

6-1

Roots and Radical Expressions



Vocabulary

Review

1. Circle the *exponents* in each expression.

21^7 $-3t^2$ $4a^7b^{12}$ $\frac{-3x^5}{4y^0}$

2. Underline the expression that is read *five to the second power*.

2^5 5^2 5×2 2×5

3. List the correct numbers or letters described by the vocabulary words for the expression $-4x^{54} + 2r^2 - 5s^5 + 7w^3$ in the correct space.

Exponents: 54, 2, 5, 3 Coefficients: -4, 2, -5, 7 Bases: x, r, s, w

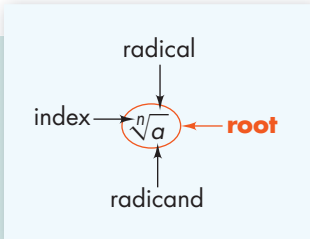
Vocabulary Builder

root (noun) **root**

Related Words: square root, cube root, *n*th root, power, radical, index, radicand

Definition: The *n*th root of a given number is a specific number that when it is used as a factor *n* times, equals the given number.

Using Symbols: $2 \times 2 \times 2 = 8$, so $\sqrt[3]{8} = 2$.



Use Your Vocabulary

4. Draw a line from the number or symbol in Column A to each term in Column B that best describes a part of $\sqrt[4]{81} = 3$.

Column A	Column B
4	radical
81	root
$\sqrt{\quad}$	index
3	radicand

Note: Lines connect 4 to index, 81 to radical, $\sqrt{\quad}$ to root, and 3 to radicand.

Key Concept The n th Root

If $a^n = b$, with a and b real numbers and n a positive integer, then a is an n th root of b .

If n is odd...

there is one real n th root of b , denoted in radical form as $\sqrt[n]{b}$.

If n is even...

- and b is positive, there are two real n th roots of b .
The positive root is the **principal root** (or principal n th root). Its symbol is $\sqrt[n]{b}$. The negative root is its opposite, or $-\sqrt[n]{b}$.
- and b is negative, there are no real n th roots of b .

The only n th root of 0 is 0.

5. Underline the correct words to complete the sentence.

Since the index of $\sqrt[4]{81}$ is even / odd and the radicand is negative / positive, there are two real fourth roots / no real fourth roots of 81.



Problem 1 Finding All Real Roots

Got It? What are the real fifth roots of 0, -1 , and 32?

6. Cross out the question that will NOT help you find the roots.

What number is the index?

Is the radicand negative or positive?

~~What number times 5 equals -1 ?~~

How many real roots of 0 are there?

7. Complete the equation to find the fifth root of 0.

$$0 \times 0 \times 0 \times 0 \times 0 = 0$$

8. The fifth root of 0 is 0.

9. Complete the equation to find the fifth root of -1 .

$$-1 \times -1 \times -1 \times -1 \times -1 = -1$$

10. The fifth root of -1 is -1 .

11. Complete the equation to find the fifth root of 32.

$$2 \times 2 \times 2 \times 2 \times 2 = 32$$

12. The fifth root of 32 is 2.



Problem 2 Finding Roots

Got It? What is each real-number root?

$$\sqrt[3]{-27}$$

$$\sqrt[4]{-81}$$

$$\sqrt{(-7)^2}$$

$$\sqrt{-49}$$

13. Circle the true equation.

$(-2)^3 = 27$

$(-3)^3 = 27$

$(-3)^3 = -27$

Underline the correct words to complete each sentence.

14. The index of $\sqrt[4]{-81}$ is even / odd and the radicand is positive / negative, so there is one real root / are no real roots.

15. The index of $\sqrt{-49}$ is even / odd, and the radicand is positive / negative, so there is one real root / are no real roots.

16. What is the principal square root of the square of a number?

Answers may vary. Sample: The principal square root of the square of a number is the absolute value of the original number.

17. Simplify each root.

$\sqrt[3]{-27}$

-3

$\sqrt[4]{-81}$

no real roots

$\sqrt{(-7)^2}$

7

$\sqrt{-49}$

no real roots

Take note

Key Concept n th Roots of n th Powers

For any real number a , $\sqrt[n]{a^n} = \begin{cases} a & \text{if } n \text{ is odd} \\ |a| & \text{if } n \text{ is even} \end{cases}$.

18. The index of $\sqrt[3]{(-2)^3}$ is even / odd, so $\sqrt[3]{(-2)^3} = -2$.



Problem 3 Simplifying Radical Expressions

Got It? What is the simplified form of the radical expression $\sqrt{81x^4}$?

19. Complete the equation to simplify the radical expression.

$$\begin{aligned}\sqrt{81x^4} &= \sqrt{9^2(\underline{x^2})^2} \\ &= 9 \underline{x^2}\end{aligned}$$



Problem 4 Using a Radical Expression

Got It? Some teachers adjust test scores when a test is difficult. One teacher's formula for adjusting scores is $A = 10\sqrt{R}$, where A is the adjusted score and R is the raw score. What are the adjusted scores for raw scores of 0 and 100?

20. Circle the first thing you should do to solve this problem.

Evaluate $A = 10\sqrt{R}$ for $R = 0$ and $R = 100$.

Solve $A = 10\sqrt{R}$ for R .

Underline the correct number to complete each sentence.

21. The lowest possible *raw score* is 0 / 10 / 15 / 100 .

The lowest possible *adjusted score* is 0 / 10 / 15 / 100 .

22. The highest possible *raw score* is 0 / 10 / 15 / 100 .

The highest possible *adjusted score* is 0 / 10 / 15 / 100 .

23. So, the curved scores for raw scores of 0 and 100 are 0 and 100 .



Lesson Check • Do you UNDERSTAND?

Error Analysis A student said *the only fourth root of 16 is 2*. Describe and correct his error.

24. If a is a fourth root of 16, which statement is true?

(A) $a \cdot a \cdot a \cdot a = 16$ (B) $a = 16^4$ (C) $4a = 16$ (D) $a = \frac{16}{4}$

25. Look at the statement you circled in Exercise 24. Circle the values of a below that make the statement true.

-64 -4 -2 2 4 64

26. There are 1 / 2 / 3 / 4 values of a that satisfy the statement you circled in Exercise 24.

Therefore, the number of fourth roots of 16 is 1 / 2 / 3 / 4 .

27. Describe the error the student made.

Answers may vary. Sample: The student may have forgotten that when n is even and b is positive, there are two n th roots of b , one positive, and one negative.

28. Complete the sentence to correct the error the student made.

The fourth root of 16 is 2 or -2 .

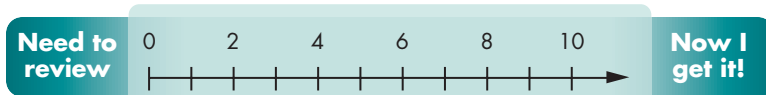


Math Success

Check off the vocabulary words that you understand.

n th root principal root radicand index

Rate how well you can *find n th roots*.



6-2

Multiplying and Dividing Radical Expressions



Vocabulary

Review

Write T for *true* or F for *false*.

T

1. All mathematical expressions can be written as an equivalent expression with a *denominator* of 1.

F

2. An expression can have a *denominator* equal to zero.

T

3. The expression above the fraction bar is the *numerator*.

T

4. Multiplying both the *numerator* and the *denominator* by the same nonzero number results in an equivalent fraction.

Circle the *numerator* and underline the *denominator* in each expression.

5. $\frac{5}{6}$

6. $\frac{-5r^2}{16}$

7. $\frac{r^2 + s^2}{c - 16}$

Vocabulary Builder

combine (verb) kum BYN

Main Idea: Combine means to put things together or to get a total.

