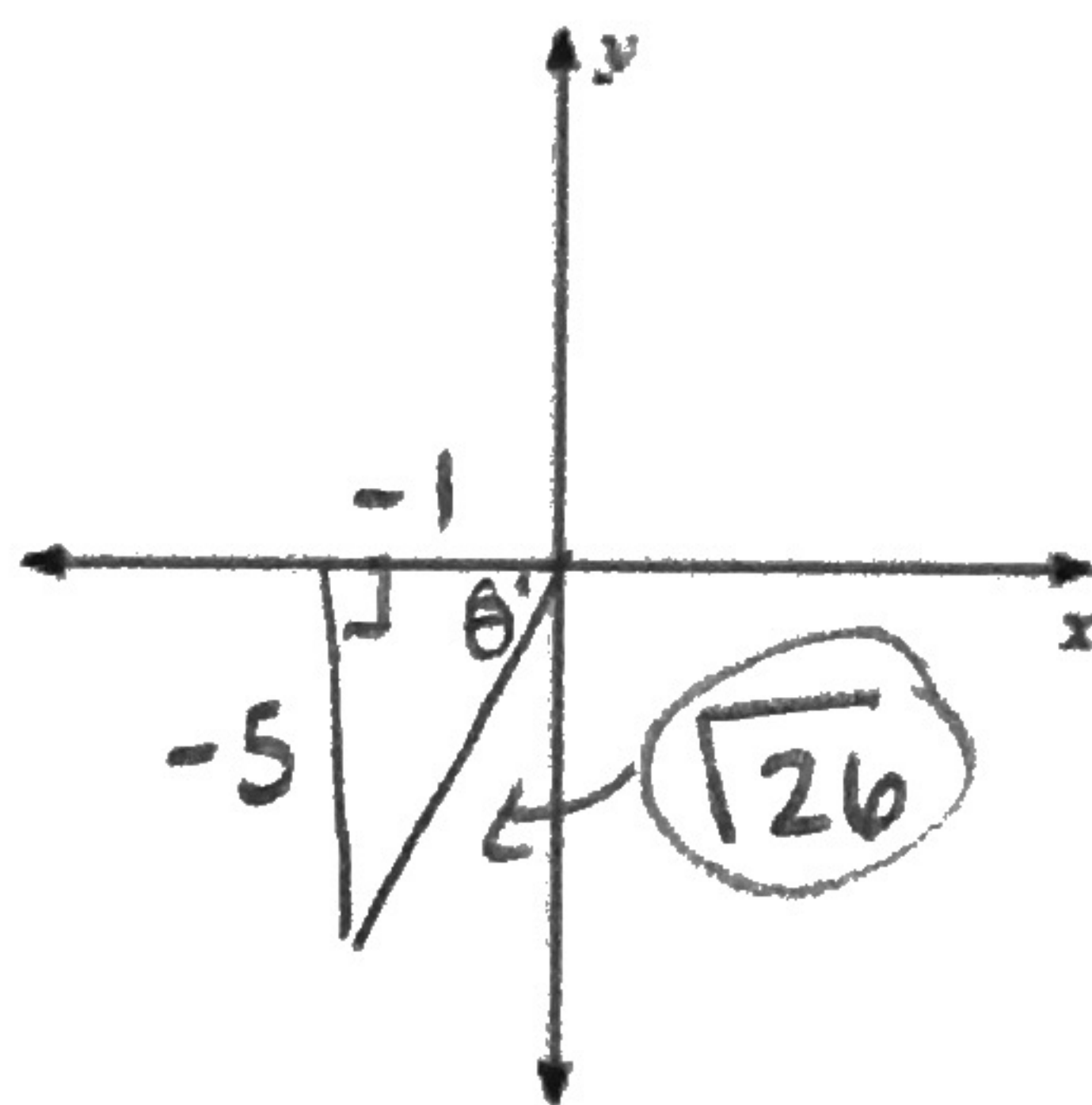
X will be its adjacent side. Y will be its opposite side.
The angle in the your triangle will be the reference angle, θ'

For each point intersected by the terminal side of θ , draw the reference triangle contain ting θ' and find the hypotenuse. * The hypotenuse is always positive.

