Name: Key - Answers + Work

Unit 13 • Serafino • Precalculus S2

201-103-RE - Calculus 1

WORKSHEET: LIMITS

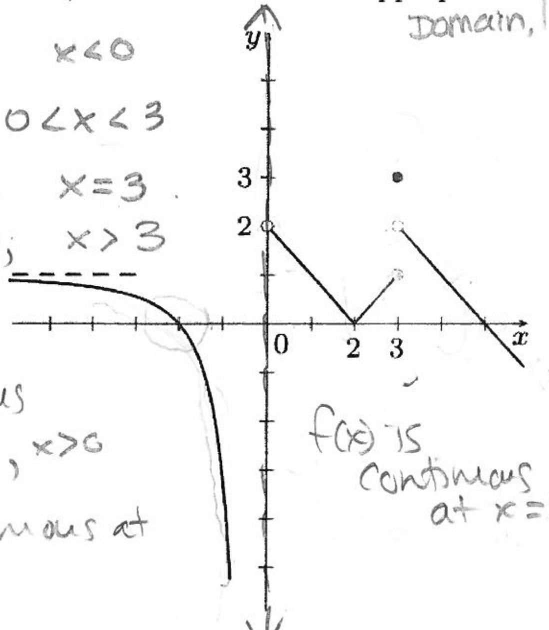
1. Use the graph of the function $f(x)$ to answer each question. Use ∞, $-\infty$ or DNE where appropriate.

Infinite Disc. $x=0$

Jump Disc. $x=3$

Domain, $\boxed{x \neq 0}$

$$f(x) = \begin{cases} \frac{x+2}{x}, & x<0 \\ |x-2|, & 0<x<3 \\ 3, & x=3 \\ -(x-5), & x>3 \end{cases}$$



f(x) is continuous @ $x<0$, $0<x<3$, $x>0$

f(x) is discontinuous at $x=0$ and $x=3$

f(x) is continuous at $x=2$

(a) $f(0) =$ DNE

(b) $f(2) =$ 0

(c) $f(3) =$ 3

(d) $\lim_{x\to 0^-} f(x) =$ DNE $(-\infty)$

(e) $\lim_{x\to 0} f(x) =$ DNE

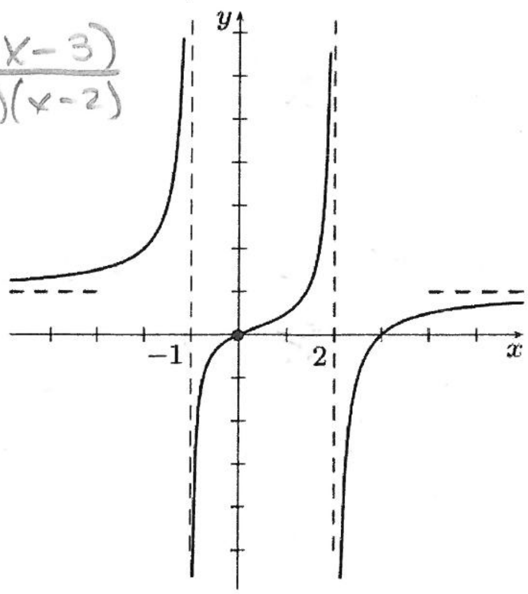
(f) $\lim_{x\to 3^+} f(x) =$ 2

(g) $\lim_{x\to 3} f(x) =$ DNE

(h) $\lim_{x\to -\infty} f(x) =$ 1

2. Use the graph of the function $f(x)$ to answer each question. Use ∞, $-\infty$ or DNE where appropriate.

$$f(x) = \frac{x(x-3)}{(x+1)(x-2)}$$



(a) $f(0) =$ 0

(b) $f(2) =$ DNE

(c) $f(3) =$ 0

(d) $\lim_{x\to -1} f(x) =$ DNE

(e) $\lim_{x\to 0} f(x) =$ 0

(f) $\lim_{x\to 2^+} f(x) =$ DNE $(-\infty)$

(g) $\lim_{x\to \infty} f(x) =$ 1

Do all work on separate paper. See my site for which ones you can skip.