Math Usage: To combine means to put together or add two like terms to get one term.

Example: The like terms $-2x^3$ and $7x^3$ can be **combined** to get $5x^3$.

Use Your Vocabulary

8. Circle the expression that shows the like terms in $3x^2 + 1 + 4x^2 - 5$ combined.

$4x^2 - 1x^2$

$7x^2 - 4$

$3x^2 - 4x^2 + 1 - 5$

Take note

Property Combining Radical Expressions: Products

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, then $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$.

Write T for *true* or F for *false*.

F 9. $(\sqrt[3]{4})(\sqrt[3]{6}) = (\sqrt[4]{24})$

T 10. $(\sqrt[3]{36}) = (\sqrt[3]{4})(\sqrt[3]{9})$

T 11. $(\sqrt[4]{3})(\sqrt[4]{8}) = (\sqrt[4]{24})$



Problem 1 Multiplying Radical Expressions

Got It? Can you simplify the product of the radical expression? Explain.

$$\sqrt[4]{7} \cdot \sqrt[5]{7}$$

$$\sqrt[5]{-5} \cdot \sqrt[5]{-2}$$

12. Circle the indexes in $\sqrt[4]{7} \cdot \sqrt[5]{7}$.

13. Circle the indexes in $\sqrt[5]{-5} \cdot \sqrt[5]{-2}$.

14. The indexes above are the same / different.

15. The indexes above are the same / different.

16. Can you simplify the product? Yes / No

17. Can you simplify the product? Yes / No

If yes, write the product.

If yes, write the product.

$$\sqrt[5]{10}$$



Problem 2 Simplifying a Radical Expression

Got It? What is the simplest form of $\sqrt[3]{128x^7}$? (*Hint: Remember $\sqrt[3]{b^3} = b$.*)

18. The index of $\sqrt[3]{128x^7}$ is 3 / 7 / 128.

19. Circle the perfect cube factors.

$$\sqrt[3]{128x^7} = \sqrt[3]{4^3 \cdot 4 \cdot x^3 \cdot x^3 \cdot x}$$

20. The simplest form of $\sqrt[3]{128x^7}$ is $4x^2\sqrt[3]{2x}$.



Problem 3 Simplifying a Product

Got It? What is the simplest form of $\sqrt{45x^5y^3} \cdot \sqrt{35xy^4}$?

21. Complete the steps to simplify $\sqrt{45x^5y^3} \cdot \sqrt{35xy^4}$.

$$\begin{aligned} \sqrt{45x^5y^3} \cdot \sqrt{35xy^4} &= \sqrt{(45x^5y^3) \cdot (35xy^4)} \\ &= \sqrt{(45 \cdot 35)(x^5y^3 \cdot xy^4)} \\ &= \sqrt{(9 \cdot 5 \cdot 5 \cdot 7)(x^6 \cdot y^7)} \\ &= \sqrt{(3^2 \cdot 5^2 \cdot 7)(x^3)^2 \cdot (y^3)^2 \cdot y} \\ &= 3 \cdot 5 \cdot x^3 \cdot y^3 \cdot \sqrt{7y} \\ &= 15x^3y^3 \sqrt{7y} \end{aligned}$$

Property Combining Radical Expressions: Quotients

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers and $b \neq 0$, then $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$.

22. Multiple Choice Which expression is equivalent to $\frac{\sqrt[3]{7}}{\sqrt[3]{4x^2}}$?

- (A) $(2\sqrt[3]{1})(x\sqrt[3]{1})$ (B) $x\sqrt[3]{\frac{7}{4}}$ (C) $\sqrt[3]{\frac{7}{4x^2}}$ (D) $21x^2$



Problem 4 Dividing Radical Expressions

Got It? What is the simplest form of $\frac{(\sqrt{50x^6})}{(\sqrt{2x^4})}$?

23. Complete the expression.

$$\frac{(\sqrt{50x^6})}{(\sqrt{2x^4})} = \sqrt{\frac{50x^6}{2x^4}}$$

Use the Combining Radicals Property for Quotients.

$$= \sqrt{25x^2}$$

Simplify under the radical sign.

$$= 5|x|$$

Simplify the square root.



Problem 5 Rationalizing the Denominator

Got It? What is the simplest form of $\frac{\sqrt[3]{7x}}{\sqrt[3]{5y^2}}$?

24. The radicand in the *denominator* needs a 5^2 and a y to make $5y^2$ a perfect cube.

25. You will need to multiply *both* the numerator and the denominator by the expression $\sqrt[3]{5^2y}$ to rationalize the denominator.

26. Complete to show the rationalization of the denominator.

$$\frac{\sqrt[3]{7x}}{\sqrt[3]{5y^2}} = \frac{\sqrt[3]{7x}}{\sqrt[3]{5y^2}} \cdot \frac{\sqrt[3]{5^2y}}{\sqrt[3]{5^2y}}$$

Rationalize the denominator.

$$= \frac{\sqrt[3]{175xy}}{\sqrt[3]{5^3 \cdot y^3}}$$

Multiply.

$$= \frac{\sqrt[3]{175xy}}{5 \cdot y}$$

Find the cube root of the denominator.

$$= \frac{\sqrt[3]{175xy}}{5y}$$

Simplify.

6-3

Binomial Radical Expressions



Vocabulary

● Review

Circle the *like terms* in each group.

1. $3y^2$ $2y$ $2y^2$ 2. b bc $4bc$ c 3. 5 18 $5a$

● Vocabulary Builder

binomial (adjective) by NOH mee ul

Definition: A **binomial** expression is an expression made up of two terms.

Related Words: monomial, binomial expression, trinomial

Examples: monomial: $a, x^2, -3, 17c^3, \sqrt{5}$

binomial: $a - 7, x^2 + 0.9, -3 - ab, 17c^3 + 1, b - \sqrt{5}$

trinomial: $a - 7 + x, x^2 + x + 0.9, -3 - ab + a, 17c^3 = c^2 + 1, b^3 + b - \sqrt{5}$

● Use Your Vocabulary

Write **M** if the expression is a *monomial*, **B** if the expression is a *binomial*, or **T** if the expression is a *trinomial*.

T 4. $37 - 100x^3y + r$

M 5. $-57t^6$

B 6. $s + 0.91r$

T 7. $18a - 1.4b^7 + 3.85c^{14}$

take note

Property Combining Radical Expressions: Sums and Differences

Use the Distributive Property to add or subtract like radicals.

$$a\sqrt[n]{x} + b\sqrt[n]{x} = (a + b)\sqrt[n]{x} \qquad a\sqrt[n]{x} - b\sqrt[n]{x} = (a - b)\sqrt[n]{x}$$

8. Underline the words that make each sentence true.

To be like radicals, their indexes must be the same / different, and their radicands must be the same / different.

To add or subtract two like radicals, you add or subtract their radicands / coefficients.



Problem 1 Adding and Subtracting Radical Expressions

Got It? What is the simplified form of each expression?

$$7\sqrt[3]{5} - 4\sqrt{5}$$

$$3x\sqrt{xy} + 4x\sqrt{xy}$$

9. Are the radicals in $7\sqrt[3]{5} - 4\sqrt{5}$ like radicals?

Yes / No

10. Are the radicals in $3x\sqrt{xy} + 4x\sqrt{xy}$ like radicals?

Yes / No

11. Is $7\sqrt[3]{5} - 4\sqrt{5}$ simplified?

Yes / No

12. Is $3x\sqrt{xy} + 4x\sqrt{xy}$ simplified?

Yes / No

13. Write the simplified form of each expression.

$$7\sqrt[3]{5} - 4\sqrt{5}$$

$$3x\sqrt{xy} + 4x\sqrt{xy}$$

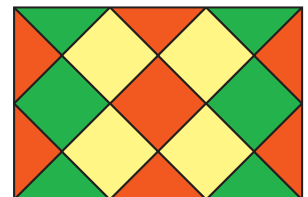
$$7\sqrt[3]{5} - 4\sqrt{5}$$

$$7x\sqrt{xy}$$



Problem 2 Using Radical Expressions

Got It? In the stained-glass window design, the side of each small square is 6 in. Find the perimeter of the window to the nearest tenth of an inch.