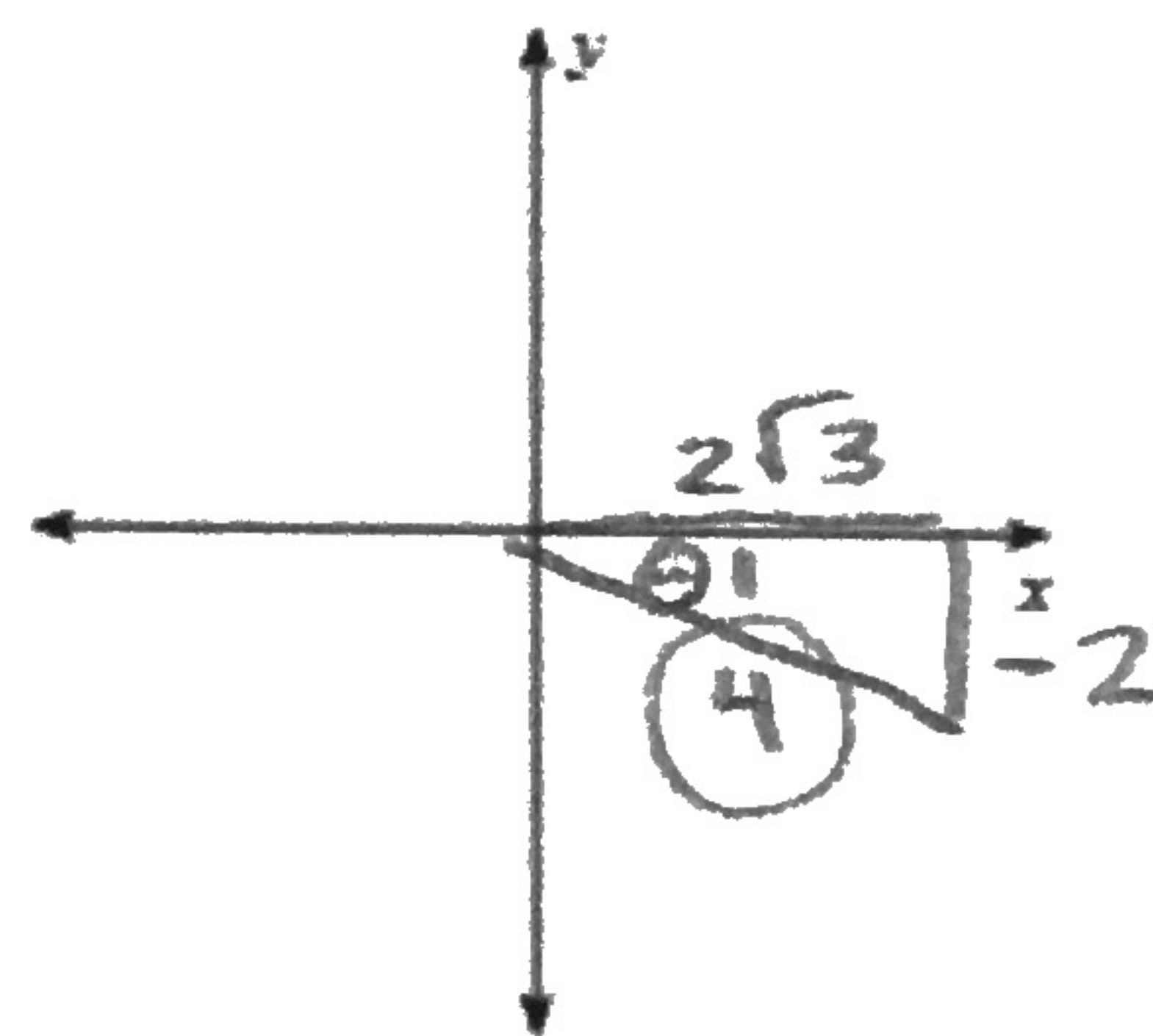
Point: $(3, 2)$



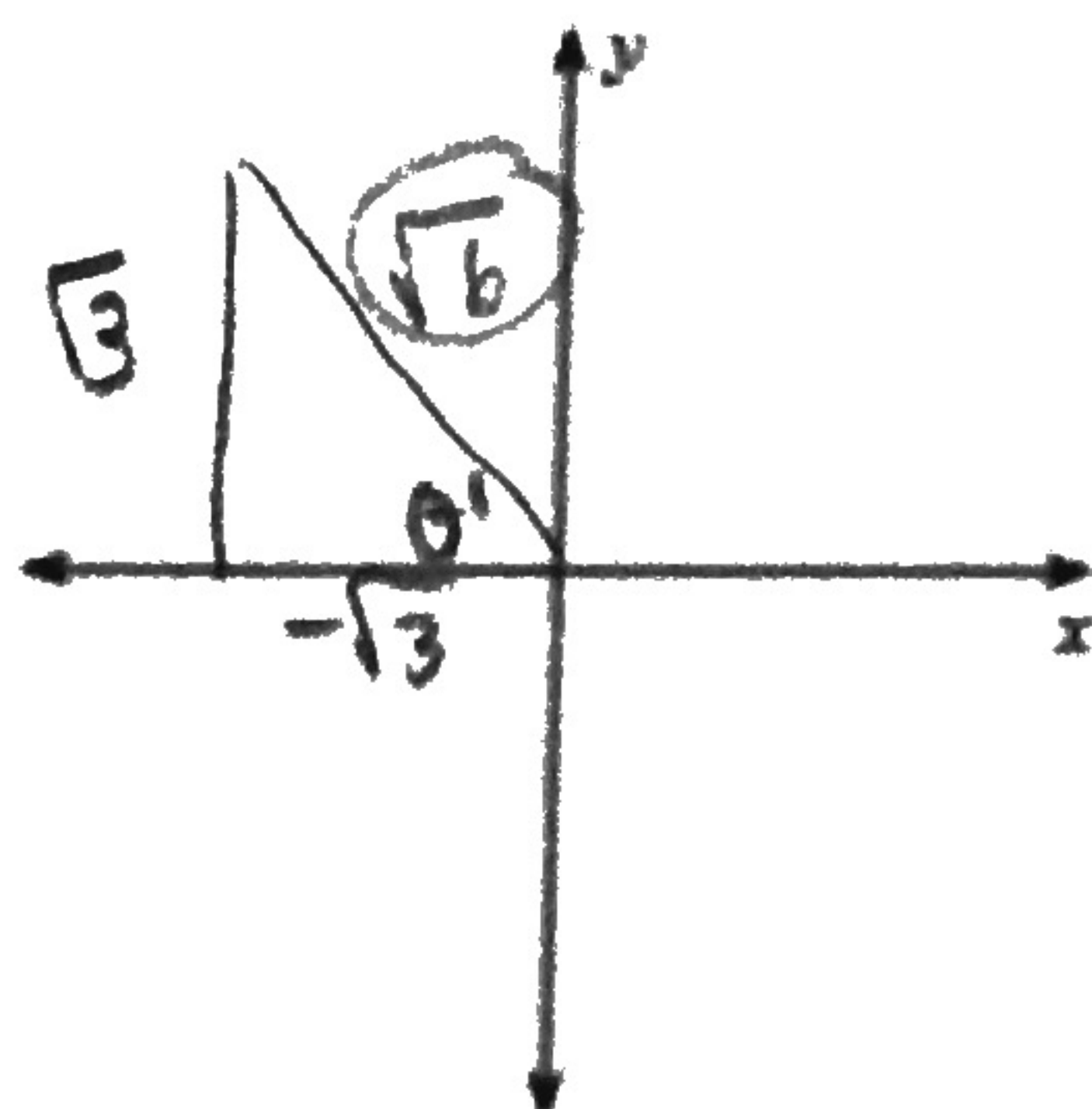
Point $(-1, -5)$



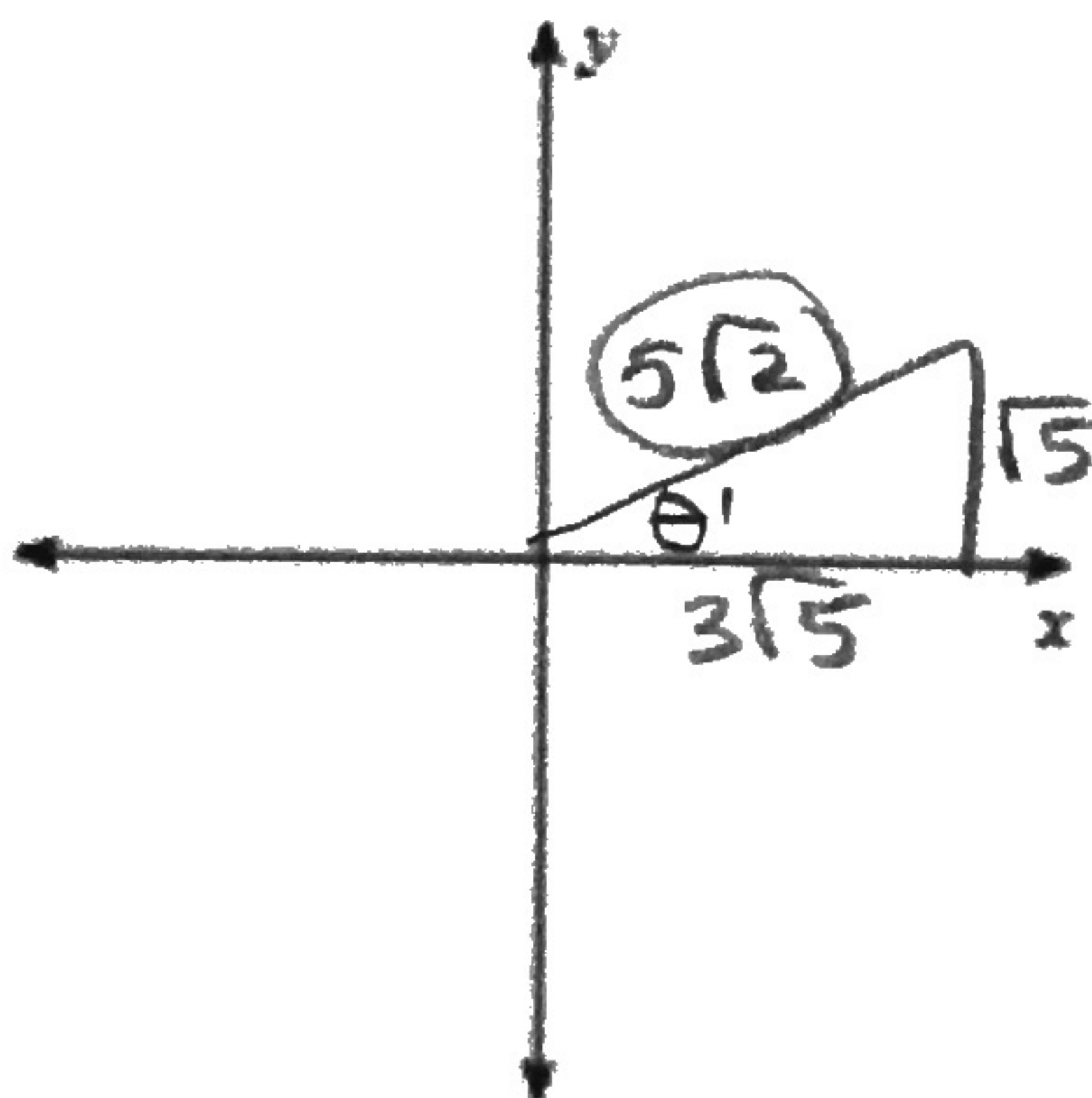
Point $(2\sqrt{3}, -2)$



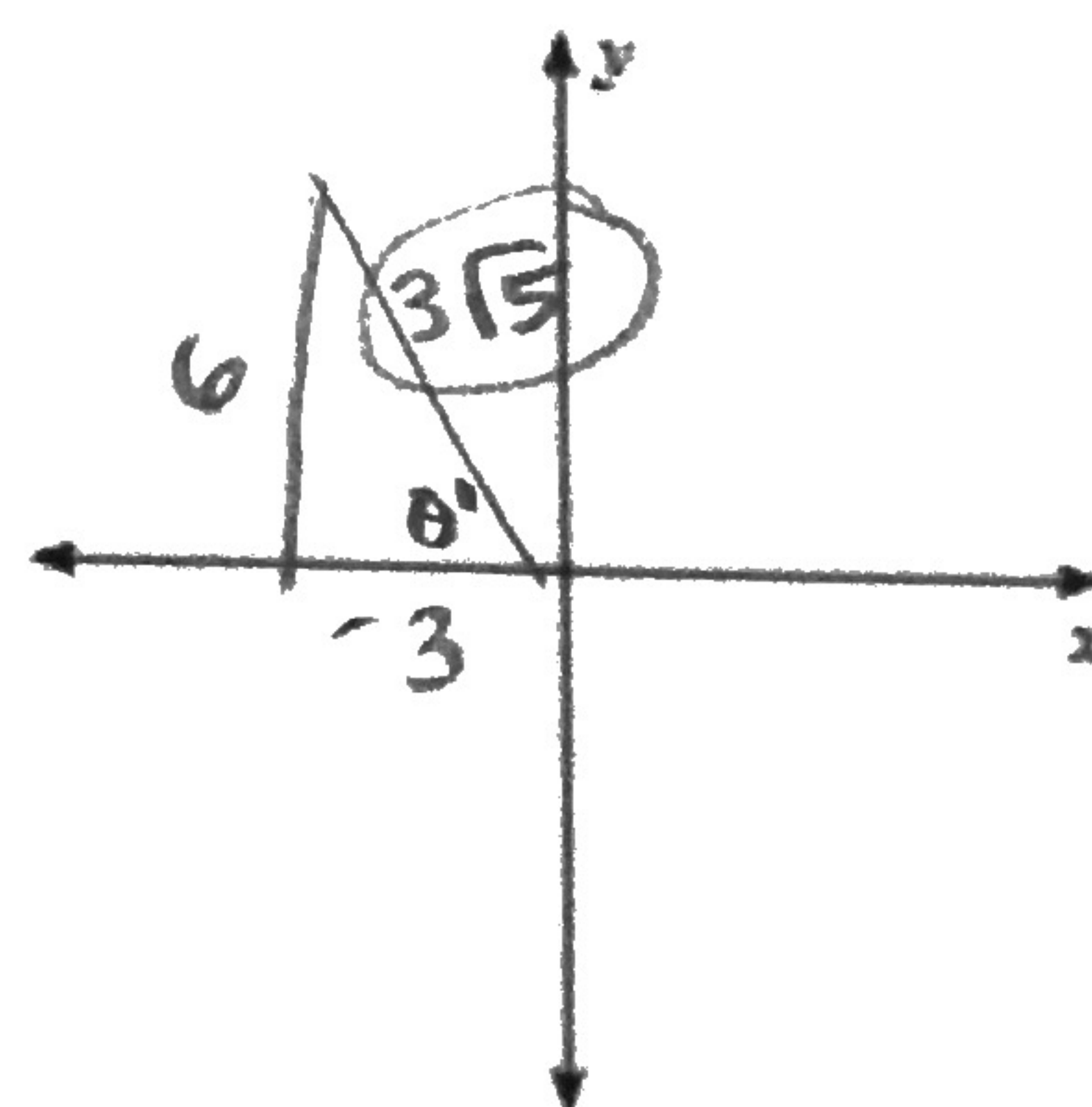
Point $(-\sqrt{3}, \sqrt{3})$



Point $(3\sqrt{5}, \sqrt{5})$



Point $(-3, 6)$



Hmmm.... you can start to see that regardless of where θ terminates, the angle we will use to do trig is θ' .

The Three Trig Ratios :

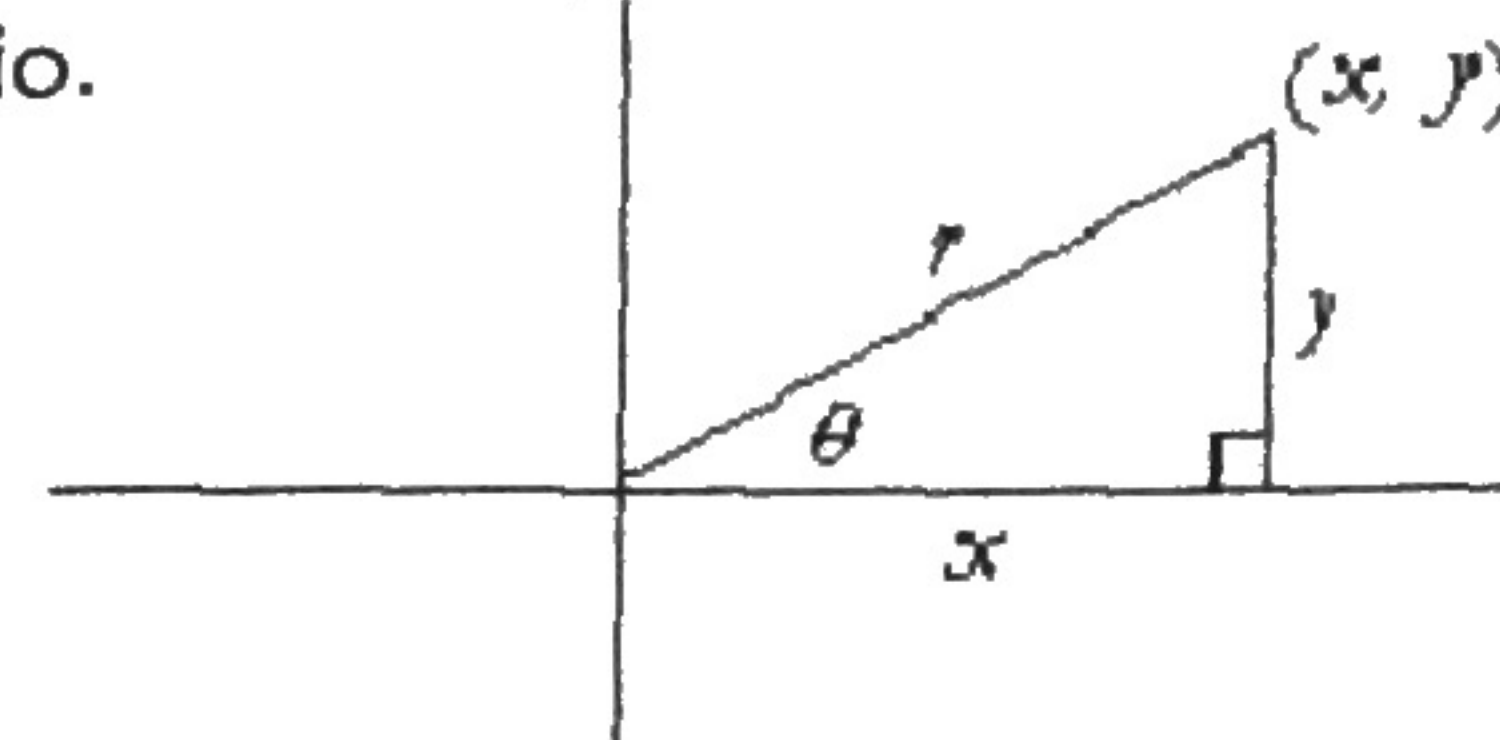
If θ intersects a point (x, y) , write the trig ratios as ratios of what distances they compare:

$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

X goes with COSINE, because it is always ADJACENT to θ . It's a HORIZONTAL ratio.
Y goes with SINE, because it is OPPOSITE of θ . It is a VERTICAL ratio.

$$\tan \theta = \frac{y}{x} = \frac{\sin \theta}{\cos \theta}$$



THIS IS HUGE!

We can look at the Sine ratio as "How FAR UP are we?" We can look at Cosine as "How far OVER are we?"
We can look at the Tangent Ratio two ways: One as "RISE / RUN"; and the other as "SINE over COSINE"

Determining the Signs of the Trig Ratios:

Draw a 30° reference angle in each of the quadrants, then draw a reference triangle.

On the x and y axes, we use the sign (\pm) associated with that side of the axis. The radius is always positive.

Draw the reference triangle and use your knowledge of SRT to get the exact ratios. Confirm with your calculator.

$$\sin 150^\circ = \frac{1}{2} (0.5)$$

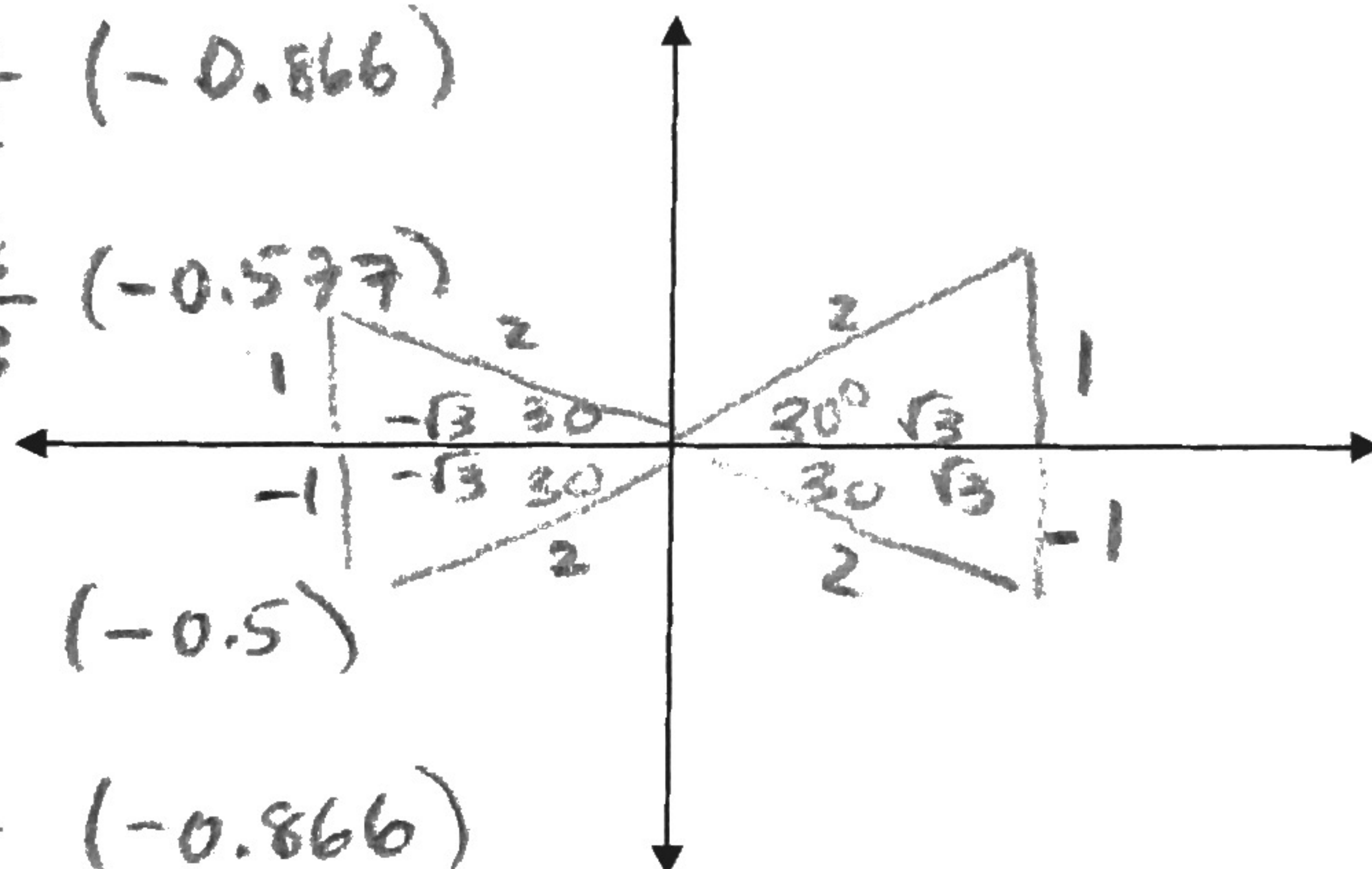
$$\cos 150^\circ = -\frac{\sqrt{3}}{2} (-0.866)$$