3. Evaluate each limit using algebraic techniques.
Use ∞, $-\infty$ or DNE where appropriate.

(a) $\displaystyle\lim_{x\to 0} \frac{x^2-25}{x^2-4x-5}$ $\boxed{=5}$

(b) $\displaystyle\lim_{x\to 5} \frac{x^2-25}{x^2-4x-5}$ $\boxed{=5/3}$

(c) $\displaystyle\lim_{x\to 1} \frac{7x^2-4x-3}{3x^2-4x+1}$ $\boxed{=5}$

(d) $\displaystyle\lim_{x\to -2} \frac{x^4+5x^3+6x^2}{x^2(x+1)-4(x+1)}$ $\boxed{=1}$

(e) $\displaystyle\lim_{x\to -3} |x+1|+\frac{3}{x}$ $\boxed{=1}$

(f) $\displaystyle\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x^2-9}$ $\boxed{=1/24}$

(g) $\displaystyle\lim_{x\to 3} \frac{\sqrt{x^2+7}-3}{x+3}$ $\boxed{=1/6}$

(h) $\displaystyle\lim_{x\to 2} \frac{x^2+2x-8}{\sqrt{x^2+5}-(x+1)}$ $\boxed{=-18}$

(i) $\displaystyle\lim_{y\to 5} \left(\frac{2y^2+2y+4}{6y-3}\right)^{1/3}$ $\boxed{=4/3}$

(j) $\displaystyle\lim_{x\to 0} \sqrt[4]{2\cos(x)-5}$ $\boxed{=\text{DNE}}$

get common denominator

* (k) $\displaystyle\lim_{x\to 0} \frac{\frac{1}{3+x}-\frac{1}{3-x}}{x}$ $\boxed{=-2/9}$

* (l) $\displaystyle\lim_{x\to -6} \frac{\frac{2x+8}{x^2-12}-\frac{1}{x}}{x+6}$ $\boxed{=1/36}$

(m) $\displaystyle\lim_{x\to \infty} \sqrt{x^2-2}-\sqrt{x^2+1}$ $\boxed{=0}$

(n) $\displaystyle\lim_{x\to -\infty} \sqrt{x-2}-\sqrt{x}$ $\boxed{=\text{DNE}}$

* (o) $\displaystyle\lim_{x\to 7} \sqrt[6]{2x-14}$ $\boxed{=\text{DNE}}$

* (p) $\displaystyle\lim_{x\to 1^-} \sqrt{3-3x}$ $\boxed{=0}$

Don't worry about the "left" part... see my expl.

(q) $\displaystyle\lim_{x\to \infty} \frac{x^4-10}{4x^3+x}$ $\boxed{=\infty}$

(r) $\displaystyle\lim_{x\to -\infty} \sqrt[3]{\frac{x-3}{5-x}}$ $\boxed{=-1}$

(s) $\displaystyle\lim_{x\to \infty} \frac{3x^3+x^2-2}{x^2+x-2x^3+1}$ $\boxed{=-3/2}$

(t) $\displaystyle\lim_{x\to \infty} \frac{x+5}{2x^2+1}$ $\boxed{=0}$

(u) $\displaystyle\lim_{x\to -\infty} \cos\left(\frac{x^5+1}{x^6+x^5+100}\right)$ $\boxed{=1}$

(v) $\displaystyle\lim_{x\to 2} \frac{2x}{x^2-4}$ $\boxed{=\text{DNE}}$

(w) $\displaystyle\lim_{x\to -1} \frac{3x}{x^2+2x+1}$ $\boxed{=\text{DNE } (-\infty)}$

(x) $\displaystyle\lim_{x\to -1} \frac{x^2-25}{x^2-4x-5}$ $\boxed{=\text{DNE}}$

(y) $\displaystyle\lim_{x\to 3} \frac{\sqrt{x^2-5}+2}{x-3}$ $\boxed{=\text{DNE}}$