14. The **length** of the window is made up of the diagonals / sides of three squares.

15. The **width** of the window is made up of the diagonals / sides of two squares.

16. **Multiple Choice** Which is the length of the diagonal of a square with side s ?

(A) $2s$

(B) $\sqrt{2}s$

(C) $s\sqrt{3}$

(D) $s\sqrt{2}$

17. Write the length of the diagonal of a square with side 6 in.

$$6\sqrt{2}$$

18. Complete the following to find the **length** and the **width** of the window.

Length of the window:

$$l = 3(6\sqrt{2}) = 18\sqrt{2}$$

Width of the window:

$$w = 2(6\sqrt{2}) = 12\sqrt{2}$$

19. Complete the steps to find the perimeter of the window.

$$\text{Perimeter} = 2\ell + 2w$$

$$= 2(18\sqrt{2}) + 2(12\sqrt{2})$$

Substitute for length and width.

$$= 36\sqrt{2} + 24\sqrt{2}$$

Simplify.

$$= 60\sqrt{2}$$

Add the coefficients of the like radicals.

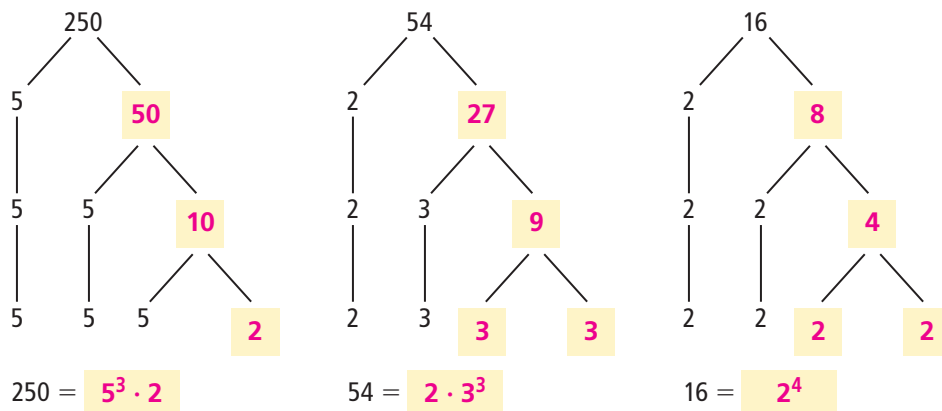
$$= 84.9$$

Use a calculator to approximate to the nearest tenth.

Problem 3 Simplifying Before Adding or Subtracting

Got It? What is the simplified form of the expression $\sqrt[3]{250} + \sqrt[3]{54} - \sqrt[3]{16}$?

20. Complete each factor tree to factor each radicand.



21. Complete the following by substituting the prime factorizations for 250, 54, and 16.

$$\sqrt[3]{250} + \sqrt[3]{54} - \sqrt[3]{16} = \sqrt[3]{5^3 \times 2} + \sqrt[3]{2 \cdot 3^3} - \sqrt[3]{2^4}$$

22. Circle the simplified form of the expression $\sqrt[3]{250} + \sqrt[3]{54} - \sqrt[3]{16}$.

$5\sqrt[3]{2} + 3\sqrt[3]{3} - 2\sqrt[3]{2}$ $6\sqrt[3]{2}$ $3\sqrt[3]{2} + 3\sqrt[3]{3}$

Problem 4 Multiplying Binomial Radical Expressions

Got It? What is the product $(3 + 2\sqrt{5})(2 + 4\sqrt{5})$?

23. Circle the expression that shows the FOIL method.

$(3 + 2\sqrt{5})(2 + 4\sqrt{5})$ $(3 + 2\sqrt{5})(2 + 4\sqrt{5})$
 $3 \cdot 2 + 3 \cdot 4\sqrt{5} + 2\sqrt{5} \cdot 2 + 2\sqrt{5} \cdot 4\sqrt{5}$ $3 \cdot 2 + 3 \cdot 4\sqrt{5} + 2\sqrt{5} \cdot 4\sqrt{5}$

24. The product $(3 + 2\sqrt{5})(2 + 4\sqrt{5}) = 46 + 16\sqrt{5}$.



Problem 5 Multiplying Conjugates

Got It? What is the product of the expression $(6 - \sqrt{12})(6 + \sqrt{12})$?

25. Use the FOIL method to find the product.

$$\begin{aligned}
 (6 - \sqrt{12})(6 + \sqrt{12}) &= 6 \cdot 6 + 6 \cdot \sqrt{12} - \sqrt{12} \cdot 6 + (-\sqrt{12}) \cdot \sqrt{12} \\
 &= 36 - 12 \\
 &= 24
 \end{aligned}$$



Problem 6 Rationalizing the Denominator

Got It? How can you write the expression $\frac{2\sqrt{7}}{\sqrt{3} - \sqrt{5}}$ with a rationalized denominator?

26. Circle the conjugate of the denominator. $\sqrt{5} - \sqrt{3} / (\sqrt{3} + \sqrt{5})$

27. Use the conjugate of the denominator to write $\frac{2\sqrt{7}}{\sqrt{3} - \sqrt{5}}$ with a rational denominator.

$$\frac{2\sqrt{7}}{\sqrt{3} - \sqrt{5}} = \frac{2\sqrt{7}}{\sqrt{3} - \sqrt{5}} \cdot \frac{\sqrt{3} + \sqrt{5}}{\sqrt{3} + \sqrt{5}} = \frac{2\sqrt{7}(\sqrt{3} + \sqrt{5})}{(\sqrt{3})^2 - (\sqrt{5})^2} = \frac{2\sqrt{21} + 2\sqrt{35}}{3 - 5} = \frac{2\sqrt{21} + 2\sqrt{35}}{-2} = -\sqrt{21} - \sqrt{35}$$



Lesson Check • Do you UNDERSTAND?

Vocabulary Determine whether each of the following is a pair of like radicals. If so, combine them.

$3x\sqrt{11}$ and $3x\sqrt{10}$

$2\sqrt{3xy}$ and $7\sqrt{3xy}$

$12\sqrt{13y}$ and $12\sqrt{6y}$

28. Cross out the pairs that do NOT have the same index and the same radicand.

~~$3x\sqrt{11}$ and $3x\sqrt{10}$~~
 $2\sqrt{3xy}$ and $7\sqrt{3xy}$
 ~~$12\sqrt{13y}$ and $12\sqrt{6y}$~~

29. The sum of the like radicals is $9\sqrt{3xy}$.



Math Success

Check off the vocabulary words that you understand.

like radicals

binomial radical expressions

Rate how well you can *add and subtract radical expressions*.