$$\tan 150^\circ = -\frac{\sqrt{3}}{3} (-0.577)$$

$$\sin 210^\circ = -\frac{1}{2} (-0.5)$$

$$\cos 210^\circ = -\frac{\sqrt{3}}{2} (-0.866)$$

$$\tan 210^\circ = \frac{\sqrt{3}}{3} (0.577)$$



$$\sin 30^\circ = \frac{1}{2} (0.5)$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2} (0.866)$$

$$\tan 30^\circ = \frac{\sqrt{3}}{3} (0.577)$$

$$\sin 330^\circ = -\frac{1}{2} (-0.5)$$

$$\cos 330^\circ = \frac{\sqrt{3}}{2} (0.866)$$

$$\tan 330^\circ = -\frac{\sqrt{3}}{3} (-0.577)$$

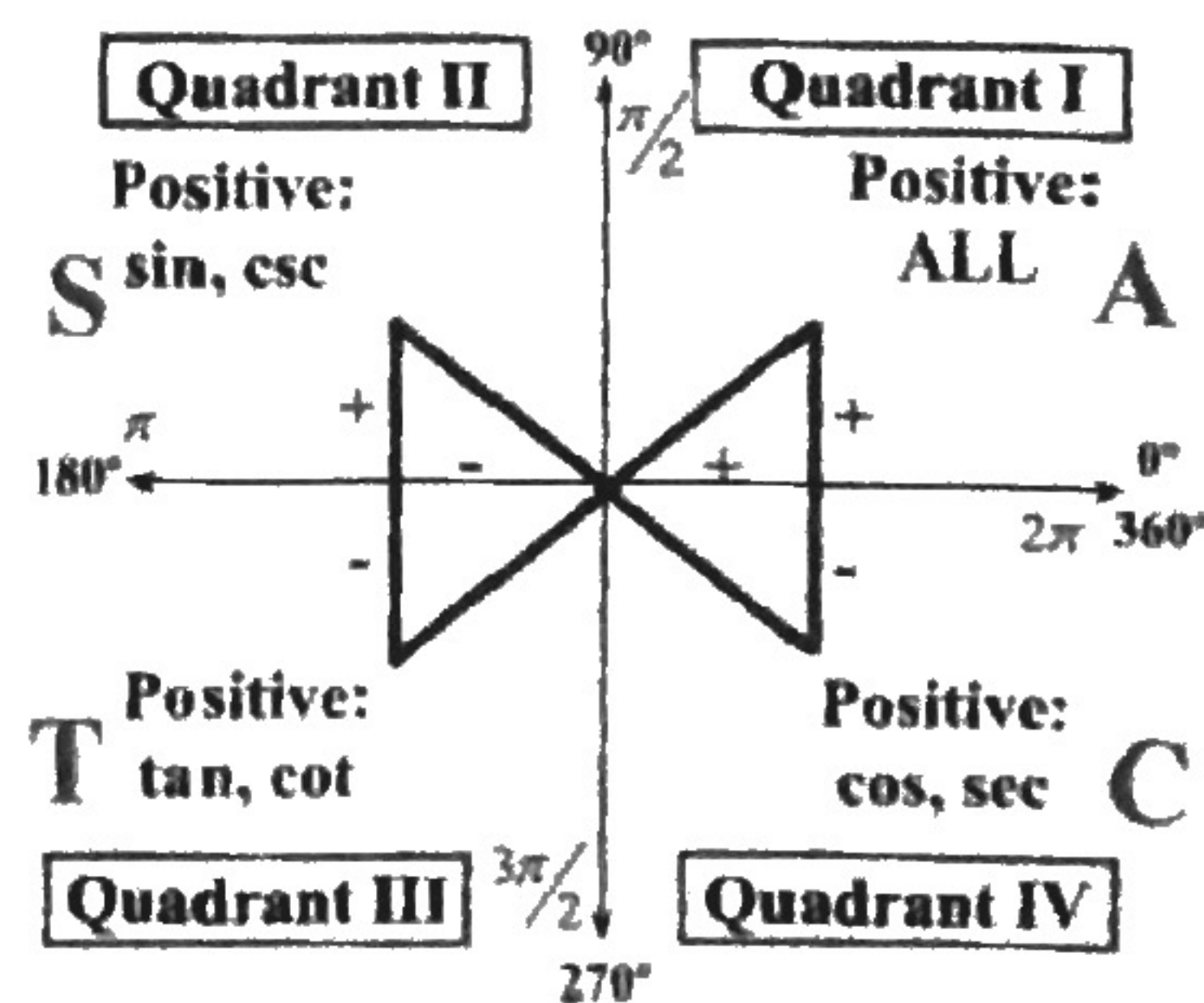
HUGE DEAL: SAME REFERENCE ANGLE = SAME RATIOS, DIFFERENT SIGNS

The X values (COS) are positive (+) in Quadrants I and IV

The X values (COS) are negative (-) in Quadrants II and III

The Y values (SIN) are positive (+) in Quadrants I and II

The Y values (SIN) are negative (-) in Quadrants III and IV



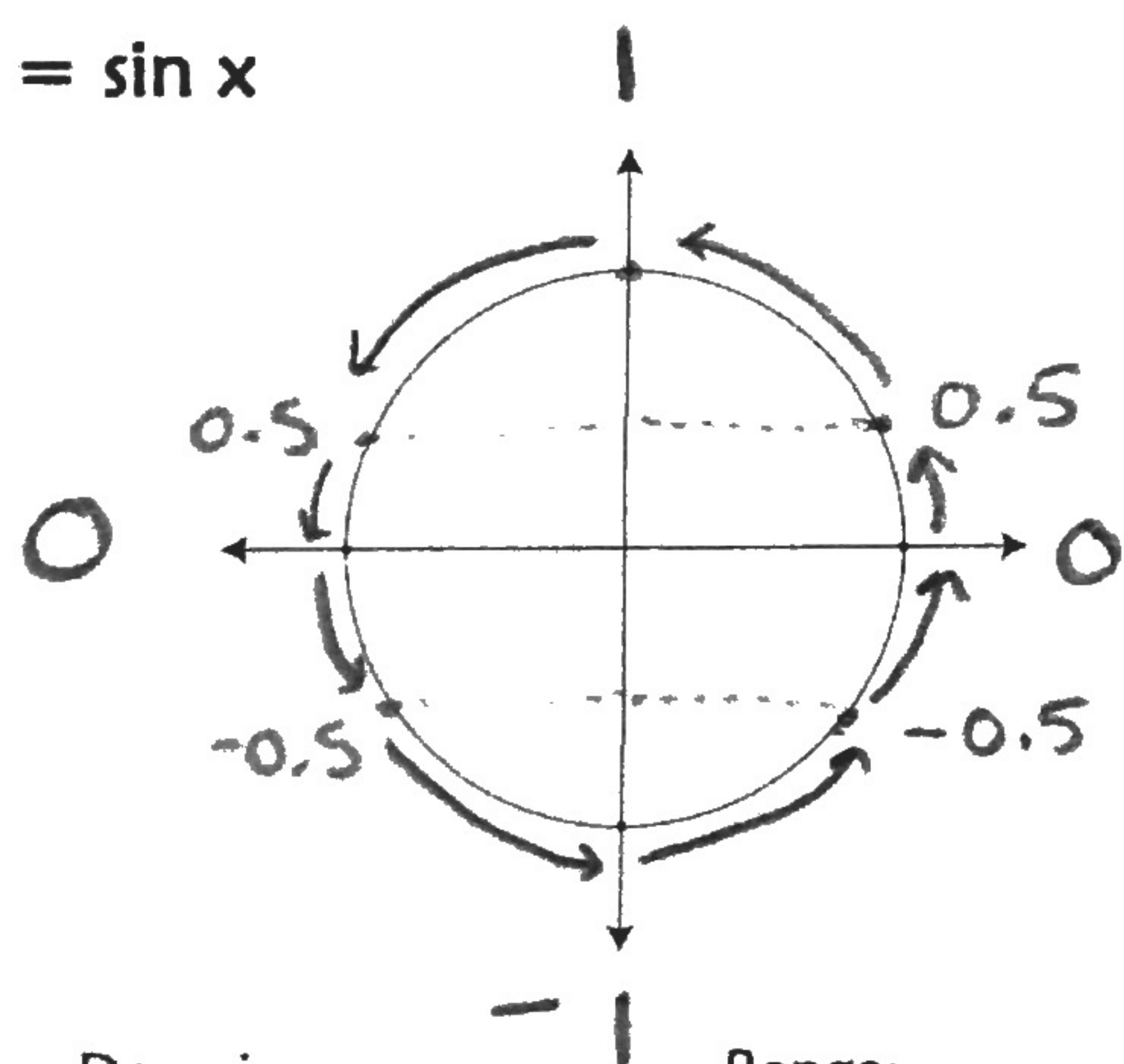
Who is positive where? Above, circle the positive trig functions in each quadrant.
I REALLY don't advise memorizing "ASTC" - but it's there if you need it.

Trig Ratios as Functions of Angles: Use your calculator to evaluate the functions. Start with these values, but put in any you like. Note any cyclical pattern.

θ	0°	10°	30°	40°	45°	50°	60°	80°	90°	100°	130°	135°	170°	180°
$\sin \theta$	0	.174	.5	.643	.707	.766	.866	.985	1	.985	.766	.707	.174	0
$\cos \theta$	1	.985	.866	.766	.707	.643	.5	.174	0	-.174	-.643	-.707	-.985	-1
$\tan \theta$	0	.176	.577	.839	1	1.192	1.732	5.67	und.	-5.67	-1.19	-1	-.176	0

θ	182°	210°	215°	225°	230°	268°	269.7°	270°	271°	300°	315°	350°	355°	360°
$\sin \theta$	-.035	-.5	-.574	-.707	-.766	-.999	-.999	-1	-.999	-.866	-.707	-.174	-.088	0
$\cos \theta$	-.999	-.866	-.819	-.707	-.642	-.035	-.005	0	.017	.5	.707	.985	.996	1
$\tan \theta$.035	0.577	.7	1	1.192	28.64	190.9	und.	-57.3	-1.73	-1	-.176	-.087	0

$f(x) = \sin x$



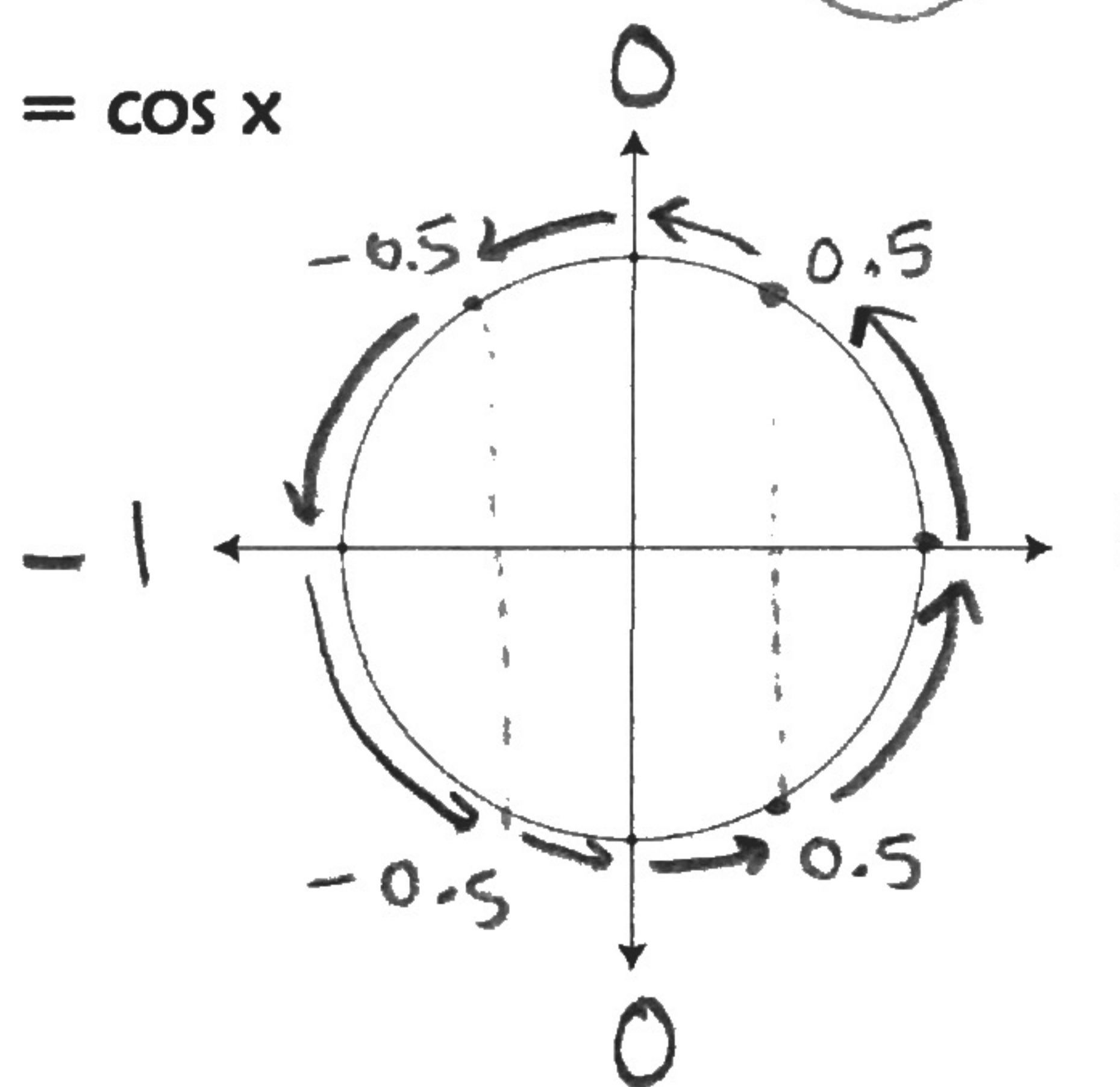
Domain:

