Name: Answer Key

9C

Modeling with Trig Functions

We can use trigonometric functions to model "circular" word problems as well. These types of word problems can be modeled by trigonometric functions.

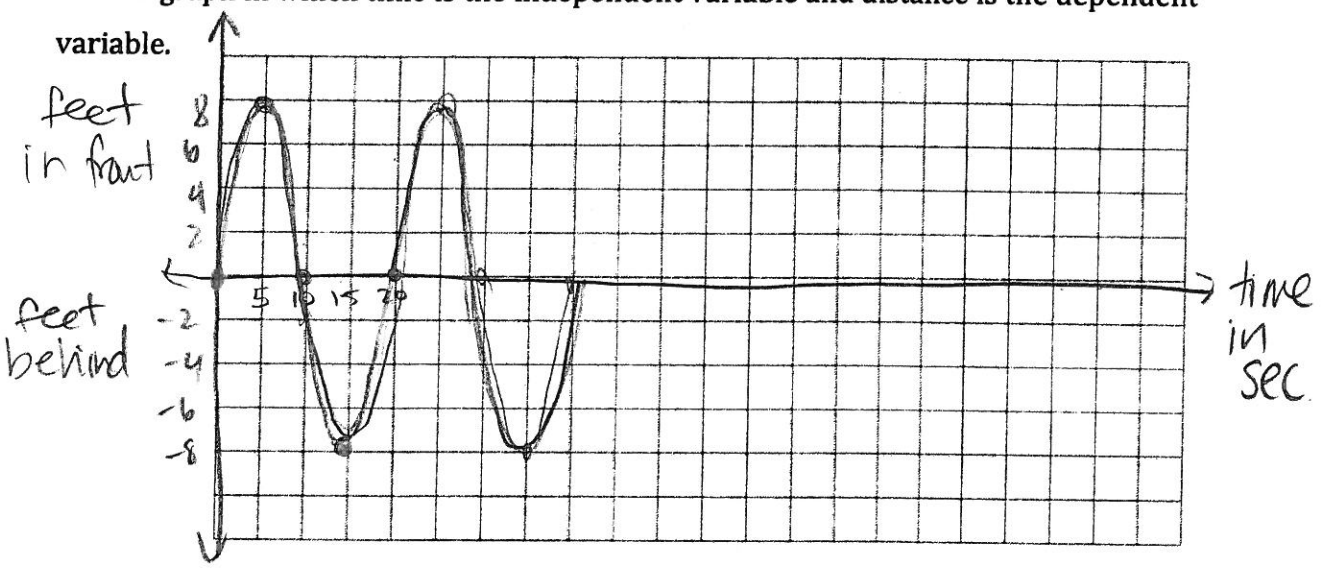
1) Jack is swinging on a swing in the park. Jill takes out her stopwatch and starts timing Jack's swing. When the watch reads 0 seconds, Jack is in the middle of the swinging path. When the watch reads 5 seconds, Jack is at the farthest point forward, 8 feet in front of the center point of the swing. Let t be the time on Jill's stopwatch and d be the distance relative to the center point of the swing.

a. What is the period of the function that relates d and t?

20 seconds

$I = 5$

b. Draw a graph in which time is the independent variable and distance is the dependent variable.



c. Find an equation that relates Jack's distance from the center point of the swing versus time, the time since Jill started her stopwatch.

$\frac{360}{20} = 18$

$d(t) = 8\sin(18t)$

d. Use your calculator to figure out what d is at times:

i. $t = 1$ 2.472

ii. $t = 8$ 4.702

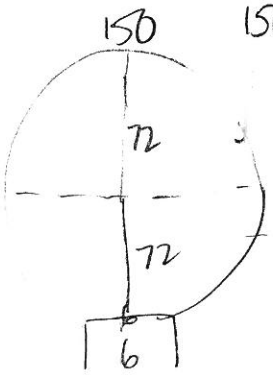
iii. $t = 11$ -2.472

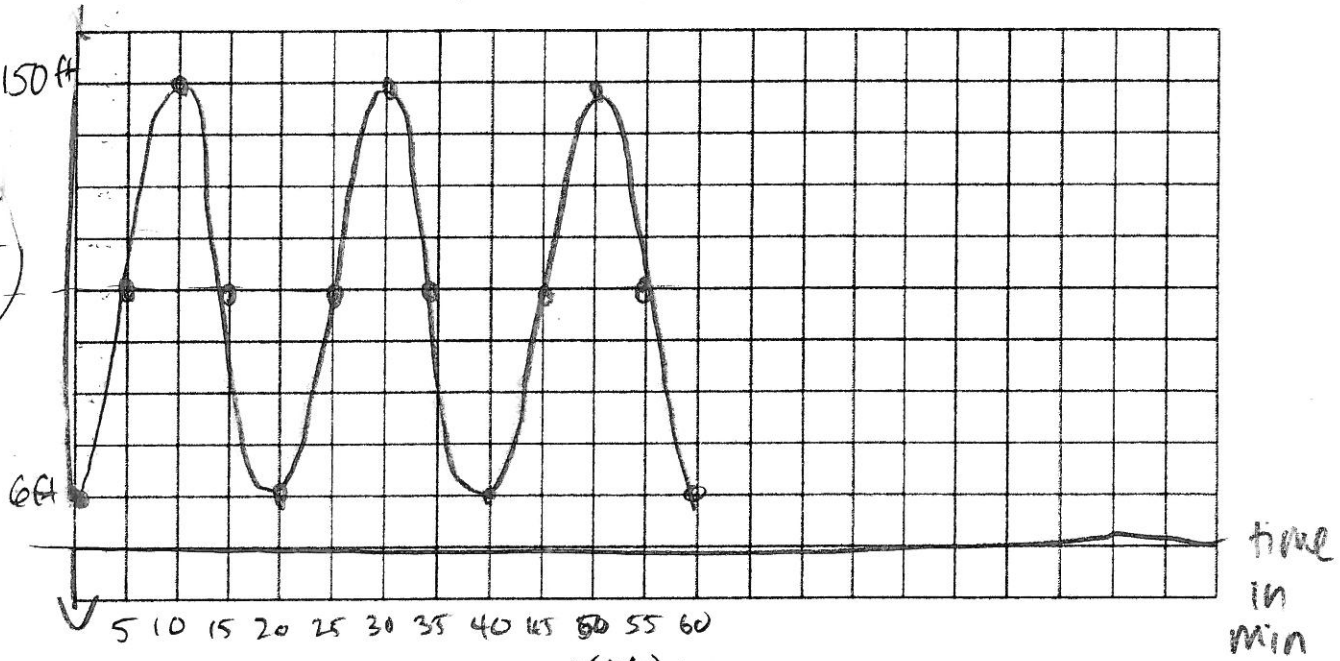
e. Using your calculator, what are the first three times after Jill started the stopwatch that Jack is 7 feet behind the center point of the swing?