~~(z)~~ $\displaystyle\lim_{x\to 0} \frac{2^x+\sin(x)}{x^4}$ ____

~~(A)~~ $\displaystyle\lim_{x\to 1^-} \frac{1}{x-1}+e^{x^2}$ ____

(B) $\displaystyle\lim_{x\to \infty} 2x^2-3x$ $\boxed{=\text{DNE } (\infty)}$

(C) $\displaystyle\lim_{x\to 0} \frac{\sqrt{x+2}-\sqrt{2-x}}{x}$ $\boxed{=\frac{\sqrt{2}}{2}}$

~~(D)~~ $\displaystyle\lim_{x\to 0^+} \frac{e^x}{1+\ln(x)}$ ____

~~(E)~~ $\displaystyle\lim_{x\to \infty} \sqrt{x^2+1}-2x$ ____

~~(F)~~ $\displaystyle\lim_{x\to 1} \frac{\sqrt[3]{x}-1}{\sqrt{x}-1}$ ____

$2(x-7)$

Worksheet: Limits

#3.

$-25/-5 = 5$

(a) $\frac{x^2-25}{x^2-4x-5}$ $\boxed{\lim_{x\to 0} f(x) = 5}$

(b) $\frac{x^2-25}{x^2-4x-5}$ $\frac{\cancel{(x-5)}(x+5)}{\cancel{(x-5)}(x+1)}$ $\boxed{\lim_{x\to 5} f(x) = 5/3}$ $10/6 = 5/3$

(c) $1 \,|\, 3 \;\; -4 \;\; 1$; $3 \;\; -1$; $3 \;\; -1 \;\; 0$ $\frac{(7x-7)(7x+3)}{(3x-3)(3x-1)}$ $\frac{\cancel{(x-1)}(7x+3)}{\cancel{(x-1)}(3x-1)}$ $10/2$

-21; $1 \,|\, 21 \;\; -1 \,|\, 3-$; $-7 \,|\, 3+$

$\boxed{\lim_{x\to 1} f(x) = 5}$

(d) $\frac{x^2(x^2+5x+6)}{x^2-4(x+1)} = \frac{x^2\cancel{(x+2)}(x+3)}{\cancel{(x+2)}(x-2)(x+1)}$ $\frac{x^2(x+3)}{(x-2)(x+1)}$ $\frac{(4)(1)}{(-4)(-1)}$

$\boxed{\lim_{x\to -2} f(x) = 1}$

(e) $|-3+1| + \frac{3}{-3} = \frac{|-2|-1}{2-1}$ $\boxed{\lim_{x\to -3} f(x) = 1}$

(f) $\frac{(\sqrt{x+1}-2)(\sqrt{x+1}+2)}{(x-3)(x+3)} = x+1-4 = \frac{\cancel{(x-3)}\,1}{\cancel{(x-3)}(x+3)(\sqrt{x+1}+2)}$

$\boxed{\lim_{x\to 3} f(x) = \frac{1}{24}}$ $\frac{1}{(3+3)(\sqrt{4}+2)} = \frac{1}{6\cdot 4}$

(g) $\frac{\sqrt{9+7}-3}{6} = \frac{4-3}{6} = \frac{1}{6}$ $\boxed{\lim_{x\to 3} f(x) = \frac{1}{6}}$

(h) $$\frac{(x+4)\cancel{(x-2)}}{\sqrt{x^2+5}-(x+1)} \quad \frac{\sqrt{x^2+5}+(x+1)}{\sqrt{x^2+5}+(x+1)}$$

$$x^2+5-(x^2+1)^2$$
$$x^2+5-(x^2+2x+1)$$
$$\cancel{x^2}+5-\cancel{x^2}-2x-1$$
$$-2x+4 \to -2\cancel{(x-2)}$$

$$\boxed{\lim_{x\to 2} f(x) = -18}$$

$$\frac{(x+4)\left[\sqrt{x^2+5}+(x+1)\right]}{-2} \to \frac{(2+4)\left[\sqrt{9}+3\right]}{-2} = \frac{6\cdot 6 -3}{-2}$$