$x \in \mathbb{R}$

Range:

$y \in [-1, 1]$

$f(x) = \cos x$



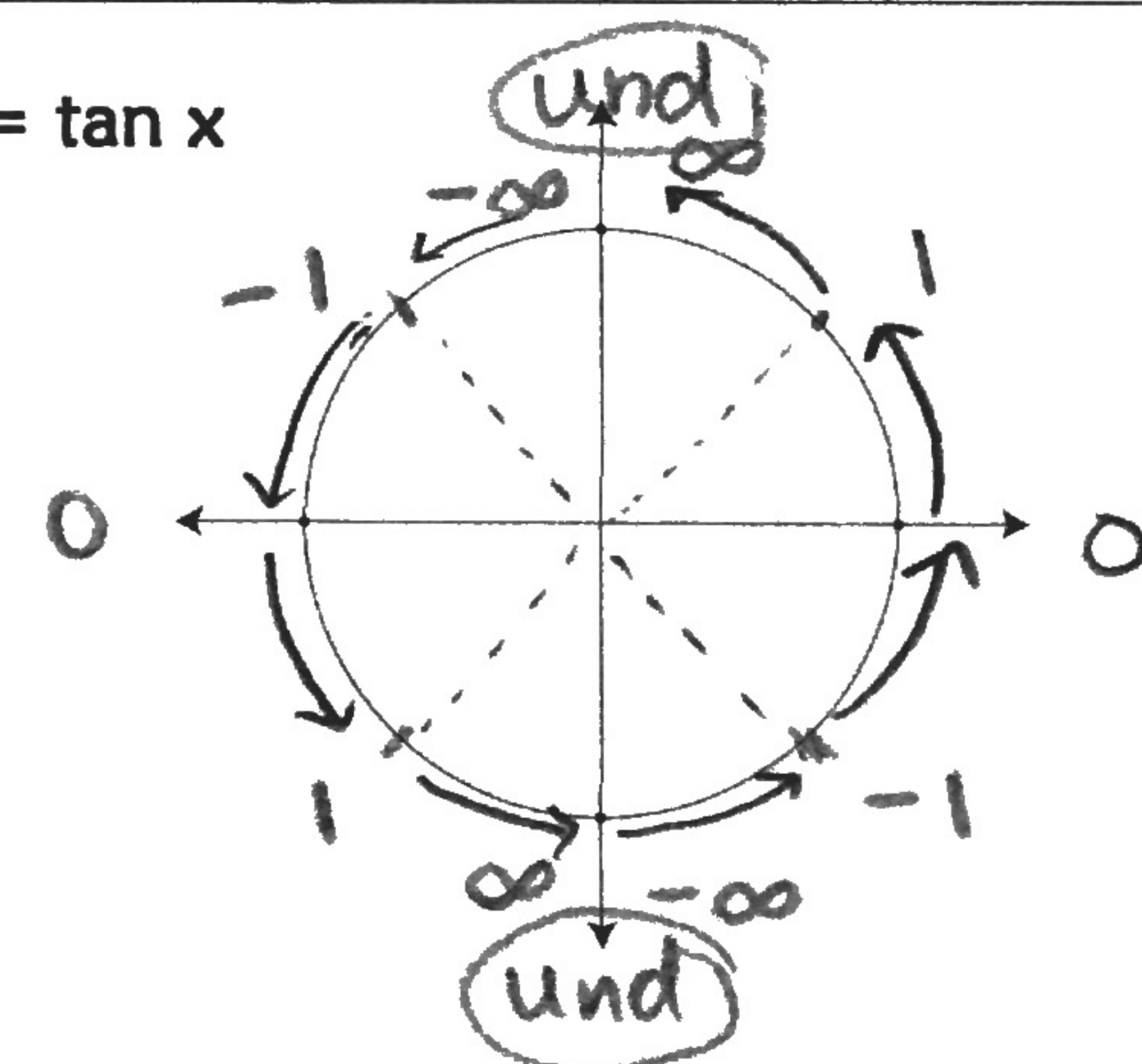
Domain:

$x \in \mathbb{R}$

Range:

$y \in [-1, 1]$

$f(x) = \tan x$



Domain:

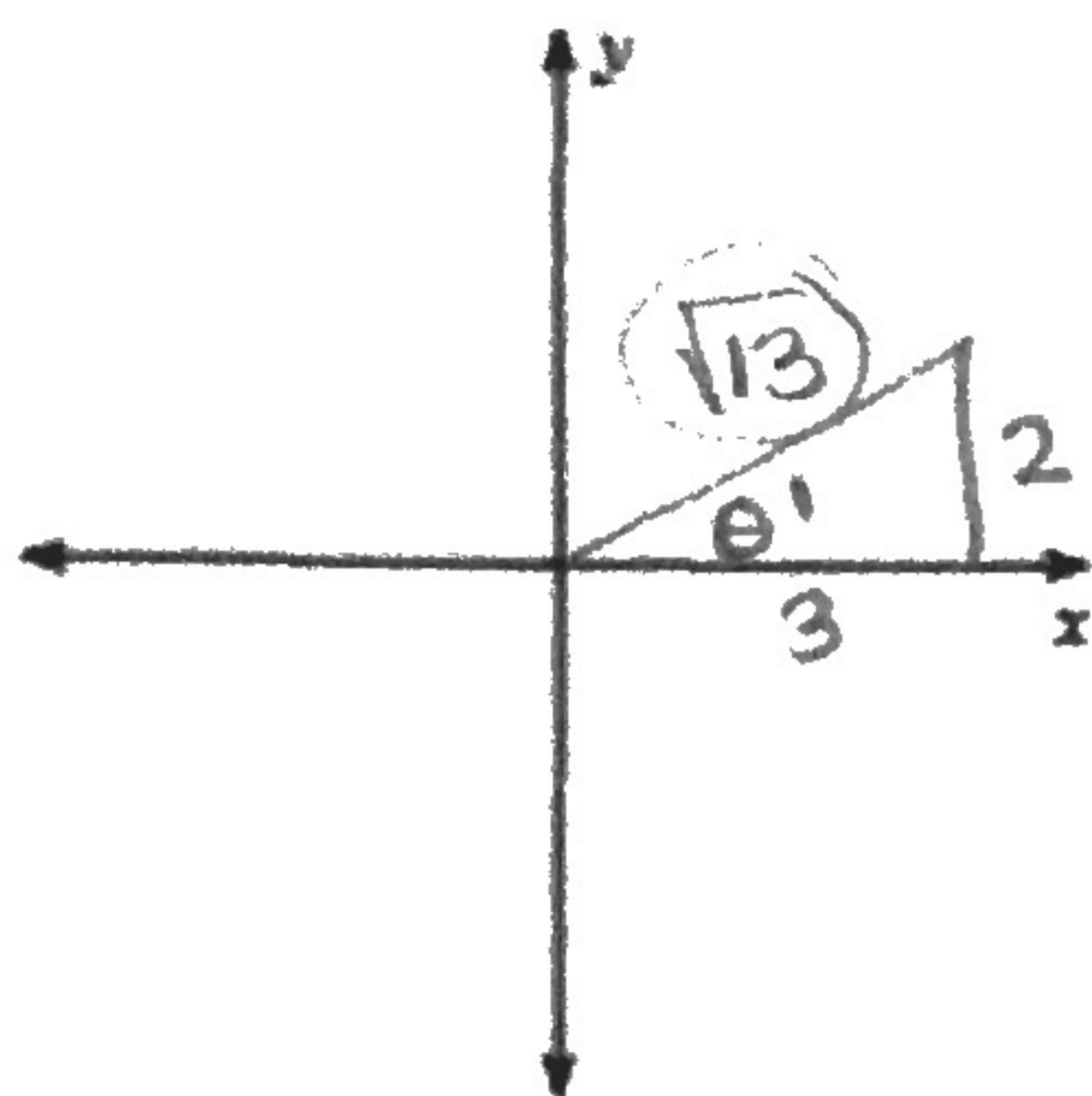
$x \neq 90 + 180k$

($x \neq -270, -90, 90, 270, \text{etc.}$)

Range:

$x \in \mathbb{R}$

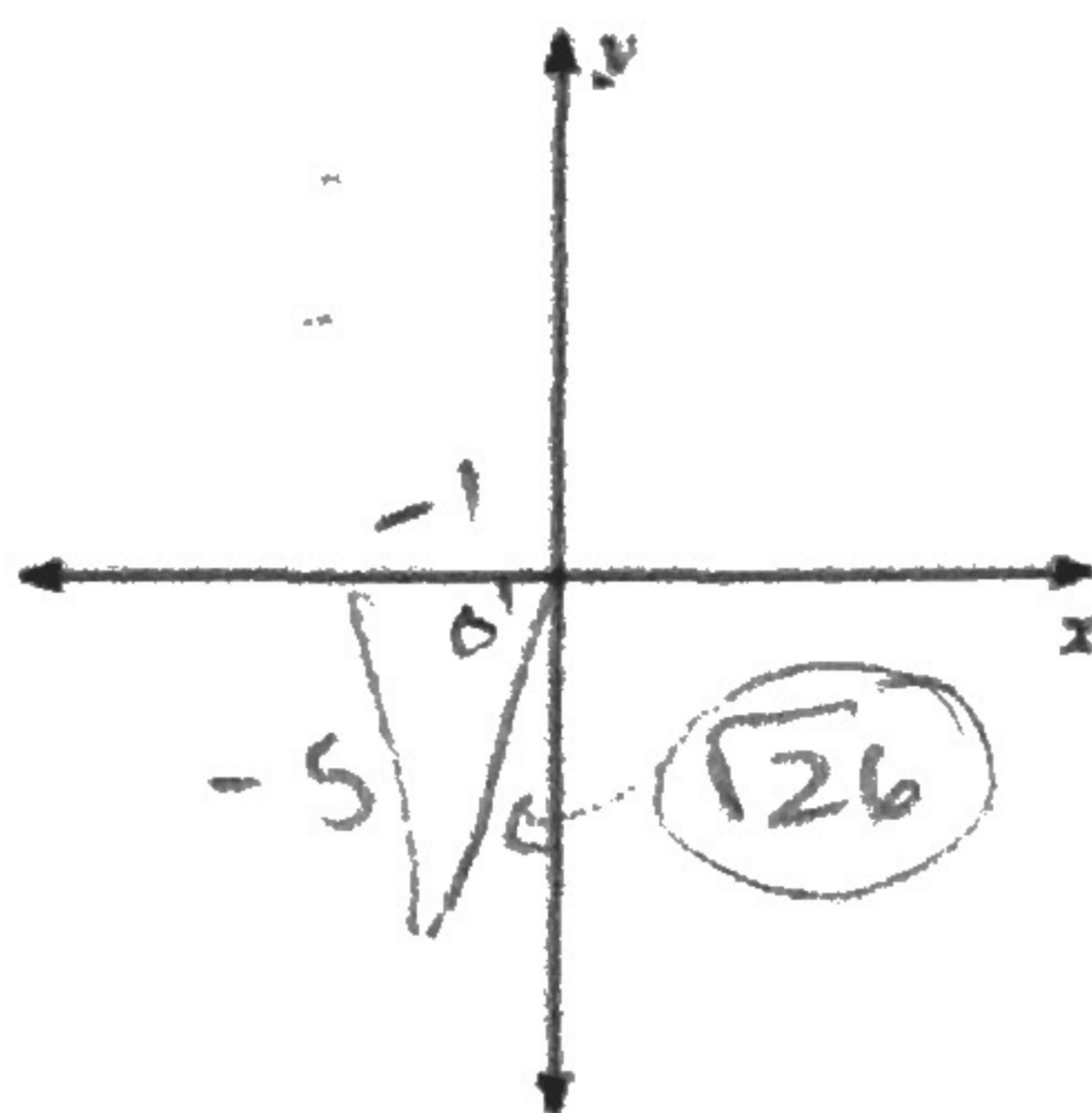
Trig Ratios in the Coordinate Plane: We can now find the trig function of any angle. We'll start when θ intersects a point (x, y) . Find the exact ratio.

