Name: Answer Key Per: ________ Date: ______________

Serafino ▪ Precalculus S2

9B Compound Probability & Conditional Probability

Notes & Classwork / Homework

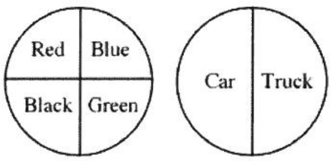
COMPOUND Probability of Independent Experiments

The probability of *several* events occurring a particular way.
These events can occur:

- In separate concurrent experiments
- One after the other

> **P(A and B) = P(A) · P(B)**
>
> *NOTE: A and B are Independent.
> The first thing doesn't affect the second *

1. You win a new car! You're on a game show and spin two spinners.
   One will determine the color, the other, the type of car you win.



P(Green Truck) $\boxed{12.5\%}$

$\frac{1}{4}\cdot\frac{1}{2} = \frac{1}{8}$

P(Black Truck or Red Car) $\boxed{25\%}$

$\frac{1}{4}\cdot\frac{1}{2} + \frac{1}{4}\cdot\frac{1}{2} = \frac{1}{4}$

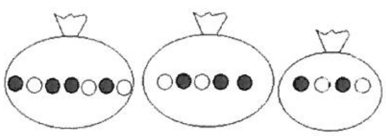
P(Blue Car) $\boxed{12.5\%}$

$\frac{1}{4}\cdot\frac{1}{2} = \frac{1}{8}$

P(Any vehicle that's not Green)

$\frac{3}{4}\cdot\frac{2}{2} = \boxed{75\%}$

2. Three different bags contain black or white marbles.
You pick one marble from each back. What is the probability you will pick marbles:



P(B, W, W) $\boxed{11.4\%}$

$\frac{4}{7}\cdot\frac{2}{5}\cdot\frac{2}{4} = \frac{4}{35}$

P(All Black) $\boxed{17.1\%}$

$\frac{4}{7}\cdot\frac{3}{5}\cdot\frac{2}{4} = \frac{6}{35}$

P(W, B, B) $\boxed{12.9\%}$

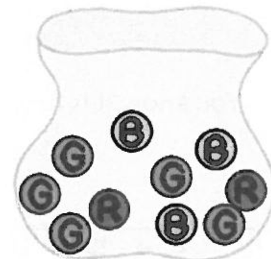
$\frac{3}{7}\,\frac{3}{5}\,\frac{2}{4} = \frac{7}{90}$

P(All White) $\boxed{8.6\%}$

$\frac{3}{7}\cdot\frac{2}{5}\cdot\frac{2}{4} = \frac{3}{35}$

COMPOUND Probability of REPEATING and Experiment With or Without Replacement
Independent vs Dependent Experiments

The bag to the right has 10 marbles: 5 Green, 3 Blue, 2 Red



You pull SEVERAL marbles out of the SAME bag, ONE AT A TIME!
Compound Probability measures that chance of pulling them out in a particular order.
But this leaves us with one key question…

"Do you put the marble back each time or not?"

If you REPLACE the marble → the events are INDEPENDENT → Event A has no effect on P(B)
If you DO NOT REPLACE the marble → the events are DEPENDENT → Event A DOES affect P(B)

3. Find the (independent) probability of selecting 3 marbles in the order given, WITH REPLACEMENT:

P(Green, Red, Blue) $\boxed{3\%}$

$\frac{5}{10}\cdot\frac{2}{10}\cdot\frac{3}{10} = \frac{3}{100}$

P(Blue, Blue, Green) $\boxed{4.5\%}$

$\frac{3}{10}\cdot\frac{3}{10}\cdot\frac{5}{10} = 9/200$

P(Red, Red, Red) $\boxed{0.8\%}$

$\frac{2}{10}\cdot\frac{2}{10}\cdot\frac{2}{10} = \frac{1}{125}$

4. Find the (dependent) probability of selecting 3 marbles in the order given, WITHOUT REPLACEMENT:

P(Green, Red, Blue) $\boxed{4.2\%}$

$\frac{5}{10}\cdot\frac{2}{9}\cdot\frac{3}{8} = \frac{1}{24}$

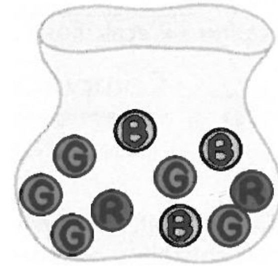
P(Blue, Blue, Green) $\boxed{4.2\%}$

$\frac{3}{10}\cdot\frac{2}{9}\cdot\frac{5}{8}$ $\frac{1}{24}$

P(Red, Red, Red)

$\frac{2}{10}\cdot\frac{1}{9}\cdot 0 = \boxed{0\%}$

CONDITIONAL Probability: The Condition is a Prior, Dependent Event



The notation P(A|B) is referring to conditional probability,
It is asking for the probability of A under the condition that B has already happened.

$$\mathbf{P(A \mid B)} = \frac{\mathbf{P(A\ and\ B)}}{\mathbf{P(B)}}$$

5G 3B 2R

If two marbles are drawn from the bag WITHOUT replacement.