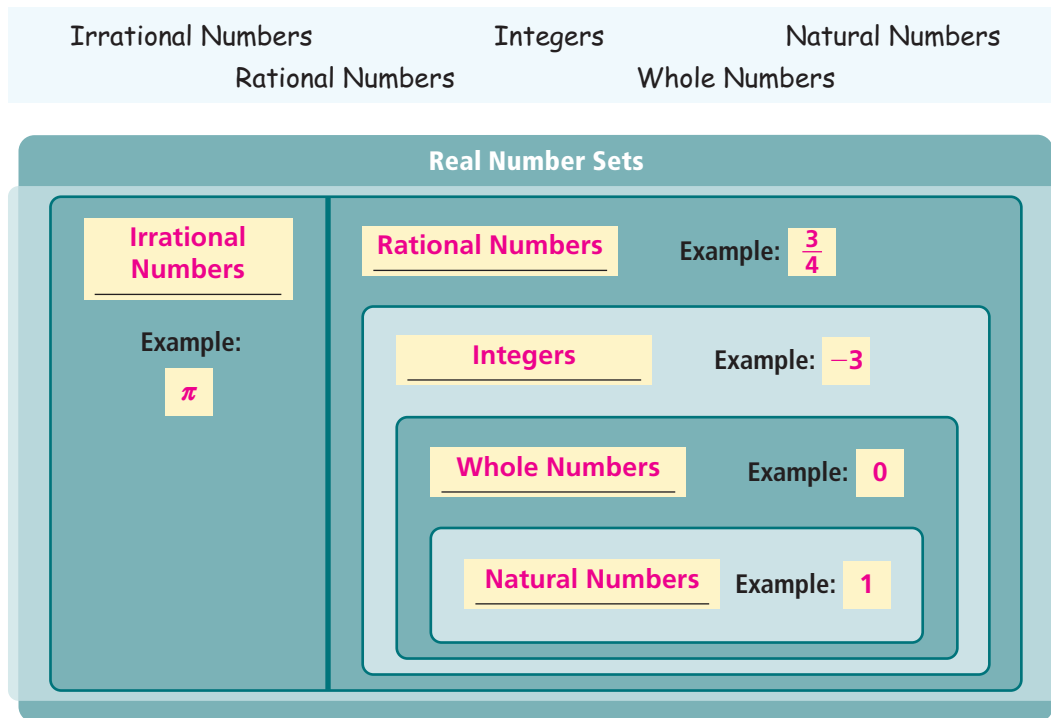


Vocabulary

Review

Examples may vary. Samples are given.

1. Use the terms in the box to complete the diagram. Then, give examples.



Vocabulary Builder

convert (verb) kun VURT

Related Words: conversion (noun), convertible (adjective)

Definition: To **convert** is to rewrite or change to another form.

Use Your Vocabulary

2. Circle the expression that shows $\frac{3}{4}$ converted to a decimal.

1.33

0.75

0.34

Complete each sentence with the correct form of the word *convert*.

- NOUN Travelers to Europe calculate the ? of dollars to euros.
- ADJECTIVE A ? sofa is a couch by day and a bed by night.
- VERB To ? fractions to decimals, divide the numerator by the denominator.

conversion

convertible

convert



Problem 1 Simplifying Expressions With Rational Exponents

Got It? What is the simplified form of the expression $64^{\frac{1}{2}}$?

- Circle the expression that shows $64^{\frac{1}{2}}$ rewritten as a radical. (Hint: $x^{\frac{1}{n}} = \sqrt[n]{x}$)

$\sqrt{64}$

$\sqrt[2]{64}$

$\frac{1}{2}\sqrt{64}$

- The simplified form of $64^{\frac{1}{2}}$ is **8**.

Take note

Key Concept Rational Exponent

If the n th root of a is a real number and m is an integer, then
 $a^{\frac{1}{n}} = \sqrt[n]{a}$ and $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$. If m is negative, $a \neq 0$.

- Draw a line from each expression in Column A to its exponent form in Column B.

Column A

Column B

$4\sqrt[3]{x}$ ————— $4x^{\frac{1}{3}}$

$\sqrt[4]{x^3}$ ————— $x^{\frac{3}{4}}$



Problem 2 Converting Between Exponential and Radical Form

Got It? What are the expressions $w^{-\frac{5}{8}}$ and $w^{0.2}$ in radical form?

- Is the exponent in $w^{-\frac{5}{8}}$ in lowest terms? **Yes**/No

- Identify the values of a , m , and n for $w^{-\frac{5}{8}}$ using the rule $a^{\frac{m}{n}} = \sqrt[n]{a^m}$.

$a = w$

$m = -5$

$n = 8$

- Use your values for a , m , and n to write $w^{-\frac{5}{8}}$ in radical form.

$\sqrt[8]{w^{-5}}$

- Convert $w^{0.2}$ to w raised to a power that is a fraction in lowest terms.

$w^{\frac{1}{5}}$

- Identify the values of a , m , and n for $w^{0.2}$ using the rule $a^{\frac{m}{n}} = \sqrt[n]{a^m}$.

$a = w$

$m = 1$

$n = 5$

- Use your values for a , m , and n to write $w^{0.2}$ in radical form.

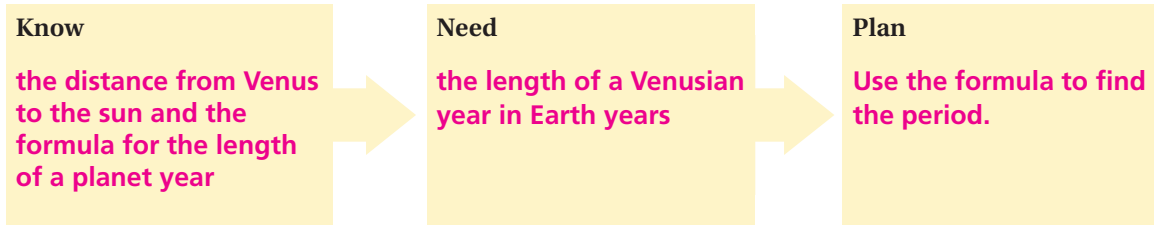
$\sqrt[5]{w}$



Problem 3 Using Rational Exponents

Got It? Kepler's third law of orbital motion states that you can approximate the period P (in Earth years) it takes a planet to complete one orbit of the sun using the function $P = d^{\frac{3}{2}}$, where d is the distance (in astronomical units, AU) from the planet to the sun. Find the approximate length (in Earth years) of a Venusian year if Venus is 0.72 AU from the sun.

15. Complete the problem-solving model below.



16. Use the formula to find the length of a Venusian year.

$$P = d^{\frac{3}{2}}$$

$$P = (0.72)^{\frac{3}{2}}$$

$$P \approx 0.610940259$$



Key Concept Properties of Rational Exponents

Let m and n represent rational numbers. Assume that no denominator equals 0.

$$a^m \cdot a^n = a^{m+n}$$

$$(a^m)^n = a^{mn}$$

$$(ab)^m = a^m b^m$$

$$a^{-m} = \frac{1}{a^m}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

Complete each example below.

17. $g^3 \cdot g^4 = g^{3+4} = g^7$

18. $(h^3)^4 = h^{3 \cdot 4} = h^{12}$

19. $(5y)^2 = 5^2 \cdot y^2$

20. $2^{-6} = \frac{1}{2^6} = \frac{1}{64}$

21. $\frac{d^8}{d^3} = d^{8-3} = d^5$

22. $\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$



Problem 4 Combining Radicals With Like Radicands

Got It? What is $\sqrt{3}(\sqrt[4]{3})$ in simplest form?

23. Convert $\sqrt{3}(\sqrt[4]{3})$ to exponential form.

$$\sqrt{3}(\sqrt[4]{3}) = 3^{\frac{1}{2}} \cdot 3^{\frac{1}{4}}$$

24. The bases of the factors are the same / different, so the Property you should use to simplify the exponential form is $(ab)^m = a^m b^m / a^m \cdot a^n = a^{m+n}$.