Point: $(3, 2)$ 

$$\sin \theta = \frac{2\sqrt{13}}{13} \approx 0.555$$

$$\cos \theta = \frac{3\sqrt{13}}{13} \approx 0.832$$

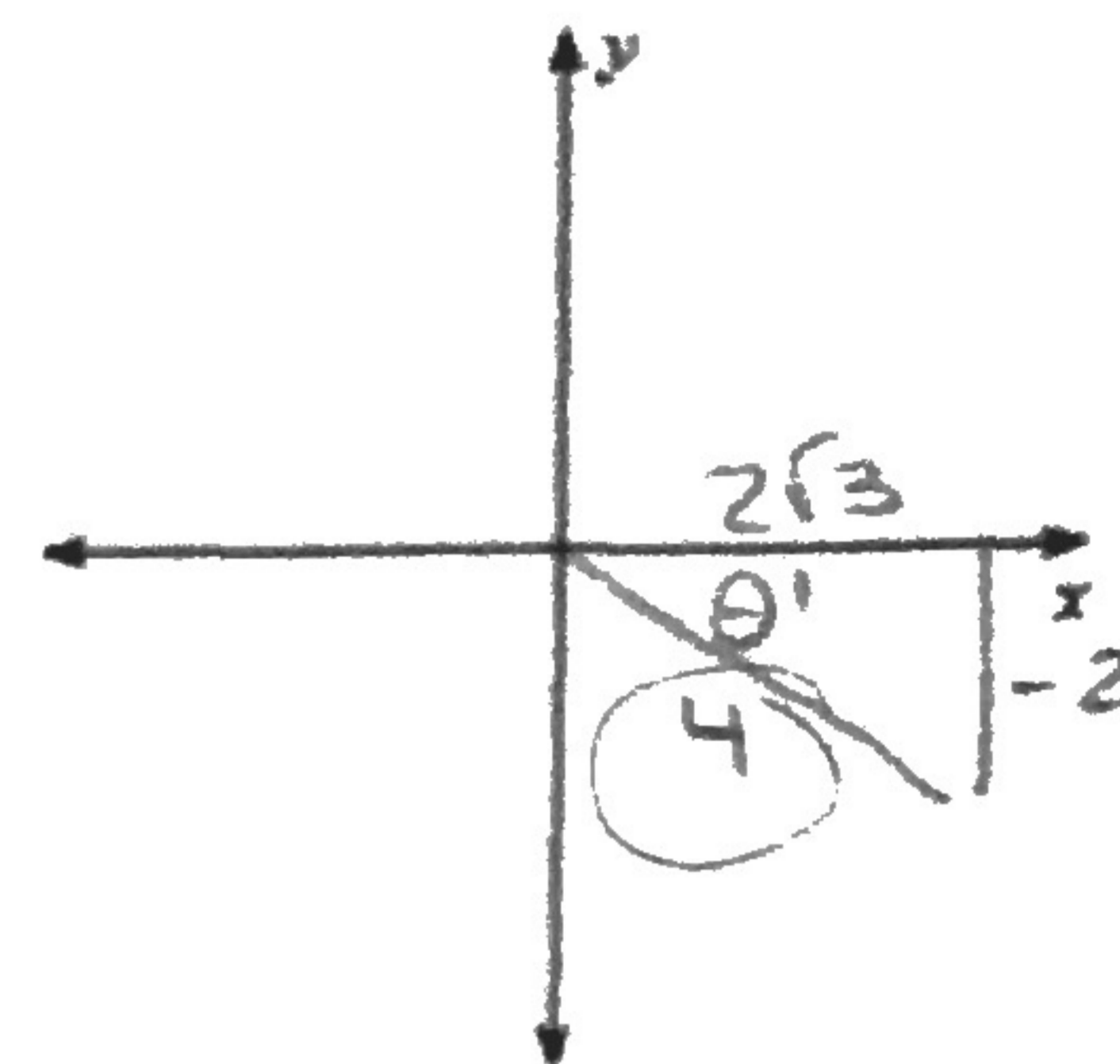
$$\tan \theta = \frac{2}{3} \approx 0.667$$

Point $(-1, -5)$ 

$$\sin \theta = -\frac{5\sqrt{26}}{26} \approx -.981$$

$$\cos \theta = -\frac{\sqrt{26}}{26} \approx 0.196$$

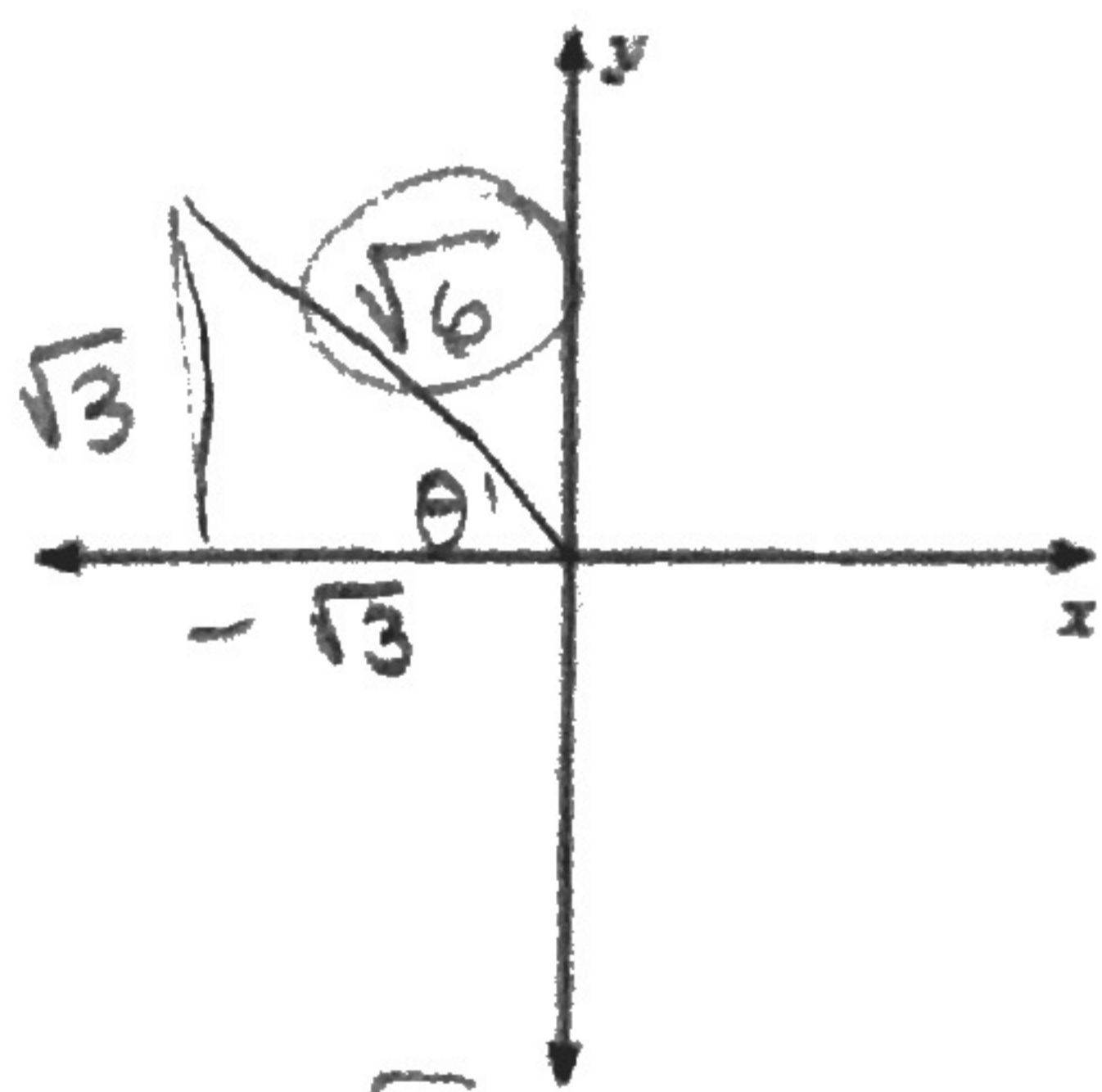
$$\tan \theta = 5 = 5$$

Point $(2\sqrt{3}, -2)$ 

$$\sin \theta = -\frac{1}{2} \approx -.5$$

$$\cos \theta = \frac{\sqrt{3}}{2} \approx 0.866$$

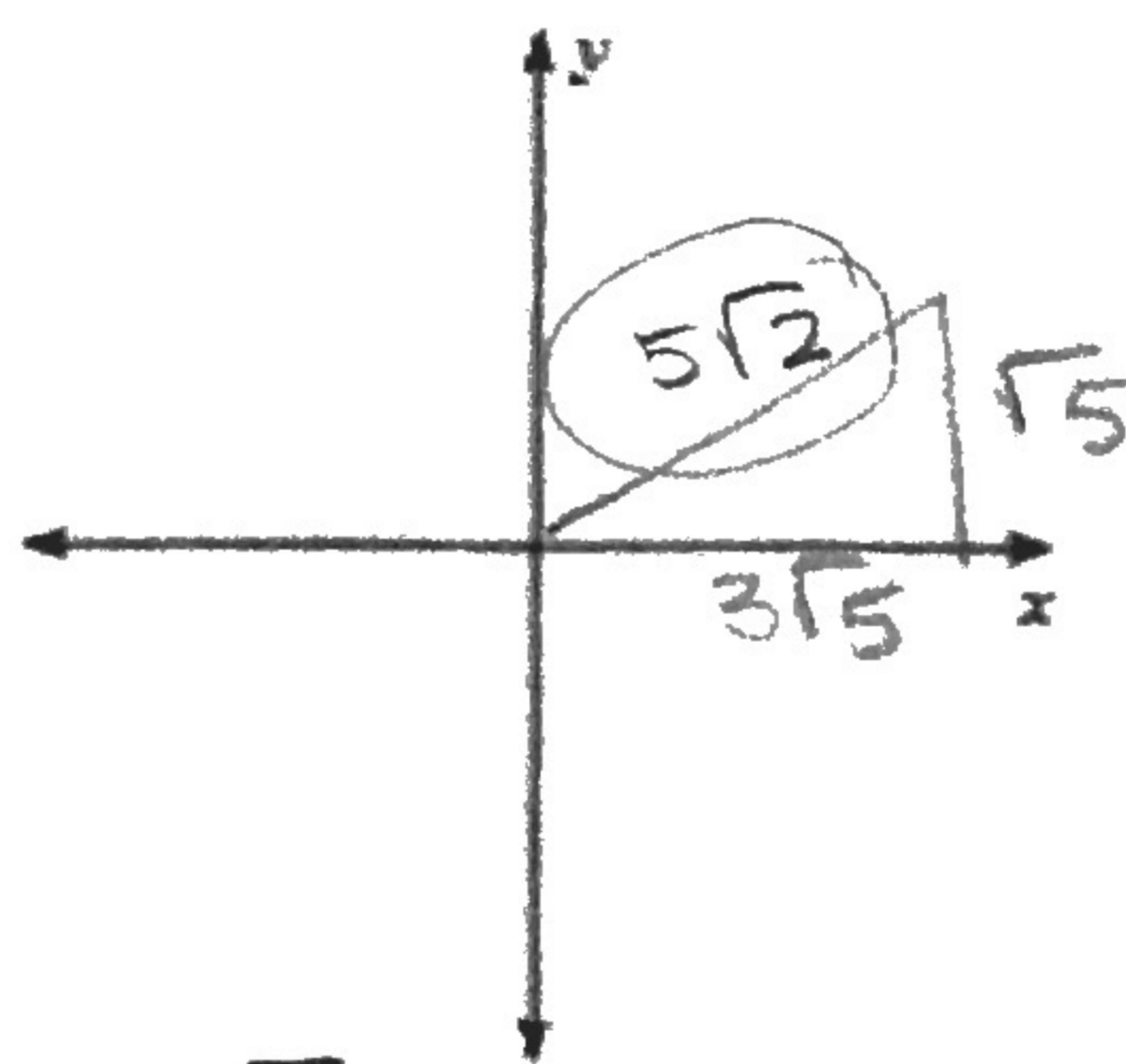
$$\tan \theta = -\frac{\sqrt{3}}{3} \approx -.577$$