P(B| R) → means "the probability of selecting a Blue, given that you have already selected a Red"

Consider that B has already happened; make the changes to your new sample space and find P(A)

Here, just out one "R", that leave you with 9 marbles. P(B) is now 3/9 = 1/3

5. Two marbles are drawn from the bag without replacement. Find the conditional probability of:

P(G | R) $\boxed{\frac{5}{9}}$ P(R | R) $\boxed{\frac{1}{9}}$ P(B | G) $\boxed{\frac{1}{3}}$ P(G | G) $\boxed{\frac{4}{9}}$

Conditional Probability: Single Event with KNOWN PROPERTIES

$$P(A|B) = \frac{P(A\ and\ B)}{P(B)}$$ "Out of all the things that are B, what is the probability it is also A?"

6. Someone is holding a hard behind their back. They ask you for the probability it is a Queen.

You know nothing about the card; P(Q) =

4/52 = $\boxed{1/13}$

You know it is NOT a 5; P(Q|5') =

4/48 = $\boxed{1/12}$

You know it is NOT a Numbered card; P(Q|N') =

4/16 = $\boxed{1/4}$

You know it is a Face: P(Q|F) =

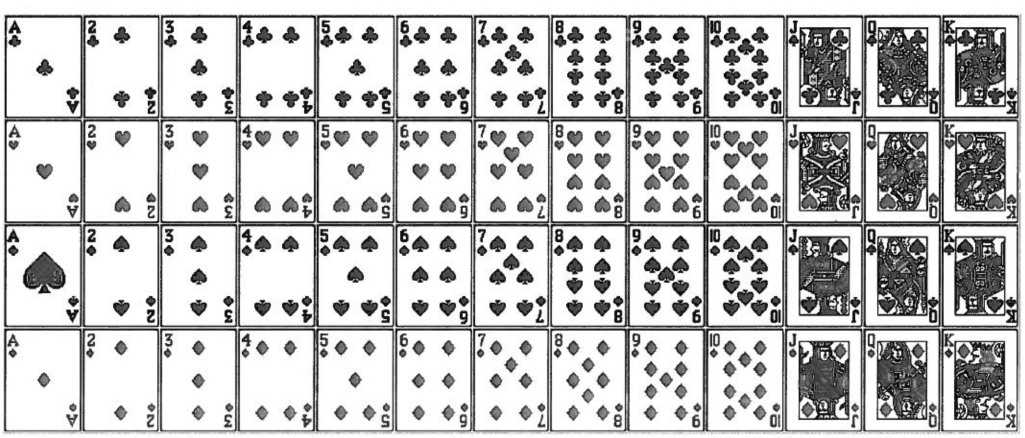
4/12 = $\boxed{1/3}$

You know it is Red; P(Q | R) =

2/26 = $\boxed{1/13}$

You know it a true Royal*: P(Q|Roy) =

4/8 = $\boxed{1/2}$



** From Wikipedia ®*
"As early as the mid-16th century, the [Jack] was known in England as the Knave <u>(meaning a male servant of royalty)</u>. ... In 1864 English cardmaker Samuel Hart published a deck using "J" instead of "Kn" to designate the lowest-ranking court card"

Conditional Probability from Situations

7. This is a contingency table. This one shows the numbers of people who survived vs. died on the Titanic.

Passenger category	Number aboard	Number saved	Number lost
Children, First Class	6	5	1
Children, Second Class	24	24	0
Children, Third Class	79	27	52
Women, First Class	144	140	4
Women, Second Class	93	80	13
Women, Third Class	165	76	89
Women, Crew	23	20	3
Men, First Class	175	57	118
Men, Second Class	168	14	154
Men, Third Class	462	75	387
Men, Crew	885	192	693
Total	**2224**	**710**	**1514**

Children	Women	Men	
109	425	1690	
Crew	3rd	2nd	1st
908	706	285	325
Died		Survived	
1514		710	

If you went back in time into the body of a random passenger on the ship ...