25. In simplest form, $\sqrt{3}(\sqrt[4]{3}) = 3^{\frac{3}{4}}$.



Problem 5 Simplifying Numbers With Rational Exponents

Got It? What is $32^{-\frac{1}{3}}$ in simplest form?

26. Solve using two different methods. Complete each method.

Method 1

$$\begin{aligned}
 32^{-\frac{1}{3}} &= (2^5)^{-\frac{1}{3}} \\
 &= (2^5)^{-\frac{1}{3}} \\
 &= (2^{-3}) \\
 &= \frac{1}{8}
 \end{aligned}$$

Method 2

$$\begin{aligned}
 32^{-\frac{1}{3}} &= \frac{1}{32^{\frac{1}{3}}} \\
 &= \frac{1}{(\sqrt[3]{32})^3} \\
 &= \frac{1}{(2)^3} \\
 &= \frac{1}{8}
 \end{aligned}$$



Lesson Check • Do you UNDERSTAND?

Error Analysis Explain why this simplification is incorrect.

27. The radical form of $5^{\frac{1}{2}}$ is $\sqrt{5}$.

The radical form of $5(5^{\frac{1}{2}})$ is $5\sqrt{5}$.

The radical form of $25^{\frac{1}{2}}$ is $\sqrt{25}$.

28. Explain why the simplification shown is incorrect.

$$\begin{aligned}
 &\cancel{5(4 - 5^{\frac{1}{2}})} \\
 &\cancel{5(4) - 5(5^{\frac{1}{2}})} \\
 &\cancel{20 - 25^{\frac{1}{2}}} \\
 &\quad 15
 \end{aligned}$$

Answers may vary. Sample: The student incorrectly simplified $5(5^{\frac{1}{2}})$.

The answer should be $20 - 5\sqrt{5}$.



Math Success

Check off the vocabulary words that you understand.

rational exponent exponential form radical form

Rate how well you can *simplify expressions with rational exponents*.



6-5

Solving Square Root and Other Radical Equations



Vocabulary

Review

1. Circle the *equation* that is equivalent to $\sqrt{x + 1} = 5$.

$\sqrt{x + 1} = 10$ $\sqrt{x + 2} = 6$ $\sqrt{x + 1} + 3 = 8$

2. Draw a line from each *equation* in Column A to its *solution* in Column B.

Column A	Column B
$3 + x = 5$	-10
$y - 4 = -6$	10
$2z = 20$	2
$\frac{w}{5} = -2$	-2

Circle the *radicand* in each expression.

3. $\sqrt{2x - 3}$

4. $7 - \sqrt{wz + 2}$

5. $\sqrt{y} + 7$

Vocabulary Builder

reciprocal (noun) rih SIP ruh kul

Related Term: multiplicative inverse

Definition: Two numbers are **reciprocals** if their product is 1.

Main Idea: To write the **reciprocal** of a fraction, switch the numerator and the denominator.

Examples: 3 and $\frac{1}{3}$, $-\frac{4}{7}$ and $-\frac{7}{4}$, $\frac{1}{6}$ and 6

Reciprocals

$$\frac{a}{b} \text{ and } \frac{b}{a}$$

where $a \neq 0$ and $b \neq 0$

Use Your Vocabulary

Write T for *true* or F for *false*.

T 6. The *reciprocal* of -1 is itself.

F 7. A decimal has no *reciprocal*.

T 8. The reciprocal of a negative number is negative.

T 9. The only real number without a reciprocal is 0.

Solving a square root equation may require that you square each side of the equation. This can introduce extraneous solutions.



Problem 1 Solving a Square Root Equation

Got It? What is the solution of $\sqrt{4x + 1} - 5 = 0$?

10. Circle the first step in solving the equation.

Isolate the square root.

Square each side.

11. Underline the correct word to complete each justification.

$$\sqrt{4x + 1} = 5$$

Isolate the square root / variable.

$$4x + 1 = 25$$

Take the square root / square of each side to remove the radical.

$$4x = 24$$

Subtract 1 to isolate the radical / variable term.

$$x = 6$$

Divide / Multiply by 4 to solve for x .



Problem 2 Solving Other Radical Equations

Got It? What are the solutions of $2(x + 3)^{\frac{2}{3}} = 8$?

12. Complete each step to find the solution.

1 Isolate the radical term.

$$2(x + 3)^{\frac{2}{3}} = 8$$

$$(x + 3)^{\frac{2}{3}} = 4$$

2 Raise each side of the equation to the reciprocal power.

$$\left((x + 3)^{\frac{2}{3}}\right)^{\frac{3}{2}} = 4^{\frac{3}{2}}$$

3 Solve for x .

$$|x + 3| = 8 \quad \text{Since the numerator of } \frac{2}{3} \text{ is even, } \left((x + 3)^{\frac{2}{3}}\right)^{\frac{3}{2}} = |x + 3|.$$

$$x + 3 = \pm 8$$

$$x = 5 \text{ or } x = -11$$



Problem 4 Checking for Extraneous Solutions

Got It? What is the solution of $\sqrt{5x - 1} + 3 = x$? Check your results.

13. Use the justifications at the right to complete each step.

$\sqrt{5x - 1} + 3 = x$	Write the original equation.
$\sqrt{5x - 1} = x - 3$	Isolate the radical.
$(\sqrt{5x - 1})^2 = (x - 3)^2$	Square each side of the equation.
$5x - 1 = x^2 + -6x + 9$	Simplify.
$0 = x^2 - 11x + 10$	Combine like terms.
$0 = (x - 1)(x - 10)$	Factor.
$x = 1$ or $x = 10$	Use the Zero-Product Property.

14. Substitute each value into the original equation to check the solutions.

$\sqrt{5x - 1} + 3 = x$	$\sqrt{5x - 1} + 3 = x$
$\sqrt{5(1) - 1} + 3 \stackrel{?}{=} 1$	$\sqrt{5(10) - 1} + 3 \stackrel{?}{=} 10$
$\sqrt{4} + 3 \stackrel{?}{=} 1$	$\sqrt{49} + 3 \stackrel{?}{=} 10$
$2 + 3 \stackrel{?}{=} 1$	$7 + 3 \stackrel{?}{=} 10$
$5 \neq 1$ false	$10 = 10$ ✓

15. The solution 1 is extraneous.

16. **Multiple Choice** What can cause an extraneous solution?

- (A) raising each side of the equation to an odd power
- (B) raising each side of the equation to an even power
- (C) adding the same number to each side of an equation
- (D) dividing each side of an equation by the same number

17. When should you check for extraneous solutions? Explain.

You should check for an extraneous solution any time you raise both sides of an equation to an even power.



Problem 5 Solving an Equation With Two Radicals

Got It? What is the solution of $\sqrt{5x + 4} - \sqrt{x} = 4$?

18. The equation has been solved below. Write the letter of the reason that justifies each step. Use the reasons in the box.

$$\sqrt{5x + 4} - \sqrt{x} = 4$$

F

$$\sqrt{5x + 4} = \sqrt{x} + 4$$

A

$$5x + 4 = (\sqrt{x} + 4)^2$$

B

$$5x + 4 = x + 8\sqrt{x} + 16$$

E

$$4x - 12 = 8\sqrt{x}$$

A

$$x - 3 = 2\sqrt{x}$$

C

$$(x - 3)^2 = 4x$$

B

$$x^2 - 6x + 9 = 4x$$

E

$$x^2 - 10x + 9 = 0$$

A

$$(x - 9)(x + 1)$$

D

$$x = 9 \text{ or } x = -1$$

G

- A Addition Property of Equality
 B Square each side.
 C Division Property of Equality
 D Factor
 E Simplify.
 F Original equation
 G Zero Product Property

19. Only the solution $x = -1 / x = 9$ satisfies the original equation.



Lesson Check • Do you UNDERSTAND?