Point $(-\sqrt{3}, \sqrt{3})$ 

$$\sin \theta = \frac{\sqrt{2}}{2} \approx .707$$

$$\cos \theta = -\frac{\sqrt{2}}{2} \approx -.707$$

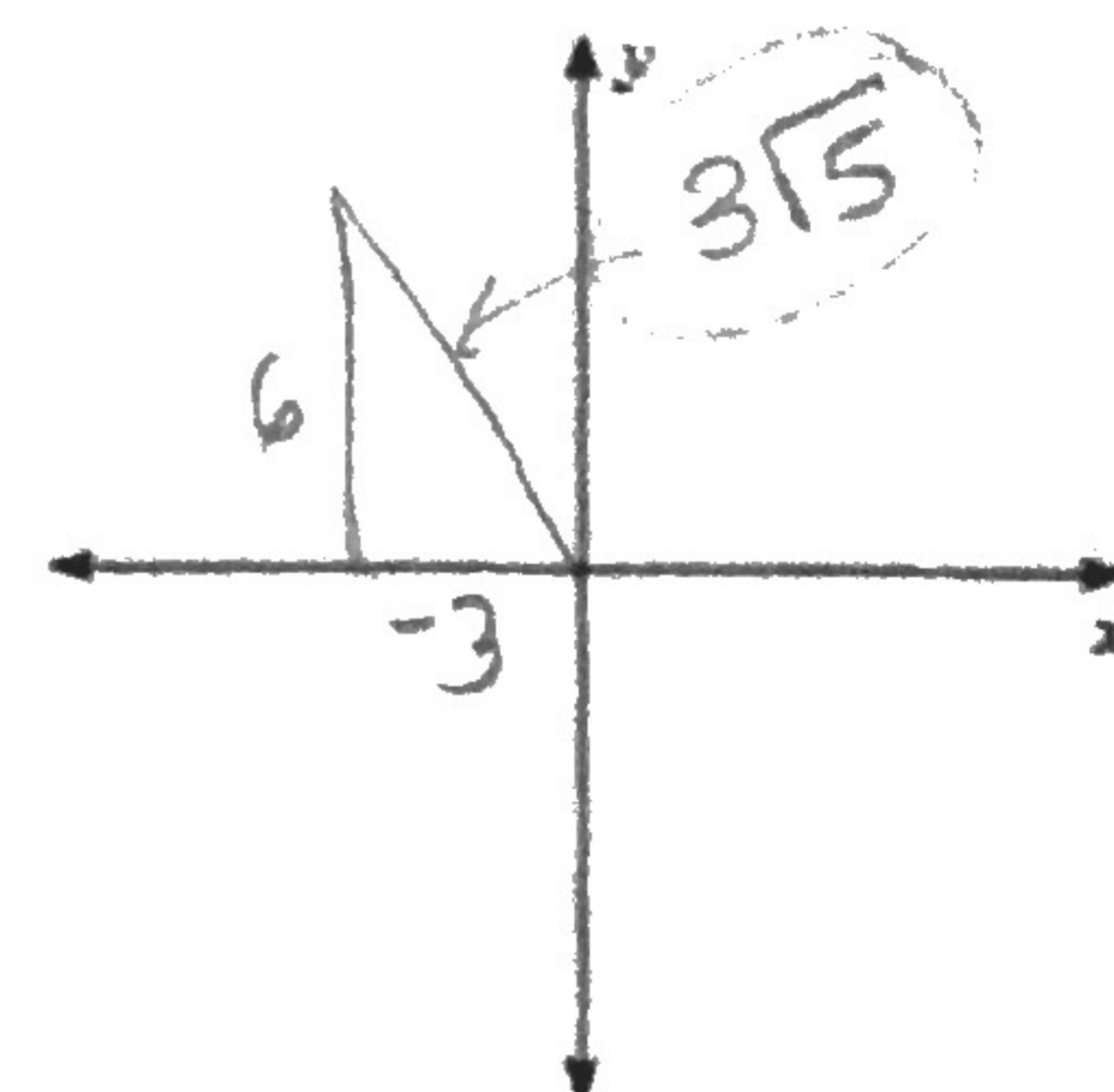
$$\tan \theta = -1 = -1$$

Point $(3\sqrt{5}, \sqrt{5})$ 

$$\sin \theta = \frac{\sqrt{10}}{10} \approx 0.316$$

$$\cos \theta = \frac{3\sqrt{10}}{10} \approx 0.949$$

$$\tan \theta = \frac{1}{3} \approx 0.333$$

Point $(-3, 6)$ 

$$\sin \theta = \frac{2\sqrt{5}}{5} \approx 0.894$$

$$\cos \theta = -\frac{\sqrt{5}}{5} \approx -.447$$

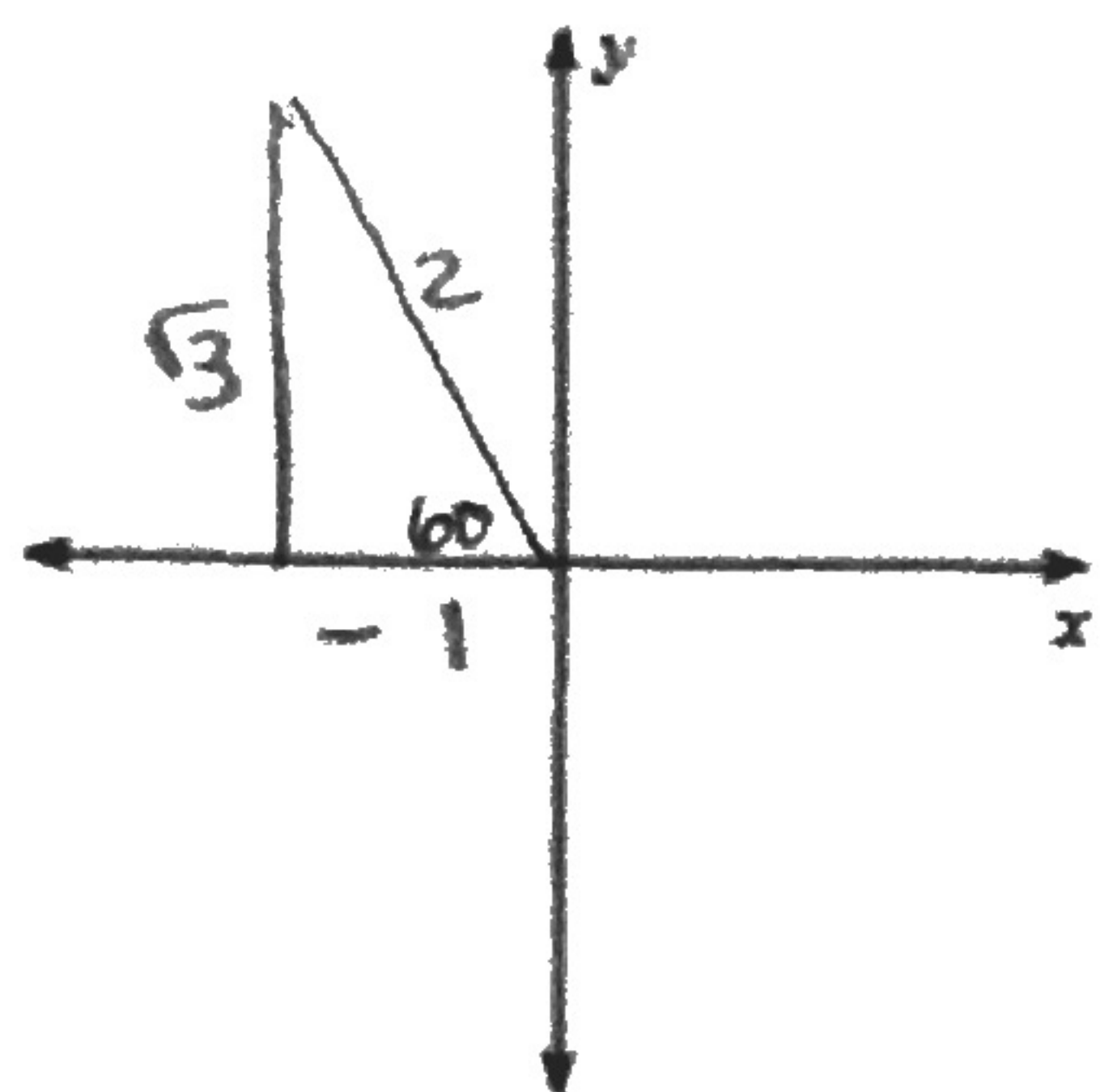
$$\tan \theta = -2 = -2$$

When you're done, approximate the decimals. Ensure the decimals make sense, based on the location of the point (is it higher up vertically or further away horizontally?)

Special Trig Ratios in the Coordinate Plane:

Now we can also find the trig ratios of any special angle! It doesn't matter what the side lengths actually are... what matters is the RATIO of the sides once the angle is locked in...