P(Died)? $\frac{1514}{2224}$ $\boxed{68\%}$

P(Woman)? $\frac{425}{2224}$ $\boxed{19.1\%}$

P(Crew)? $\frac{908}{2224}$ $\boxed{40.8\%}$

P(1st Class)? $\frac{325}{2224}$ $\boxed{14.6\%}$

P(Woman or Child)? $\frac{534}{2224}$ $\boxed{24\%}$

P(3rd or Crew)? $\frac{1614}{2224}$ $\boxed{72.6\%}$

P(Man or 2nd Class)? $\frac{1807}{2224}$ $\boxed{81.3\%}$

P(1st class child)? $\frac{6}{2224}$ $\boxed{0.3\%}$

P(Woman on Crew)? $\frac{23}{2224}$ $\boxed{1\%}$

P(Man who Died)? $\frac{1352}{2224}$ $\boxed{60.8\%}$

P(Male Crew who Survived)? $\frac{192}{2224}$ $\boxed{8.6\%}$

P(2nd Class Child who Died?) $\boxed{0\%}$

P(C | W) = $\frac{23}{425}$ $\boxed{5.4\%}$

P(W | C) = $\frac{23}{908}$ $\boxed{2.5\%}$

P(1st | S) $\frac{202}{710}$ $\boxed{28.5\%}$

P(S | 1st) $\frac{202}{325}$ $\boxed{62.2\%}$

P(C | S) = $\frac{192}{710}$ $\boxed{27\%}$

~~P(1st | S) =~~ P(S | C) $192/908$ $\boxed{21\%}$

~~P(Crew | S) =~~

We can use CONDITIOAL probability to analyze breakdowns of the demographics:

What is the probability you'll die if you go back as a...

Child? $\frac{53}{109}$ $\boxed{48.6\%}$

Woman? $\frac{109}{425}$ $\boxed{25.6\%}$

Man? $\frac{1352}{1690}$ $\boxed{80\%}$

What is the probability that you'll survive, if you go back as someone in...

1st class? P(S | 1st) $\frac{202}{325}$ $\boxed{62.2\%}$

2nd class? P(S | 2nd) $\frac{118}{285}$ $\boxed{41.4\%}$

3rd class? P(S | 3rd) $\frac{178}{706}$ $\boxed{25.2\%}$

Crew? P(S | C) $\frac{212}{908}$ $\boxed{23.3\%}$

What is the probability that you'll survive, given that you are going back as an adult the SAME GENDER you are now in...

1st class? F: $\boxed{97\%}$ M: $\boxed{33\%}$

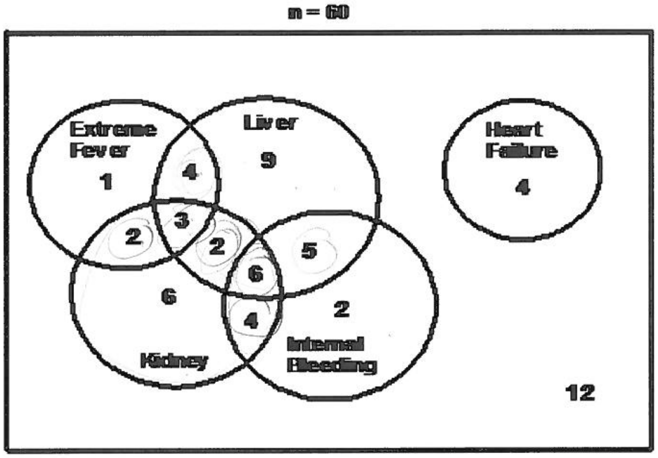
2nd class? F: $\boxed{86\%}$ M: $\boxed{8.3\%}$

3rd class? F: $\boxed{46\%}$ M: $\boxed{16.2\%}$

Crew? F: $\boxed{87\%}$ M: $\boxed{22\%}$

8. Rhesus monkeys are used to test disease vaccinations and medications. 60 monkeys were tested for vaccines and drugs resistant to the Ebola Virus. The 48 who died had autopsies performed.

