Name Answer key

Algebra 2E Final Exam Review

Calculator Section

1. Find the domain and range of the relation given by the ordered pairs (-2, 3), (-1, 2), (2, 4), and (5, 8).

Domain: $\boxed{x \in \{-2, -1, 2, 5\}}$ Range: $\boxed{y \in \{2, 3, 4, 8\}}$

2. Find the solution to each of the following systems of equations.

a. $\begin{cases} 2x - y + 3z = 8 \\ x - 6y - z = 0 \\ -6x + 3y - 9z = 24 \end{cases}$ $\boxed{\text{no solution}}$

b. $\begin{cases} 2x - 3y = 4 \\ 4x + y = -6 \end{cases}$ $\boxed{(-1, -2)}$

3. Find the inverse of each of the following.

a. $f(x) = \sqrt{x-2} + 1$ $\boxed{f^{-1}(x) = (x-1)^2 + 2}$

$x = \sqrt{y-2} + 1$

$(x-1)^2 = (\sqrt{y-2})^2$

$(x-1)^2 = y - 2$

$y = (x-1)^2 + 2$

b. $g(x) = \frac{1}{2}x + 7$ $\boxed{g^{-1}(x) = 2x - 14}$

$x = \frac{1}{2}y + 7$

$x - 7 = \frac{1}{2}y$

$y = 2(x-7)$

4. A certain radioactive element decays over time according to the equation $y = A\left(\frac{1}{2}\right)^{\frac{t}{300}}$ where A is the number of grams present initially and *t* is the time in years. $y = ab^x$

$\left(\frac{1}{2}\right)^{\frac{1}{300}t} \rightarrow 0.9976922$

a. By what percent does the radioactive element decay per year? $\boxed{\approx 0.2308\%}$

b. If 1000 grams were present initially, how many grams will remain after 900 years?

$y = ab^x$ $\quad y = 1000(.9976922)^{900} \approx \boxed{125g}$

5. Bacteria in a culture are growing exponentially with time, as shown in the table to the right. Write an equation that expresses the number of bacteria, *y*, present at any time, *t*.

Day	Bacteria
0	100
1	200
2	400

$\boxed{y = 100(2)^t}$

6. Find the domain of each of the following:

a. $f(x) = 3\ln(3x+4)$

$3x + 4 > 0$

$\boxed{x > -4/3}$

$x \in (-4/3, \infty)$

b. $f(x) = \frac{1}{2x+3}$

$2x + 3 \neq 0$

$\boxed{x \neq -3/2}$

7. Write an explicit and recursive rule for the following arithmetic sequence: $-2, -\frac{5}{4}, -\frac{1}{2}, \frac{1}{4}, \ldots$

$\frac{-8}{4}$ $\frac{-5}{4}$ $\frac{-2}{4}$ $\frac{1}{4}$

$d = 3/4$

R: $a_n = a_{n-1} + 3/4$; $a_1 = -2$

E: $a_n = -2 + 3/4(n-1)$

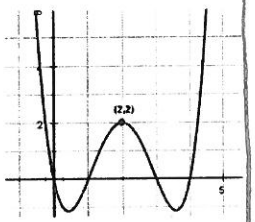
8. Write an explicit and recursive rule for the following geometric sequence: $112, 56, 28, 14, \ldots$

R: $a_n = a_{n-1} \cdot 1/2$; $a_1 = 112$

E: $a_n = 112(1/2)^{n-1}$

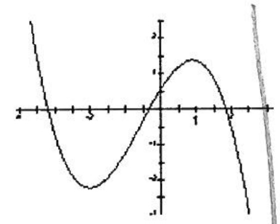
9. State the degree and the end behavior of each of the following.

a.



Deg: 4
As $x \to -\infty$, $f(x) \to \infty$
$x \to \infty$, $f(x) \to \infty$
Even/odd?
Neither!

b.



Deg: 3
As $x \to -\infty$, $f(x) \to \infty$
$x \to \infty$, $f(x) \to -\infty$
Even/odd?
Neither

10. Find the sum of each infinite geometric series, if it exists.

a. $2+\frac{6}{4}+\frac{18}{16}+\frac{54}{64}+\ldots$ $r = 3/4$ $|r|<1$ ✓

$S_\infty = \frac{a_1}{1-r}$ $\frac{2}{1-3/4} = \boxed{8}$

b. $\frac{1}{2}-\frac{5}{3}+\frac{50}{9}-\frac{500}{27}+\ldots$ $r=-10/3$ $|r|>1$ ✗

$\boxed{\text{No sum possible}}$

11. Find the average rate of change of the function $g(x) = 2x^3 + 3$ from x = 1 to x = 4.

$(1,5)$ $(4,131)$ $\frac{126}{3} = \boxed{42}$

12. The data in the table to the right shows the relationship between temperatures measured in Fahrenheit and Celsius. What is the equation of the regression **line** for the data?