Vocabulary Which value, 12 or 3, is an extraneous solution of $x - 6 = \sqrt{3x}$?

20. The solution $x = 12$ satisfies / does not satisfy the original equation.
 21. The solution $x = 3$ satisfies / does not satisfy the original equation.
 22. The solution $x = 12 / x = 3$ is an extraneous solution of $x - 6 = \sqrt{3x}$.

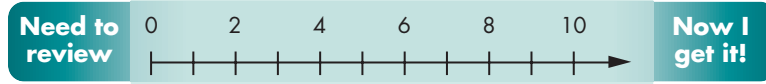


Math Success

Check off the vocabulary words that you understand.

- radical equation square root equation

Rate how well you can *solve square root and other radical equations*.





Vocabulary

Review

- In function notation, $gx / g(x) / x(g)$ is read g of x .
- Circle the equation that shows a *function rule*.

$$x + 17y = -4.7$$

$$f(x) = 14x - 0.3$$

$$15z(13t)$$

- The function rule $f(t) = 1.83t$ represents the cost of a number of tons of wheat t .

The number of tons of wheat is the input / output .

The output is the cost of the wheat / number of tons of wheat .

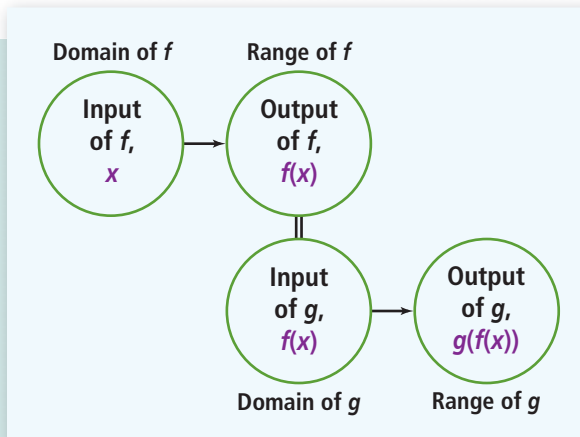
Vocabulary Builder

composite (adjective) kum PAHZ it

Related Words: composite function, composite number

Main Idea: Something that is **composite** is made up of more than one thing.

Math Usage: A **composite** function uses the output of one function as the input of a second function.



Use Your Vocabulary

Complete each sentence with the correct form of the word *composite*.

composite

composition

compose

4. VERB The musician worked to ? a new piece of music.

compose

5. ADJECTIVE A ? number has more than two factors.

composite

6. NOUN The poster was a ? of photos and famous quotes.

composition

Key Concept Function Operations

Addition	$(f + g)(x) = f(x) + g(x)$	Subtraction	$(f - g)(x) = f(x) - g(x)$
Multiplication	$(f \cdot g)(x) = f(x) \cdot g(x)$	Division	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$

The domains of the sum, difference, product, and quotient functions consist of the x -values that are in the domains of *both* f and g . Also, the domain of the quotient function does not contain any x -value for which $g(x) = 0$.

7. Let $f(x) = x^2 + 2$ and $g(x) = x - 6$. Complete the steps for finding $f(x) + g(x)$.

$$f(x) + g(x) = (x^2 + 2) + (x - 6) = x^2 + x - 4$$



Problem 1 Adding and Subtracting Functions

Got It? Let $f(x) = 2x^2 + 8$ and $g(x) = x - 3$. What are $f + g$ and $f - g$? What are their domains?

8. Circle $f + g$.

$$2x^2 + x + 8 \quad (2x^2 + x + 5) \quad x^2 + 5 \quad x^2 + 8$$

9. Circle $f - g$.

$$(2x^2 - x + 11) \quad 2x^2 - x + 5 \quad x^2 + 5 \quad x^2 + 11$$

10. Write the domain of f .

all real numbers

11. Write the domain of g .

all real numbers

12. The domain of both $f + g$ and $f - g$ is all real numbers.



Problem 2 Multiplying and Dividing Functions

Got It? Let $f(x) = 3x^2 - 11x - 4$ and $g(x) = 3x + 1$. What are $f \cdot g$ and $\frac{f}{g}$ and their domains?

13. Circle $f \cdot g$.

$$3x^2 - 11x - 4 + 3x + 1 \quad (3x^2 - 11x - 4)(3x + 1) \quad \frac{3x^2 - 11x - 4}{3x + 1}$$

14. Circle $\frac{f}{g}$.

$$\frac{3x + 1}{3x^2 - 11x - 4} \quad (3x^2 - 11x - 4)(3x + 1) \quad \frac{3x^2 - 11x - 4}{3x + 1}$$

15. Simplify the product.

$9x^3 - 30x^2 - 23x - 4$

16. Simplify the quotient.

$x - 4$

17. For what values of x does $g(x) = 0$?

$-\frac{1}{3}$

18. Write the domain of $f \cdot g$.

all real numbers

19. Write the domain of $\frac{f}{g}$.

all real numbers except $x = -\frac{1}{3}$

take note

Key Concept Composition of Functions

The composition of function g with function f is written $g \circ f$ and is defined as $(g \circ f)(x) = g(f(x))$. The domain of $g \circ f$ consists of the x -values in the domain of f for which $f(x)$ is in the domain of g .

$$(g \circ f)(x) = \underbrace{g}_{\substack{1 \\ 2}}(f(x))$$

1. Evaluate $f(x)$ first.

2. Then use $f(x)$ as the input for g .

Function composition is not commutative, since $f(g(x))$ does not always equal $g(f(x))$.

If $f(x) = x^2$ and $g(x) = x + 3$, find each composition.

20. $g \circ f = g(x^2) = x^2 + 3$

21. $f \circ g = f(x + 3) = (x + 3)^2 = x^2 + 6x + 9$



Problem 3 Composing Functions

Got It? Let $f(x) = x - 5$ and $g(x) = x^2$. What is $(f \circ g)(-3)$?

Method 1

22. Find $g(-3)$.

9

23. Use $g(-3)$ as the input for $f(x)$.

$$(f \circ g)(-3) = f(g(-3)) = f(9) = 9 - 5 = 4$$

Method 2

24. Simplify $(f \circ g)(x) = f(g(x))$.

$$f(g(x)) = f(x^2) = x^2 - 5$$

25. Use the rule for $f(g(x))$ to find $f(g(-3))$.

$$(f \circ g)(-3) = (-3)^2 - 5 = 4$$



Problem 4 Using Composite Functions

Got It? A store is offering a 15% discount on all items. Also, employees get a 20% employee discount. Write composite functions.

Model taking the 15% discount and then the 20% discount.

Model taking the 20% discount and then the 15% discount.

26. Let x be the price of an item. Write functions to model each discount.

$$C(x) = \text{cost using the storewide discount} = x - 0.15x = 0.85x$$

$$D(x) = \text{cost using the employee discount} = x - 0.2x = 0.8x$$

27. Draw a line from each composition to its rule.

$$(D \circ C)(x) \quad \text{---} \quad 0.85(0.8x)$$

$$(C \circ D)(x) \quad \text{---} \quad 0.8(0.85x)$$

28. Simplify each function.

$$(D \circ C)(x) = 0.68x$$

$$(C \circ D)(x) = 0.68x$$

29. If you were an employee, which discount would you take first? Why?

Either discount can be taken first, since in both cases the result is that you pay 68% of the original price.