↓ should be on next page (sorry)



Kidney Damage: 23 Live Damage: 29 Fever: 10
Int. Bleeding: 17 Heart Failure: 4

Liver & Kidney: 11 Kidney & Internal Bleeding: 10
Liver & Fever: 7 Kidney & Fever: 5
Liver & Internal Bleeding: 11

Liver, Kidney & Bleeding: 6
Livery, Kidney & Fever: 3

If one monkey out of the 60 was selected at random, find the probability that monkey would eventually…

P(Die) $\frac{48}{60}$ $\boxed{80\%}$

P(Die with liver damage) $\frac{29}{60}$ $\boxed{48\%}$

P(Die with Heart Failure) $\frac{4}{60}$ $\boxed{6.7\%}$

P(Die with a Fever) $\frac{10}{60}$ $\boxed{16.7\%}$

P(Died with Liver or Kidney Damage) $\frac{41}{60}$ $\boxed{68\%}$

P(Died with Liver and Kidney Damage) $\frac{11}{60}$ $\boxed{18.3\%}$

P(Liver | Die) $\frac{29}{48}$ $\boxed{60.4\%}$

P(Kidney | Die) $\frac{23}{48}$ $\boxed{47.9\%}$

P(Fever | Die) $\frac{10}{48}$ $\boxed{20.8\%}$

P(Bleeding | Liver Damage) $\frac{11}{29}$ $\boxed{37.9\%}$

P(Bleeding | Kidney damage) $\frac{10}{23}$ $\boxed{43.5\%}$

P(Bleeding| Heart Failure) $\boxed{0\%}$

P(Died, but NOT with fever nor bleeding) $\frac{21}{60}$ $\boxed{35\%}$

P(Liver Damage and one other Symptom) $\frac{11}{60}$ $\boxed{18.3\%}$

P(Kidney Damage and one other Symptom) $\frac{8}{60}$ $\boxed{13.3\%}$

P(Liver Damage and two other symptoms) $\frac{9}{60}$ $\boxed{15\%}$

P(Kidney Damage and two other symptoms) $\frac{9}{60}$ $\boxed{15\%}$

Odds (Didn't Die in experiment) $\boxed{1:4}$

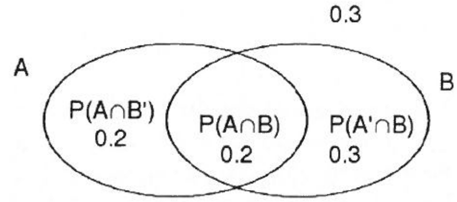
What symptom had the highest probability of leading to death? Liver Damage

What symptom had the lowest probability of leading to death? Heart Failure

What symptom ALONE had the lowest probability of leading to death? Extreme Fever

Which 2–symptom combination was the most deadly? Liver + Kidney Damage

APPLICATIONS OF PROBABILITY WITH INDEPENDENT EVENTS



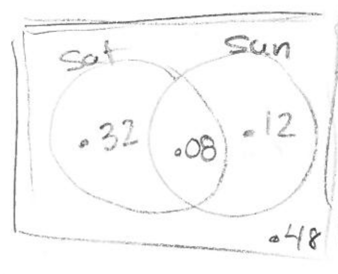
9. This Venn Diagram can be any TWO INDEPENDENT Events:
P(A) doesn't affect P(B). Like spinning a spinner vs. rolling a die.

a) P(A) 40%　c) P(A ∩ B) 20% (.5)(.4)　e) P(A∩B') 20%　g) P(A|B) 40% .2/.5　i) P(A|B') 40% .2/.5

b) P(B) 50%　d) P(A U B) 70% .5+.4-.2　f) P(B∩A') 30%　h) P(B|A) 50% .2/.4　j) P(B|A') 50% .3/.6

NOTE: We see the same probabilities in the first two questions as the last two because … the fact that A happened should have no effect on the probability that B will happen. **P(A) = P(A|B) if A and B are INDEPENDENT**

10. There is a 40% chance of rain Saturday'; 20% chance of rain Sunday. If the two events are independent, what is the probability it will rain....



P(Sat) 40%　P(Sun) 20%　P(Sat AND Sun) 8% (.4)(.2)　P(Sat OR Sun) 52% .4+.2-.08

P(Sat and Not Sun) 32%　P(Sun and Not Sat) 12%　P(Only one day) 44%

P(Sun|Sat) 20% .08/.4　P(Sat|Sun) 40% .08/.2　P(Sat|Not Sun) 40% .32/.8　P(Sun|Not Sat) 20% .12/.6

Same b/c independent

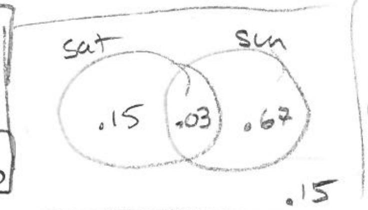
APPLICATIONS OF PROBABILITY WITH DEPENDENT EVENTS

11. Not all weather events are independent. Let's say there's a Tornado coming. There's a 85% chance it will hit this weekend. Since tornados only last for about 10 minutes, there is a 3% chance it will hit BOTH days. There is an 18% chance it will be on Saturday.