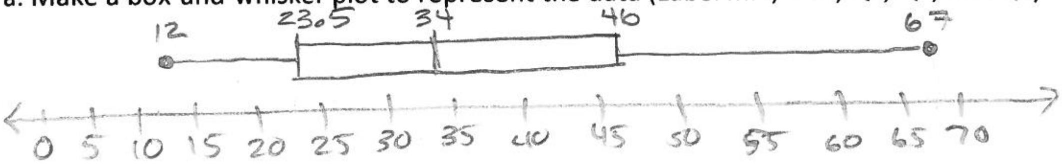
Fahrenheit degrees (°F)	Celsius degrees (°C)
32	0
68	20
86	30
122	50
158	70
194	90
212	100

$\boxed{y = 0.555x - 17.778}$

13. Given: ~~56~~, ~~32~~, ~~54~~, ~~34~~, ~~23~~, ~~67~~, ~~23~~, ~~45~~, ~~12~~, ~~32~~, ~~34~~, ~~24~~, ~~36~~, ~~47~~, ~~19~~, and ~~43~~.

12, 19, 23, 23 | 24, 32, 32, 34 | 34, 36, 43, 45 | 47, 54, 56, 67

a. Make a box-and-whisker plot to represent the data (Label min, max, Q1, Q2, and Q3)



b. Find and interpret the range of the data.

Range: 55

c. Find and interpret the interquartile range of the data.

IQR: 22.5

14. Identify the y-intercept, x-intercept(s), holes, vertical asymptotes (VA), and horizontal (HA)/slant (SA) asymptotes of each of the following.

a. $h(x) = \frac{x^3 - 16x}{-3x^2 + 3x + 18}$ $\frac{x(x-4)(x+4)}{-3(x-3)(x+2)}$

HA: none
VA: $x = 3$; $x = -2$
SA: $y = -1/3\,x + ?$
xint: $(0,0)$, $(4,0)$ $(-4,0)$
y-int: $(0,0)$

b. $g(x) = \frac{-3x^2 - 12x - 9}{x^2 + 5x + 4}$ $-3(x^2+4x+3)$ $\frac{-3(x+3)\cancel{(x+1)}}{(x+4)\cancel{(x+1)}}$

HA: $y = -3$
VA: $x = -4$
SA: none
hole: $(-1, -2)$
x-int: $(-3,0)$
y-int: $(0, -9/4)$

$$\left[\frac{2}{2}\right]\frac{1}{2} + \frac{3}{4} = \frac{5}{4}$$

15. Simplify each of the following.

a. $\dfrac{x+3}{x+5}+\dfrac{6}{x^2+3x-10}$

$$\boxed{\frac{x^2+x}{(x+5)(x-2)}}$$

$$\frac{x+3}{x+5}\left[\frac{x-2}{x-2}\right] + \frac{6}{(x+5)(x-2)}\left[\;\right]$$

$$x^2+x-\cancel{6} + \cancel{6}$$

b. $\dfrac{2x^2-10x}{x^2+8x+16}\cdot\dfrac{4x+16}{x^2-25}$

$$\frac{2x(\cancel{x-5})}{(x+4)(\cancel{x+4})}\cdot\frac{4(\cancel{x+4})}{(\cancel{x-5})(x+5)}$$

$$\boxed{\frac{8x}{(x+4)(x+5)}}$$

c. $\dfrac{x^2+4x}{x+3}\div\dfrac{x^2+x-12}{x^2-9}$

$$=\boxed{x}$$

$$\frac{x(\cancel{x+4})}{\cancel{x+3}}\cdot\frac{(\cancel{x+3})(\cancel{x-3})}{(\cancel{x+4})(\cancel{x-3})}$$

d. $\dfrac{m}{m-n}-\dfrac{m}{n-m}$

$$\frac{m}{m-n} - \frac{m}{(-m+n)} \Rightarrow \frac{m}{m-n} + \frac{m}{(m-n)}$$

$$\boxed{\frac{2m}{m-n}}$$

16. Solve each of the following equations:

a. $\dfrac{6}{p}=\dfrac{1}{p-5}-\dfrac{p+4}{p^2-5p}$

$$\frac{6}{p}\left[\frac{p-5}{p-5}\right] = \frac{1}{p-5}\left[\frac{p}{p}\right] - \frac{(p+4)}{p(p-5)}\left[\;\right]$$