Lesson Check • Do you UNDERSTAND?

Open-Ended Find two functions f and g such that $f(g(x)) = x$ for all real numbers x .

30. If $g(x) = x + 3$, write a function $f(x)$ to give $f(g(x)) = x$.

$$f(x) = x - 3$$

31. If $g(x) = 2x$, write a function $f(x)$ to give $f(g(x)) = x$.

$$f(x) = \frac{x}{2}$$

32. Write a function $g(x)$. Then, find $f(x)$ such that $f(g(x)) = x$.

Answers will vary. The functions should be inverses of one another.



Math Success

Check off the vocabulary words that you understand.

composite function

function operations

Rate how well you can *find the composition of two functions*.



6-7

Inverse Relations and Functions



Vocabulary

Review

- Underline the correct term to complete each sentence.

The *domain* of a relation is the set of inputs, also called the x- / y- coordinates of the ordered pair.

The *range* of a relation is the set of outputs, also called the x- / y- coordinates of the ordered pair.

- Use the relation $\{(4, 5), (6, 7), (12, 20), (8, 3), (2, 7)\}$. Write the domain and range of the relation.

Domain

$\{2, 4, 6, 8, 12\}$

Range

$\{3, 5, 7, 20\}$

Vocabulary Builder

inverse (noun) in VURS

Related Words: opposite, reverse

Math Usage: The **inverse** of a function is found by reversing the order of the elements in the ordered pairs.

Example: The **inverse** of the function $\{(1, 2), (2, 4), (3, 6)\}$ is $\{(2, 1), (4, 2), (6, 3)\}$.

Use Your Vocabulary

- Complete the diagram below. Use relation $r \{(0, 1), (2, 3), (4, 1), (8, 3)\}$.

Relation r

Domain	Range
$0, 2, 4, 8$	$1, 3$

Inverse of Relation r

Domain	Range
$1, 3$	$0, 2, 4, 8$



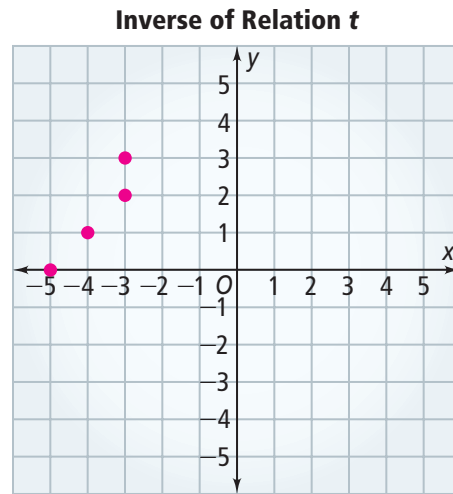
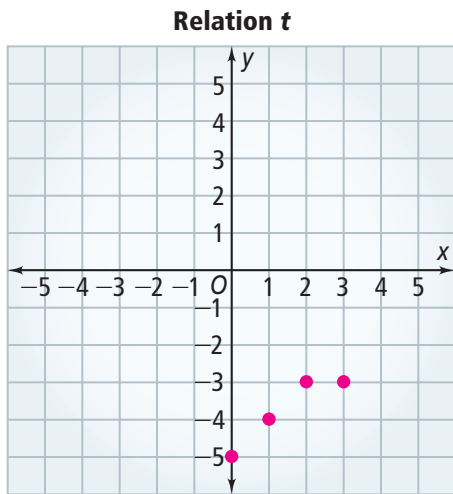
Problem 1 Finding the Inverse of a Relation

Got It? What are the graphs of t and its inverse?

4. Complete the table of values for the inverse of relation t .

Relation t		Inverse of Relation t	
x	y	x	y
0	-5	-5	0
1	-4	-4	1
2	-3	-3	2
3	-3	-3	3

5. Plot the points from the Relation t table and from the Inverse of Relation t table.



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Problem 2 Finding an Equation for the Inverse

Got It? What is the inverse of $y = 2x + 8$?

6. Switch the x and y values in the function.

function $y = 2x + 8$

inverse $x = 2y + 8$

7. Solve the inverse equation for y .

$$x = 2y + 8$$

$$x - 8 = 2y$$

$$\frac{x - 8}{2} = y$$

Problem 3 Graphing a Relation and Its Inverse

Got It? What are the graphs of $y = 2x + 8$ and its inverse?

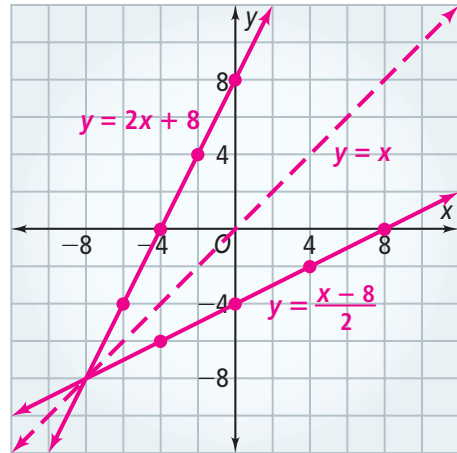
8. Complete the table for $y = 2x + 8$.

x	-6	-4	-2	0
y	-4	0	4	8

9. Complete the table for the inverse of $y = 2x + 8$.

x	-4	0	4	8
y	-6	-4	-2	0

- Plot and draw a line through the points from the $y = 2x + 8$ table.
- On the same grid, plot and draw a line through the points from the inverse of $y = 2x + 8$ table.
- Draw a dashed line to show the line that reflects the equation $y = 2x + 8$ to its inverse.



Problem 5 Finding the Inverse of a Formula

Got It? The function $d = \frac{v^2}{19.6}$ relates the distance d , in meters, that an object has fallen to its velocity v , in meters per second. Find the inverse of this function. What is the velocity of the cliff diver in meters per second as he enters the water?

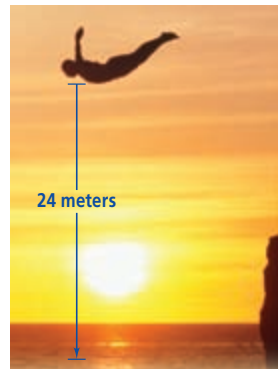
13. Solve the function for v .

$$d = \frac{v^2}{19.6}$$

$$19.6d = v^2 \rightarrow \sqrt{19.6d} = v$$

14. Let $d = 24$ meters. Write the value of the velocity v , to the nearest hundredth meter per second, of the diver as he enters the water.

$$21.69$$



Key Concept Composition of Inverse Functions

If f and f^{-1} are inverse functions, then $(f^{-1} \circ f)(x) = x$ and $(f \circ f^{-1})(x) = x$ for x in the domains of f and f^{-1} , respectively.



Problem 6 Composing Inverse Functions

Got It? Let $g(x) = \frac{4}{x+2}$. What is $g^{-1}(x)$?

15. Complete each step.

$$x = \frac{4}{y+2}$$

Write the inverse function.

$$x(y+2) = 4$$

Multiply.

$$(y+2) = \frac{4}{x}$$

Divide.

$$y = \frac{4}{x} - 2$$

Subtract.

16. So, $g^{-1}(x) = \frac{4}{x} - 2$.



Lesson Check • Do you UNDERSTAND?

Reasoning A function consists of the pairs (2, 3), (x, 4), and (5, 6). What values, if any, may x not assume?

17. Each x -value in the domain of a function corresponds to

exactly one y -value / many y -values in the range.

18. What values, if any, may x not assume?

Answers may vary. Sample: x cannot equal 2 or 5.