P(A or B) = P(A) + P(B) + P(A∩B)
.85 = .18 + x − .03
x = .7

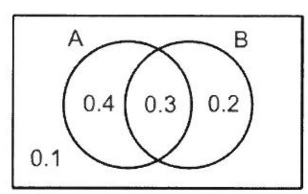
P(Sat) 18%　P(Sun) 70%　P(Sat AND Sun) 3%　P(Sat OR Sun) 85%

P(Sat and Not Sun) 15%　P(Sun and Not Sat) 67%　P(Only one day) 82%



P(Sun|Sat) 16.7% .03/.18　P(Sat|Sun) 4.3% .03/.7　P(Sat|Not Sun) 50% .15/.3　P(Sun|Not Sat) 81.7% .67/.82

12. Let's revisit grades & report cards.
These are the percentage of students with As and/ or B's.



P(As) = 70%　P(Bs) = 50%　P(As and Bs) = 30%

You can see that here, the intersection (A and B) is NOT the product of A and B because grades are not totally independent. If you're the type of student who gets As, you probably also have more Bs than Cs and Ds.

P(Bs|As) = 42.9% .3/.7　P(As|Bs) = 60% .3/.5　P(Bs|No As) = 66.7% .2/.3　P(As|No Bs) = 80% .4/.5

This is a chart of graduate students enrolled in a Teaching course.
Let W = Female, M = Male, Ft = Full Time, P = Part Time
Find the conditional probability if one person is chosen at random.
Be sure you're able to articulate WHAT the question is asking.

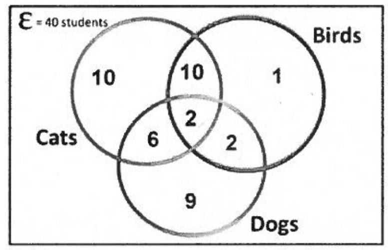
	Full-time	Part-Time	Total
Female	28	15	43
Male	12	16	28
Total	40	31	71

P(W) = $\boxed{\frac{43}{71}}$ P(M) = $\boxed{\frac{28}{71}}$ P(Ft) = $\boxed{\frac{40}{71}}$ P(P) = $\boxed{\frac{31}{71}}$

P(M ∩ P) = $\boxed{\frac{16}{71}}$ P(W ∩ Ft) = $\boxed{\frac{28}{71}}$ P(Ft | W) = $\boxed{\frac{28}{43}}$ P(Ft | M) = $\boxed{\frac{12}{28}}$

P(W | Ft) = $\boxed{\frac{28}{40}}$ P(W | P) = $\boxed{\frac{15}{31}}$ P (P| M) = $\boxed{\frac{16}{28}}$ P(M|P) = $\boxed{\frac{16}{31}}$

13. Here are the pets owned by 40 students.



P(C) = $\frac{28}{40}$ $\boxed{70\%}$ P(B ∩ D) = $\frac{4}{40}$ $\boxed{10\%}$ P(D|C) = $\frac{8}{28}$ $\boxed{28.6\%}$

P(D) = $\frac{19}{40}$ $\boxed{47.5\%}$ P(B ∩ C) = $\frac{12}{40}$ $\boxed{30\%}$ P(C | D) = $\frac{8}{19}$ $\boxed{42.1\%}$

P(one other animal | Dog) P(one other animal | Cat) P(one other animal | B) =

$\frac{8}{19} = \boxed{42.1\%}$ $\frac{16}{40} = \boxed{40\%}$ $\frac{12}{15} = \boxed{80\%}$

Owning which animal gives you the highest probability of having the other two there as well?

Cats $2/40$ $\boxed{5\%}$ Dog $2/19$ $\boxed{10.5\%}$ Bird $2/15$ $\boxed{13.3\%}$ $\boxed{\text{Birds!}}$

14. The chance that you will pass a Spanish test is 60%. The chance that you will study for it is 45%. The chance that you study and pass the test is 40%. If you study, what is the probability you pass?

$$P(\text{Pass} | \text{Study}) = \frac{\text{Study \& Pass}}{\text{Study}} = \frac{.4}{.45} = \boxed{88.9\%}$$

15. The probability that Rich Aunt Sue will go to Mexico in the winter is 60%. The probability that Sue will go to Mexico in the winter AND to France in the summer is 40%. Find the probability that she will go to France this summer, given that she just returned from her winter vacation in Mexico.

$$\frac{.4}{.6} = \boxed{66.7\%}$$

16. Looking at the stats, the probability that a person smokes is 30%. The probability that they will smoke and develop lung disease is 27%. Find the probability that that person will develops lung disease, given that they smoke.

$$\frac{.27}{.3} = \boxed{90\%}$$