$$\theta = 120^\circ$$

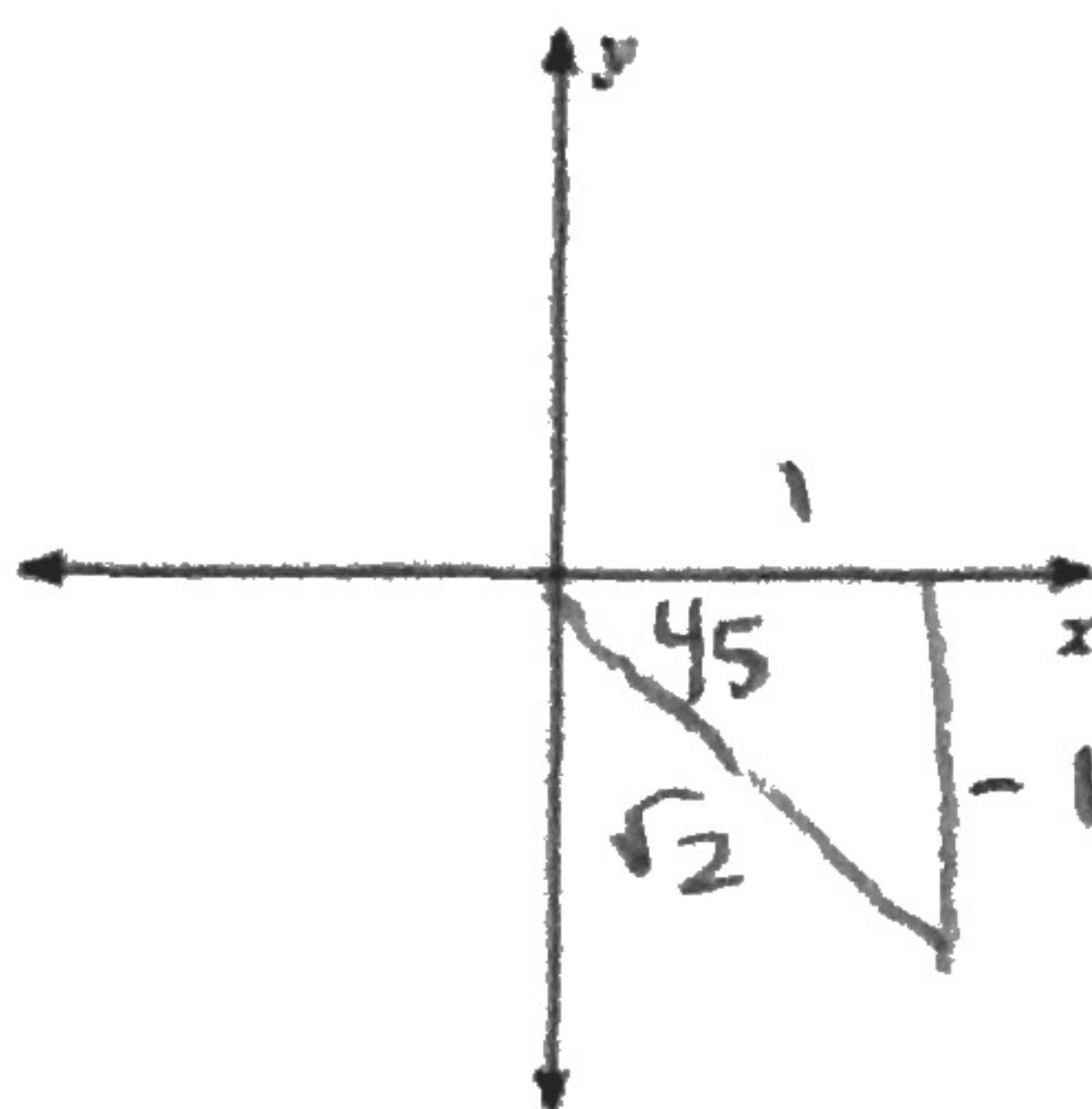


$$\sin \theta = \frac{\sqrt{3}}{2}$$

$$\cos \theta = -\frac{1}{2}$$

$$\tan \theta = -\sqrt{3}$$

$$\theta = -45^\circ$$

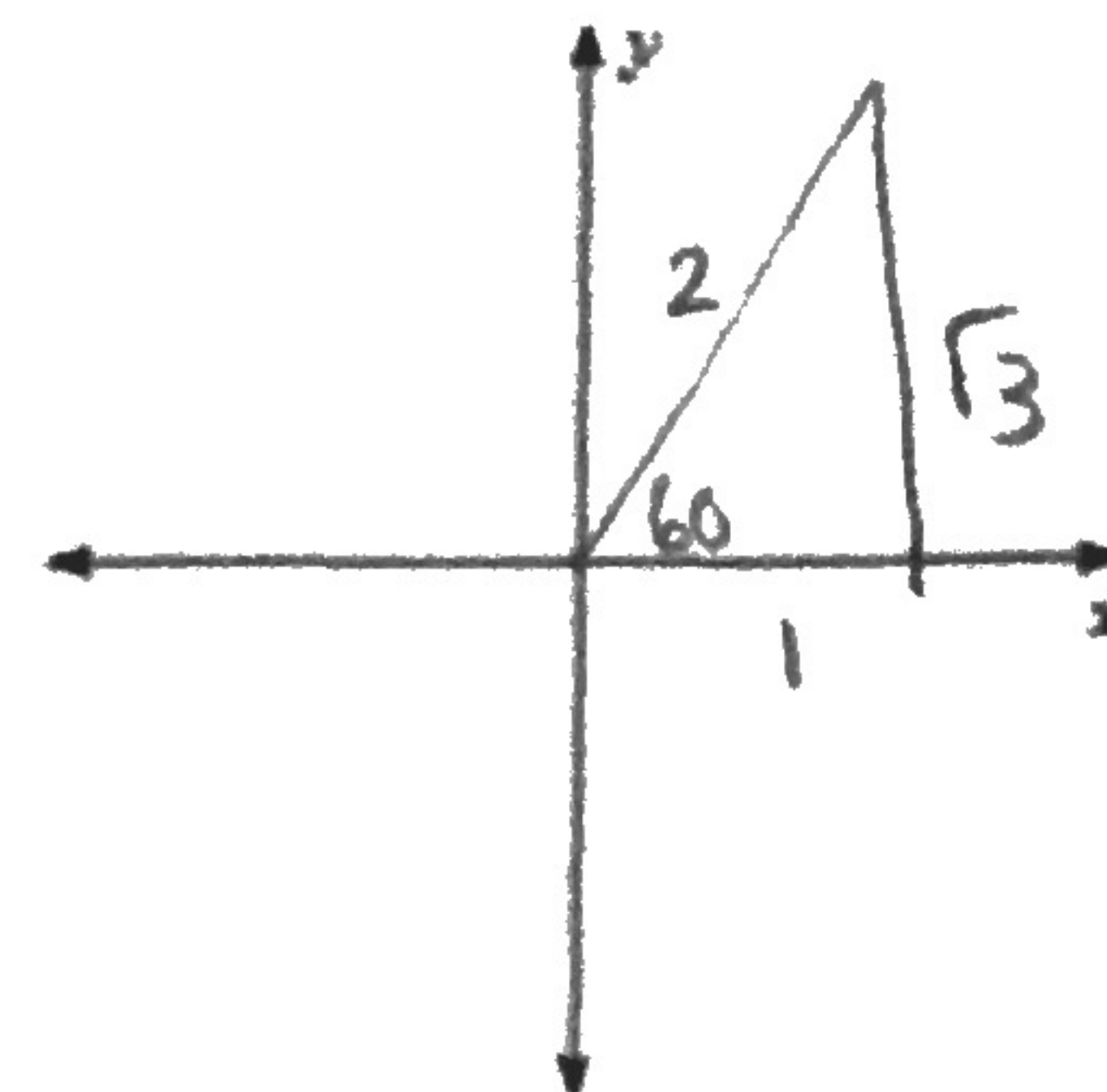


$$\sin \theta = -\frac{\sqrt{2}}{2}$$

$$\cos \theta = \frac{\sqrt{2}}{2}$$

$$\tan \theta = -1$$

$$\theta = 60^\circ$$

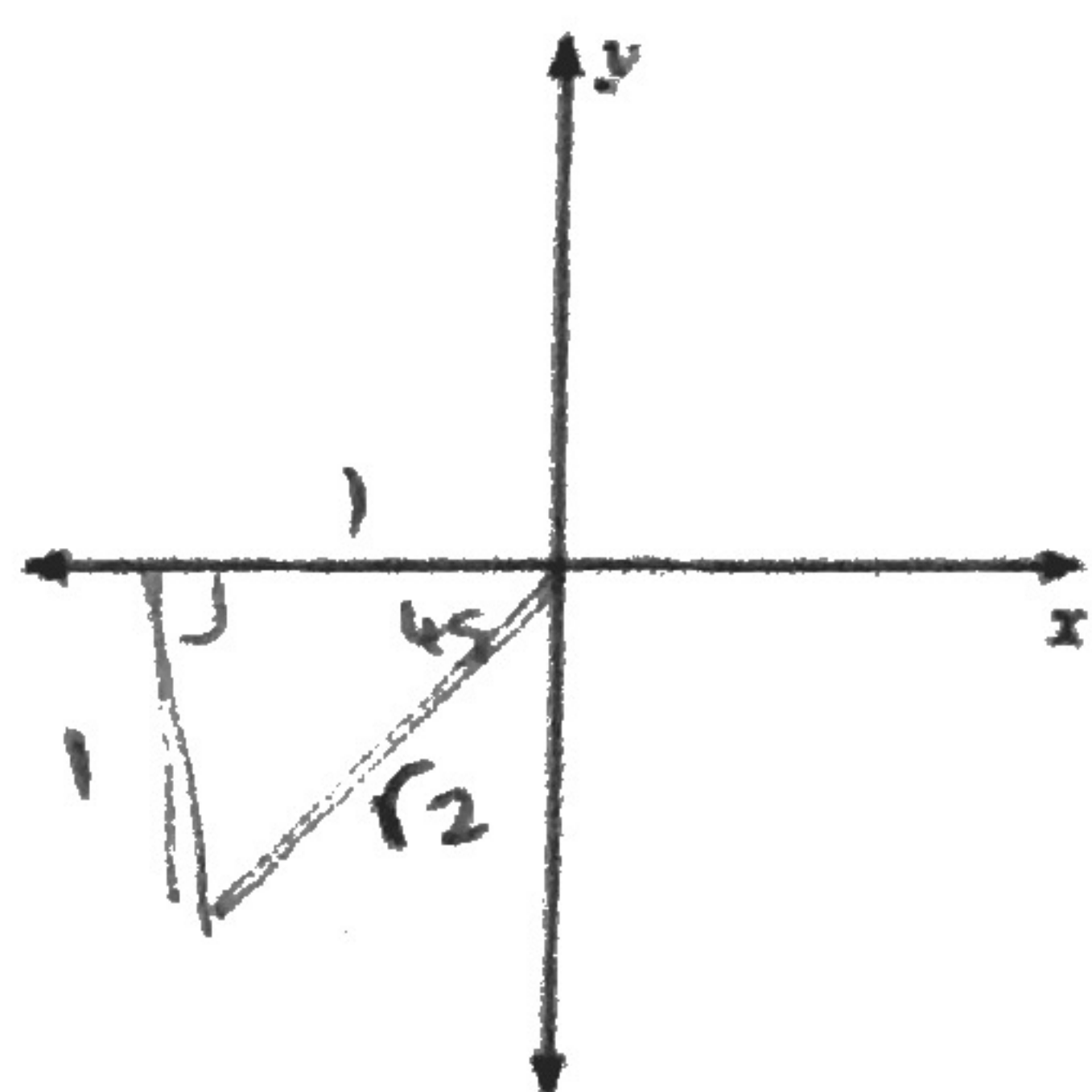


$$\sin \theta = \frac{\sqrt{3}}{2}$$

$$\cos \theta = \frac{1}{2}$$

$$\tan \theta = \sqrt{3}$$

$$\theta = 225^\circ$$

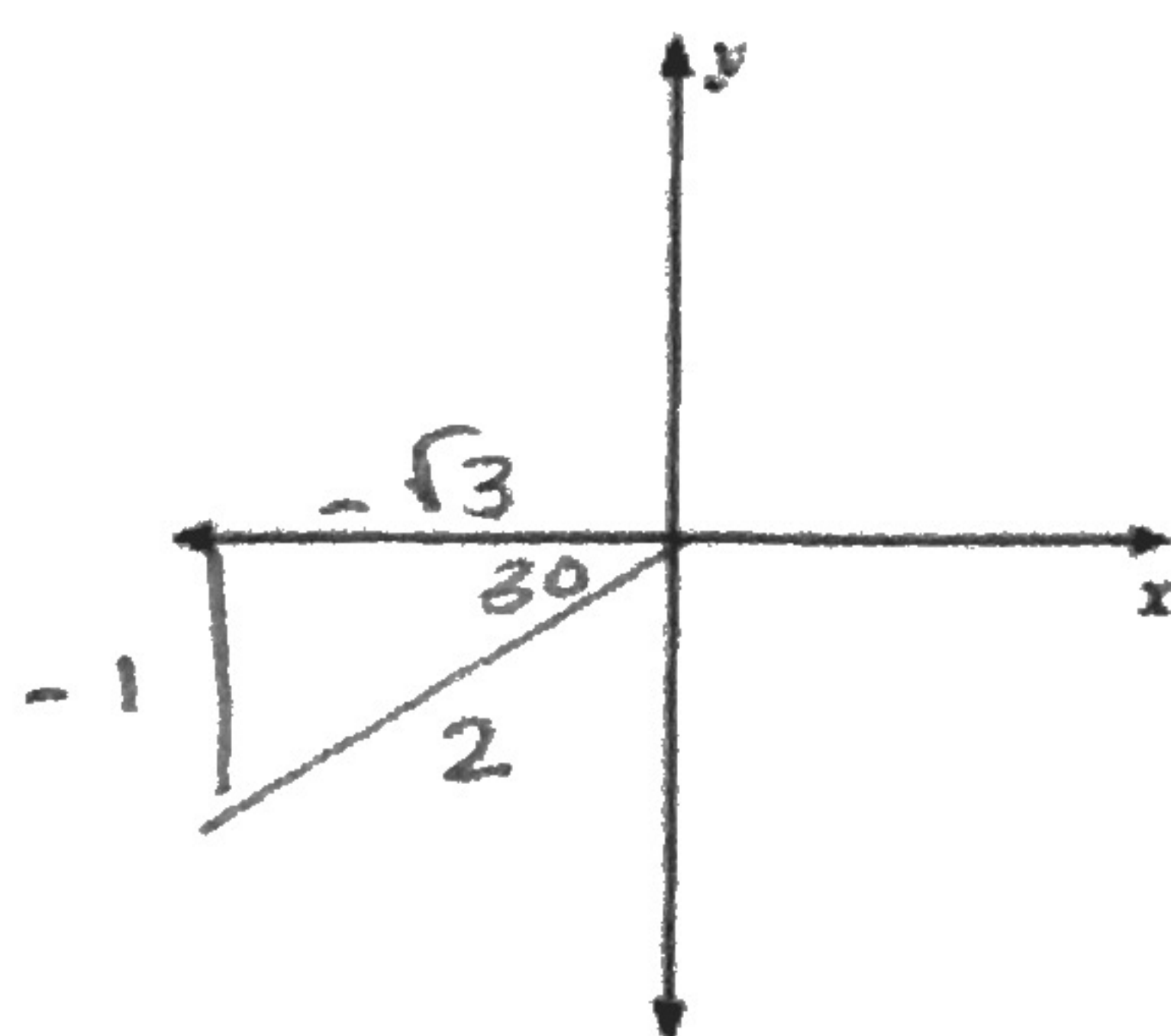


$$\sin \theta = -\frac{\sqrt{2}}{2}$$

$$\cos \theta = -\frac{\sqrt{2}}{2}$$

$$\tan \theta = 1$$

$$\theta = -150^\circ$$

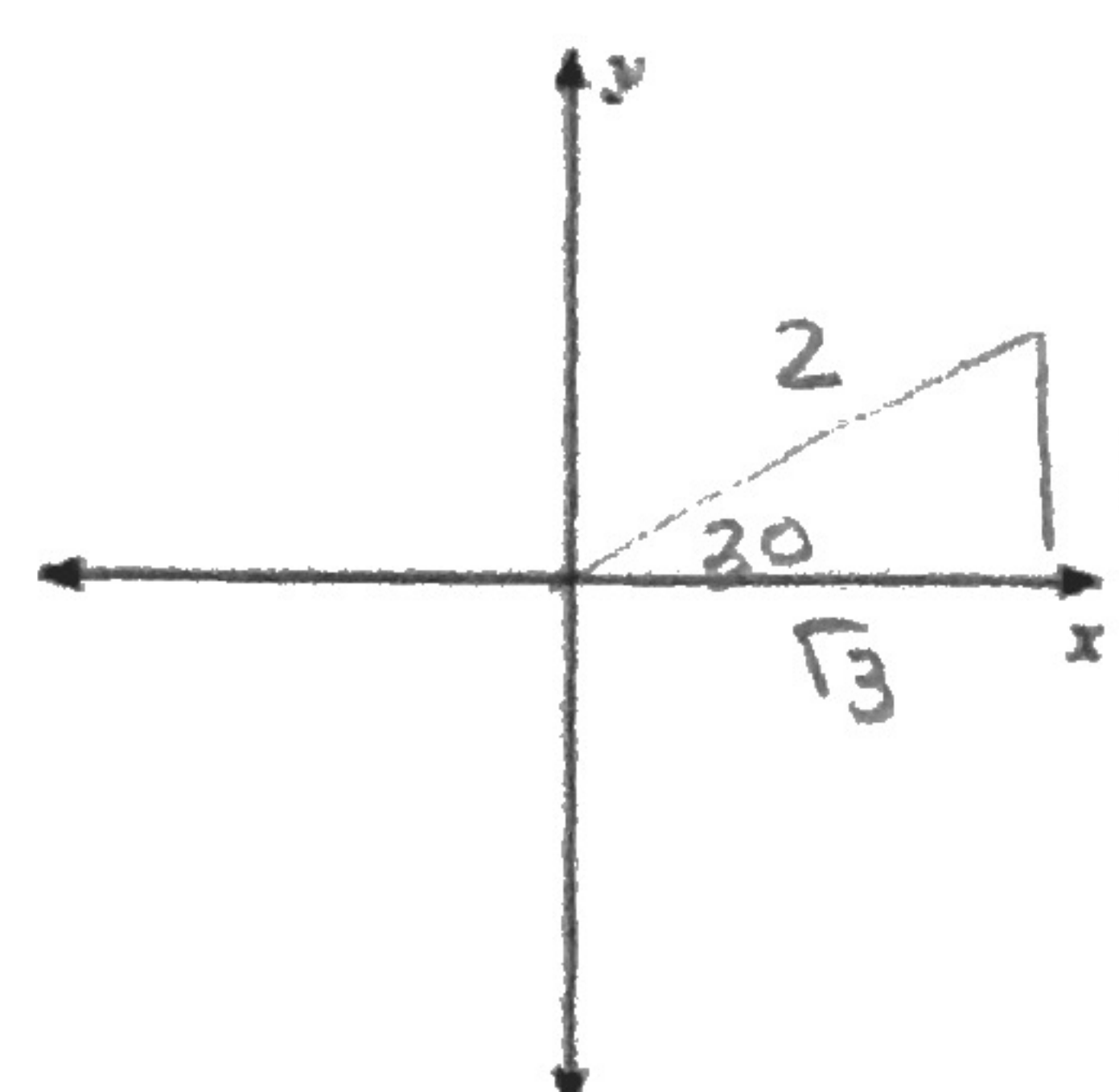


$$\sin \theta = -\frac{1}{2}$$

$$\cos \theta = -\frac{\sqrt{3}}{2}$$

$$\tan \theta = \frac{\sqrt{3}}{3}$$

$$\theta = 390^\circ$$



$$\sin \theta = \frac{1}{2}$$

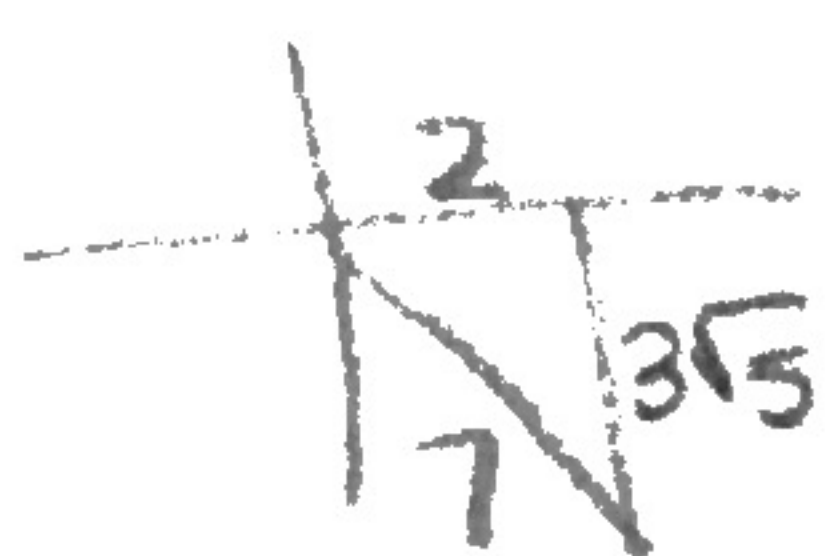
$$\cos \theta = \frac{\sqrt{3}}{2}$$

$$\tan \theta = \frac{\sqrt{3}}{3}$$

Find the other two trigonometric ratios, given one ratio.