Math Success

Check off the vocabulary words that you understand.

inverse relation

inverse function

one-to-one function

Rate how well you can *find the inverse of a relation or function*.

Need to review

0 2 4 6 8 10



Now I get it!



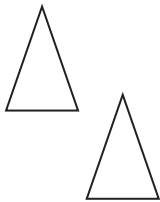
Vocabulary

Review

Write T if the figures show a *translation* or N if they do NOT show a translation.

1.

T



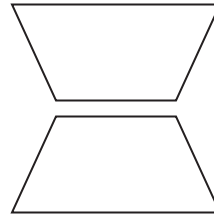
2.

N



3.

N



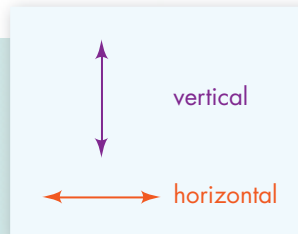
Vocabulary Builder

vertical (adjective) *VUR tih kul*

Related Word: horizontal

Main Idea: A vertical line is straight up and down.
A vertical shift moves something up or down.

Math Usage: A vertical translation moves the graph of a relation parallel to the y-axis.



Use Your Vocabulary

Underline the correct word to complete each sentence.

- A helicopter takes off vertically / horizontally .
- A package on a flat conveyor belt moves vertically / horizontally .
- Stepping side-to-side is a vertical / horizontal movement.

Key Concepts Families of Radical Functions

	Square Root	Radical
Parent function	$y = \sqrt{x}$	$y = \sqrt[n]{x}$
Reflection in x -axis	$y = -\sqrt{x}$	$y = -\sqrt[n]{x}$
Stretch ($a > 1$), shrink ($0 < a < 1$) by factor a	$y = a\sqrt{x}$	$y = a\sqrt[n]{x}$
Translation: horizontal by h , vertical by k	$y = \sqrt{x-h} + k$	$y = \sqrt[n]{x-h} + k$



Problem 1 Translating a Square Root Function Vertically

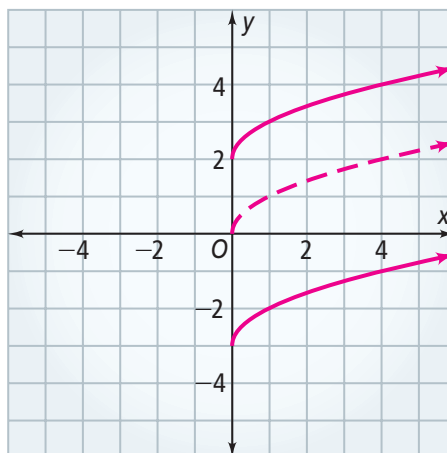
Got It? What are the graphs of $y = \sqrt{x} + 2$ and $y = \sqrt{x} - 3$?

7. The graph of $y = \sqrt{x} + 2$ is a horizontal / vertical translation of $y = \sqrt{x}$
up / down / left / right 2 units.

8. What does the translation $y = \sqrt{x} - 3$ look like? **Answers may vary. Sample:**

It is a vertical translation of $y = \sqrt{x}$. It is shifted down 3 units.

9. Draw the graph of the function $y = \sqrt{x}$.
 Then, use that graph to draw the graphs of
 $y = \sqrt{x} + 2$ and $y = \sqrt{x} - 3$ on the same grid.



Problem 2 Translating a Square Root Function Horizontally

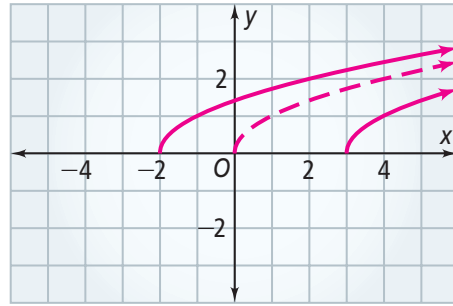
Got It? What are the graphs of $y = \sqrt{x-3}$ and $y = \sqrt{x+2}$?

10. The graph of $y = \sqrt{x-3}$ is a horizontal / vertical translation of $y = \sqrt{x}$
up / down / left / right 3 units.

11. What does the translation $y = \sqrt{x+2}$ look like?

It is a horizontal translation of $y = \sqrt{x}$. It is shifted 2 units left.

12. Draw the graph of the function $y = \sqrt{x}$. Then, use that graph to draw the graphs of $y = \sqrt{x - 3}$ and $y = \sqrt{x + 2}$ on the same grid.



Problem 3 Graphing a Square Root Function

Got It? What is the graph of $y = 3\sqrt{x + 2} - 4$?

13. Complete the table.

x	1	4	16
\sqrt{x}	1	2	4
$3\sqrt{x}$	3	6	12

14. Multiplying the y-coordinates of $y = \sqrt{x}$ by 3 shrinks / stretches the graph.
15. Explain how the graph of $y = 3\sqrt{x + 2} - 4$ relates to the graph of $y = \sqrt{x}$.

Answers may vary. Sample: It is a translation of $y = \sqrt{x}$ that is 2 units left and 4 units down, and stretched by a factor of 3.

Problem 5 Graphing a Cube Root Function

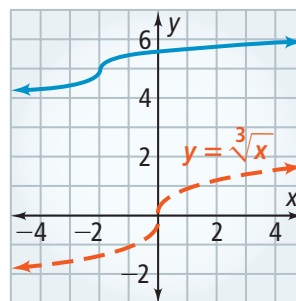
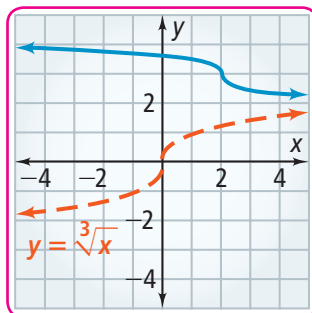
Got It? What is the graph of $y = 3 - \frac{1}{2}\sqrt[3]{x - 2}$?

16. Write $y = 3 - \frac{1}{2}\sqrt[3]{x - 2}$ in standard form.

$$y = -\frac{1}{2}\sqrt[3]{x - 2} + 3$$

Underline the correct numbers or words to complete each sentence.

17. The shift is 2 / 3 units left / right and 2 / 3 units up / down.
18. There is a shrink / stretch by a factor of 1 / 2 and the result is / is not reflected.
19. Circle the graph of $y = 3 - \frac{1}{2}\sqrt[3]{x - 2}$.





Problem 6 Rewriting a Radical Function

Got It? How can you rewrite $y = \sqrt[3]{8x + 32} - 2$ so you can graph it using transformations? Describe the graph.

20. Factor out the GCF of the radicand.

$$8x + 32 = 8(x + 4)$$

21. Complete the equation.

$$\sqrt[3]{8x + 32} - 2 = 2\sqrt[3]{x + 4} - 2$$

22. Underline the correct word to complete each phrase.

translated left / right by 4 units

stretched / shrunk by a factor of 2

translated up / down by 2 units



Lesson Check • Do you UNDERSTAND?

Error Analysis Your friend states that the graph of $g(x) = \sqrt{-x - 1}$ is a reflection of the graph of $f(x) = -\sqrt{x + 1}$ across the x -axis. Describe your friend's error.

23. The graph of the function $\frac{-f(x)}{f(-x)}$ is a reflection of $f(x)$ over the x -axis.

24. Write the expression that is equal to $-f(x)$.

$$\sqrt{x + 1}$$

25. Explain the error your friend made.

Answers may vary. Sample: The student multiplied the radicand

by -1 instead of multiplying the entire radical by -1 .



Math Success

Check off the vocabulary words that you understand.

radical function

square root function

Rate how well you can graph square root functions.