$$6p-30 = \cancel{p} - \cancel{p} - 4$$
$$+30$$
$$6p = 26$$

$$\boxed{p = 13/3}$$

b. $\dfrac{5x-20}{x^2-9x+18}+\dfrac{1}{x-6}=\dfrac{x-4}{x^2-9x+18}$

$$\frac{5x-20}{(x-6)(x-3)} + \frac{1}{(x-6)}\left[\frac{x-3}{x-3}\right] = \frac{(x-4)}{(x-6)(x-3)}$$

$$5x-20+(x-3) = x-4$$
$$5x-20+\cancel{x}-3 = \cancel{x}-4$$
$$5x = 19$$
$$\boxed{x = 19/5}$$

17. If $\sin\theta=\dfrac{3}{5}$ and θ lies in Quadrant II, find $\cos\theta$ and $\tan\theta$.

3, 5, −4

$$\boxed{\cos\theta = -4/5\;;\;\tan\theta = -3/4}$$

18. Given that $\cos\theta=-\dfrac{8}{9}$ and $\sin\theta<0$, find $\sin\theta$.

−8, $-\sqrt{17}$, 9

$$\boxed{\sin\theta = \frac{-\sqrt{17}}{9}}$$

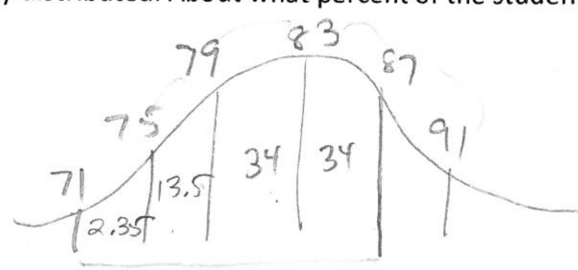
19. Given that $\sin\theta=-\dfrac{1}{4}$ and $\cos\theta>0$, find $\cos\theta$.

$$\sqrt{16-1}$$
$$\sqrt{15}$$

$\sqrt{15}$, −1, 4

$$\cos\theta = \boxed{\frac{\sqrt{15}}{4}}$$

20. The average test score on a college exam was 83 points, and the standard deviation was 4 points, where the scores are normally distributed. About what percent of the students who took the test scored between 71 points and 87 points?



$$= \boxed{83.85\%}$$

21. You survey the senior class to see what programs they prefer the most. The results are shown in the table below.

Gender	Preferred Program			
	Dance	Sports	Movies	Total
Women	16	6	8	30
Men	2	10	8	20
Total	18	16	16	50

a. If a senior is randomly selected, what is the probability that they are a male, given that they like movies?

$\frac{8}{16} = \boxed{1/2}$

b. If a senior is randomly selected, what is the probability that they like dance, given that they are female?

$\frac{16}{30} = \boxed{\frac{8}{15}}$

22. An equation that relates the height of the rider on a Ferris wheel from the ground in feet vs. time in seconds is $h = -20\cos\left(\frac{\pi}{4}t\right) + 23$. How many solutions are there to the equation $h = 33$ feet on the interval $0 < t < 16$?

use calc! graph it with a good window $x: 0 \to 16$ $y: 0 \to 50$

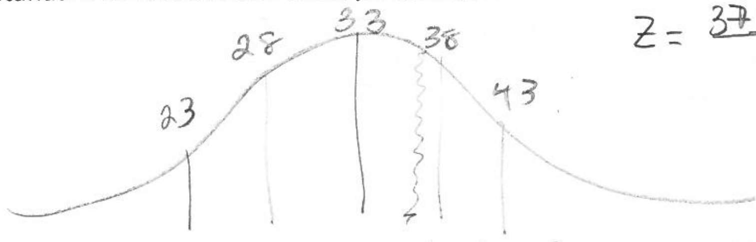
$\boxed{4 \text{ solutions}}$

23. What is the average rate of change of the function $f(x) = \frac{1}{x+2}$ from x = 2 to x = 6?

$(2, \frac{1}{4})$ $(6, \frac{1}{8})$

$\frac{-1/8}{4} = \boxed{\frac{-1}{32}}$

24. The amount of water that high school students drink is normally distributed with a mean of 33 ounces and a standard deviation of 5. What percentage of students in the high school drink more than 37 ounces?



$Z = \frac{37-33}{5}$ $z = 0.8$ $.7881$

$\boxed{21.19\%}$

25. Given the polynomial $g(x) = 2x^4 + x^3 - 4x^2 - 1$, fill in the blank: As $x \to \infty$, $f(x) \to$ $\boxed{\infty}$.