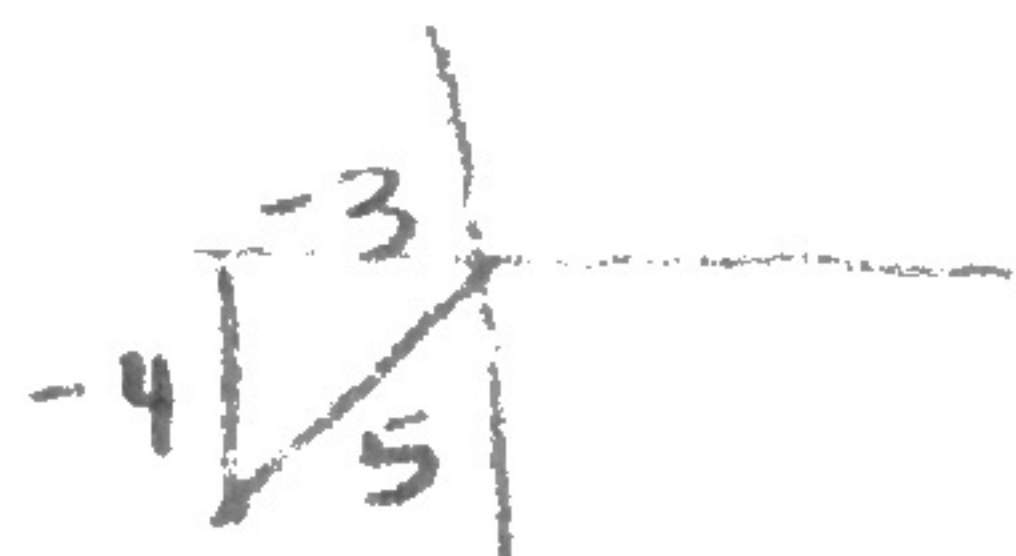
Draw it out, solve for what's missing, then evaluate the other trig ratios. Watch your signs...

$\cos \theta = 2/7, \theta \in \text{QIV}$



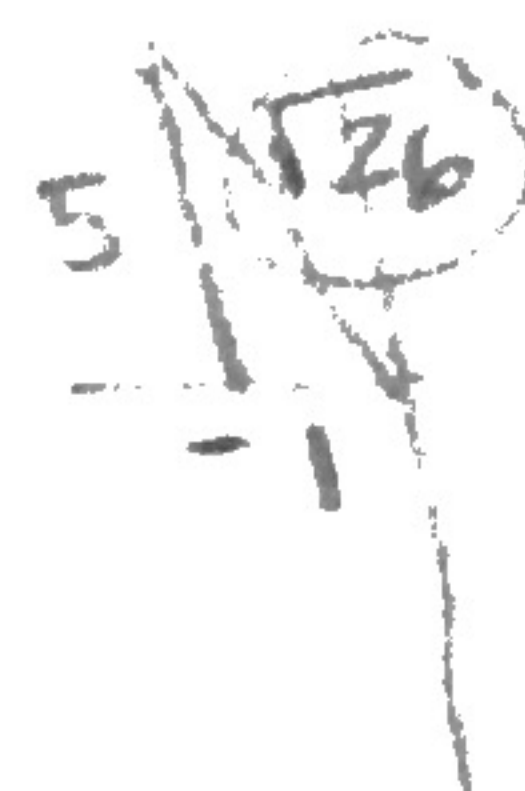
$\sin \theta = \frac{-3\sqrt{5}}{7}; \tan \theta = \frac{-3\sqrt{5}}{2}$

$\sin \theta = -4/5; \theta \in \text{QIII}$



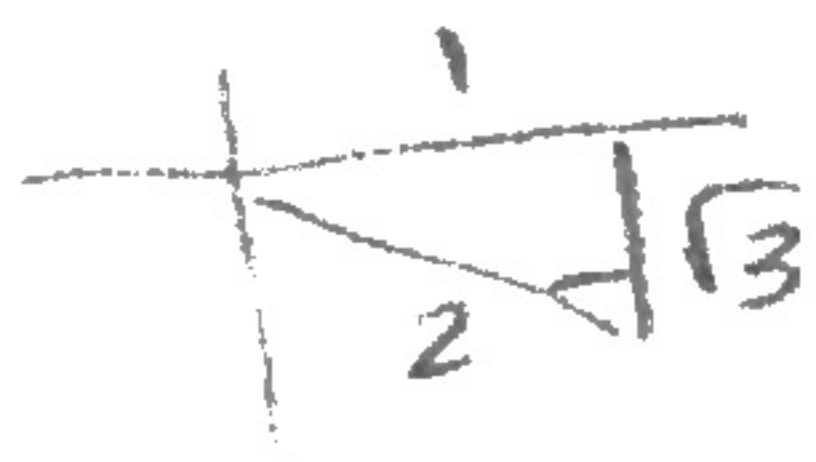
$\cos \theta = -3/5$
 $\tan \theta = 4/3$

$\tan \theta = -5; \theta \in \text{QII}$



$\sin \theta = \frac{5\sqrt{26}}{26}$
 $\cos \theta = \frac{-\sqrt{26}}{26}$

$\cos \theta = 1/2, \sin \theta < 0$



$\sin \theta = \frac{-\sqrt{3}}{2}; \tan \theta = -\sqrt{3}$

$\tan \theta = -1; \cos \theta < 0$



$\sin \theta = \frac{\sqrt{2}}{2}$
 $\cos \theta = \frac{-\sqrt{2}}{2}$

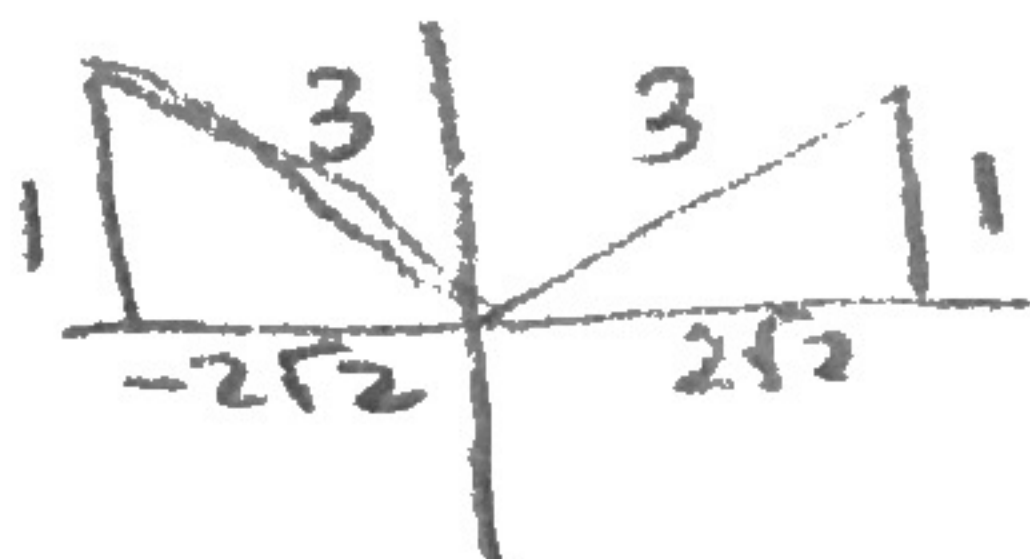
$\tan \theta = \sqrt{3}; \sin \theta > 0$



$\sin \theta = \frac{\sqrt{3}}{2}$
 $\cos \theta = \frac{1}{2}$

IF NO QUADRANT IS SPECIFIED, you must give TWO sets of answers: Since each trig function is positive or negative in two quadrants, you have to draw your reference triangles in BOTH quadrants and give both scenarios.

$\sin \theta = 1/3$



$\tan \theta = -\sqrt{3}$



if $\theta \in \text{QI}$

$\cos \theta = \frac{2\sqrt{2}}{3}$

$\tan \theta = \frac{\sqrt{2}}{4}$

if $\theta \in \text{QII}$

$\cos \theta = \frac{-2\sqrt{2}}{3}$

$\tan \theta = \frac{-\sqrt{2}}{4}$

if $\theta \in \text{QII}$

$\sin \theta = \frac{\sqrt{3}}{2}$

$\cos \theta = \frac{-1}{2}$

if $\theta \in \text{QIV}$

$\sin \theta = \frac{-\sqrt{3}}{2}$

$\cos \theta = \frac{1}{2}$

The Special Angle Chart:

You'll see this chart MANY times in the future.

Right now, draw some special right triangles and start to get familiar with the trig ratios of special reference angles.

Can you see the Cofunction theorem in action?

yes! $\sin 30 = \cos 60$
 $\sin 45 = \cos 45$

See any other neat patterns?

yes... \cup

	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan θ	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	und

Practice: Given the information, provide all three exact trig ratios (signs are important!!)

1. $\sin \theta = 12/13$; and $\theta \in \text{QI}$

$$\begin{aligned}\sin \theta &= 12/13 \\ \cos \theta &= 5/13 \\ \tan \theta &= 12/5\end{aligned}$$

2. $\cos \theta = -8/15$; $\theta \in \text{QII}$

$$\begin{aligned}\sin \theta &= \sqrt{161}/15 \\ \cos \theta &= -8/15 \\ \tan \theta &= -\sqrt{161}/8\end{aligned}$$

3. $\tan \theta = -24/7$; $\cos \theta < 0$

$$\begin{aligned}\sin \theta &= 24/25 \\ \cos \theta &= -7/25 \\ \tan \theta &= -24/7\end{aligned}$$

4. $\sin \theta = -4/5$; $\theta \in \text{QIV}$

$$\begin{aligned}\sin \theta &= -4/5 \\ \cos \theta &= 3/5 \\ \tan \theta &= -4/3\end{aligned}$$

5. $\cos \theta = 12/37$; $\theta \in \text{QIV}$

$$\begin{aligned}\sin \theta &= -35/37 \\ \cos \theta &= 12/37 \\ \tan \theta &= -35/12\end{aligned}$$

6. θ intersects $(-3, 5)$

$$\begin{aligned}\sin \theta &= \frac{5\sqrt{34}}{34} \\ \cos \theta &= -\frac{3\sqrt{34}}{34} \\ \tan \theta &= -5/3\end{aligned}$$

7. $\cos \theta = -5/13$ and $\sin \theta < 0$

$$\begin{aligned}\sin \theta &= -12/13 \\ \cos \theta &= -5/13 \\ \tan \theta &= 12/5\end{aligned}$$

8. θ intersects $(3, -1)$

$$\begin{aligned}\sin \theta &= -\sqrt{10}/10 \\ \cos \theta &= 3\sqrt{10}/10 \\ \tan \theta &= -1/3\end{aligned}$$

9. θ intersects $(-2, -\sqrt{5})$

$$\begin{aligned}\sin \theta &= -\sqrt{5}/3 \\ \cos \theta &= -2/3 \\ \tan \theta &= -\sqrt{5}/2\end{aligned}$$

10. $\cos \theta = 12/13$ and $\sin \theta < 0$

$$\begin{aligned}\sin \theta &= -5/13 \\ \cos \theta &= 12/13 \\ \tan \theta &= -5/12\end{aligned}$$

11. $\theta = 210^\circ$

$$\begin{aligned}\sin \theta &= -\frac{1}{2} \\ \cos \theta &= \frac{\sqrt{3}}{2} \\ \tan \theta &= \sqrt{3}/3\end{aligned}$$

12. $\cos \theta = 1/2$, $\theta \in \text{QIV}$:

$$\begin{aligned}\sin \theta &= -\sqrt{3}/2 \\ \cos \theta &= 1/2 \\ \tan \theta &= -\sqrt{3}\end{aligned}$$

13. $\theta = -45^\circ$

$$\begin{aligned}\sin \theta &= -\sqrt{2}/2 \\ \cos \theta &= \sqrt{2}/2 \\ \tan \theta &= -1\end{aligned}$$

14. $\tan \theta = 1$ and $\sin \theta < 0$

$$\begin{aligned}\sin \theta &= -\sqrt{2}/2 \\ \cos \theta &= -\sqrt{2}/2 \\ \tan \theta &= 1\end{aligned}$$