≈ 13.5 sec, 16.5 sec, ≈ 23.5 sec

2) Deno's Wonder Wheel, a famous Ferris wheel at Coney Island in New York is 150 feet tall. It has a radius of 72 feet and therefore the lowest point of the wheel is 6 feet from the ground. When running at full speed, the wheel completes 3 revolutions every minute. Charlie is watching his friends Linus and Lucy riding on the wheel. He starts his stopwatch when Linus and Lucy are at their highest point above the ground. Let h be the height of the friends from the ground at time t seconds after Charlie starts his stopwatch.

a. Sketch a graph of at least two periods of the function that relates the height of the friends from the ground vs. time. (optional)





b. Write the equation that relates the height of the friends vs. time. $h(t) = -72\cos(18t) + 78$

$\frac{360}{20} =$

c. When are the first three times that the wheel is 6 feet off the ground?

0 min, 20 min, 40 min

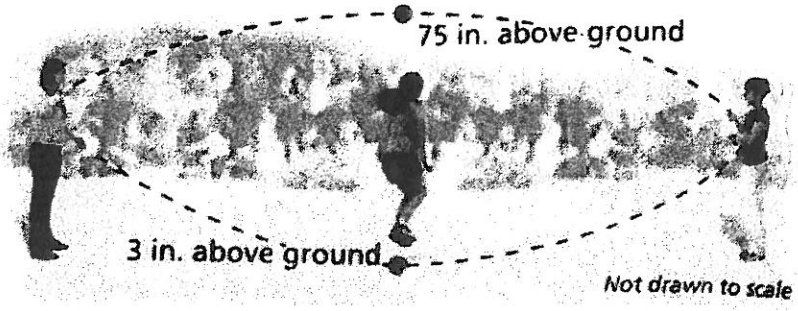
~~d.~~ How many times within the first two revolutions will the friends be 100 feet off the ground?

4 times

~~e.~~ Using your calculator, determine the time within the first two revolutions the friends will be 100 feet off the ground.

$\approx$ 6 min, 14 min, 26 min, 34 min

3) Two people swing jump ropes, as shown in the diagram. The highest point of the middle of each rope is 75 inches above the ground, and the lowest point is 3 inches. The rope makes 2 revolutions per second.

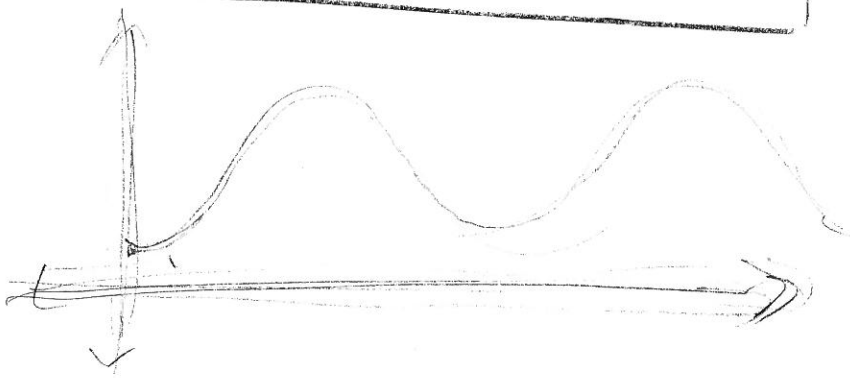


a. Write a model for the height h (in inches) of a rope as a function of the time t (in seconds) given that the rope is at its lowest point when $t = 0$.

$h(t) = -36\cos(720x) + 39$

per: 1/2 sec

$\frac{360}{1/2} = 720$



b. Describe how the model would change when the lowest point of a rope is 5 inches above the ground and the highest point is 70 inches above the ground.

$36\begin{pmatrix} 75 & 70 \\ 39 & 37.5 \end{pmatrix}32.5$

3 5

Amplitude decreases

Sinusoidal axis shifts down

$h(t) = -32.5\cos(720x) + 37.5$

c. Order the average rates of change for the function from part (a) from least to greatest.

① $0 \le x \le 0.5$

3 3

$\frac{0}{.5} = 0$

② $0.75 \le x \le 0.9$

75 27.87

$\frac{47.13}{0.15} = 314.2$

① $0.4 \le x \le 0.6$

27.87 27.87

$\frac{0}{.2} = 0$

③ $0.5 \le x \le 0.6$

75 27.87

$\frac{47.13}{.1} = 471.3$