(i) $$\left(\frac{2(5)^2+2(5)+4}{6(5)-3}\right)^{\frac{1}{3}}$$

$$\boxed{\lim_{x\to 5} f(x) = \frac{4}{3}}$$

$$=\left(\frac{50+10+4}{27}\right)^{\frac{1}{3}} = \left(\frac{64}{27}\right)^{1/3} = \frac{4}{3}$$

(j) $$\sqrt[4]{2\cos(x)-5}$$

$$\sqrt[4]{2\cos(0)-5} \to \sqrt[4]{-3}$$

imaginary

$$\boxed{\lim_{x\to 0} f(x) = \text{DNE}}$$

(k) $$\frac{\frac{1}{3+x}-\frac{1}{3-x}}{x} = \frac{\left[\frac{3-x}{3-x}\right]\frac{1}{3+x}-\frac{1}{3-x}\left[\frac{3+x}{3+x}\right]}{x}$$

$$=\frac{\frac{(3-x)-(3+x)}{(3-x)(3+x)}}{x} \Rightarrow \frac{\frac{\cancel{3}-x-\cancel{3}-x}{(3-x)(3+x)}}{x} \quad \frac{-2\cancel{x}}{\cancel{x}(3-x)(3+x)}$$

$$\frac{-2}{(3+0)(3-0)} = -2/9$$

$$\boxed{\lim_{x\to 0} f(x) = \frac{-2}{9}}$$

(l) $$\frac{\frac{2x+8}{x^2-12} - \frac{1}{x}}{x+6} \rightarrow \frac{\left[\frac{x}{x}\right]\frac{2x+8}{x^2-12} - \frac{1}{x}\left(\frac{x^2-12}{x^2-12}\right)}{x+6}$$

$$(2x^2+8x)-(x^2-12) = 2x^2+8x-x^2+12$$

$$= x^2+8x+12 = \frac{\frac{(x+6)(x+2)}{x(x^2-12)}}{x+6} = \frac{(x+\cancel{6})(x+2)}{x(x^2-12)(x\cancel{+6})}$$

$$\frac{(-6+2)}{-6(36-12)} = \frac{\cancel{-4}\ 1}{\cancel{+6}(\cancel{24})6}$$

$$\boxed{\lim_{x\to -6} f(x) = 1/36}$$

(m) $$\frac{\sqrt{x^2-2} - \sqrt{x^2+1}}{1} \frac{\left(\sqrt{x^2-2}+\sqrt{x^2+1}\right)}{\left(\sqrt{x^2-2}+\sqrt{x^2+1}\right)}$$

$$= \frac{(\cancel{x^2}-2)-(\cancel{x^2}+1)}{\sqrt{x^2-2}+\sqrt{x^2+1}} = \frac{-3}{\sqrt{x^2-2}+\sqrt{x^2+1}}$$

$$\boxed{\lim_{x\to\infty} f(x) = 0}$$

higher degree on bottom

(n) $\sqrt{x-2} - \sqrt{x}$

Can't take sq. root of neg. #

$$\boxed{\lim_{x\to -\infty} f(x) = DNE}$$

(o) $\sqrt[6]{2(7)-14}$

$\sqrt[6]{0} = 0$ but

The domain for $\sqrt[6]{x}$ is not all reals. It looks like

so the limit at $x=7$ DNE from the left... so...

$$\boxed{\lim_{x\to 7} f(x) = DNE}$$

(p) $\sqrt{3-3(1)} = \sqrt{0} = 0$

$$\boxed{\lim_{x\to 1^-} f(x) = 0}$$

* it has to be the left-hand limit b/c $\sqrt{-x}$ opens to the LEFT, so it is only left-continuous

(q) $\dfrac{x^4+16}{4x^3+x}$ $\rightarrow$ end behavior is $y=\frac{1}{4}x$ $\boxed{\lim_{x\to\infty} f(x) = \infty}$

(r) $\sqrt[3]{\dfrac{x-3}{5-x}} = \sqrt[3]{\dfrac{x-3}{-x+5}}$ $\rightarrow$ end behavior is $y=-1$ $\rightarrow$ $\sqrt[3]{-1} = -1$

$\boxed{\lim_{x\to -\infty} f(x) = -1}$

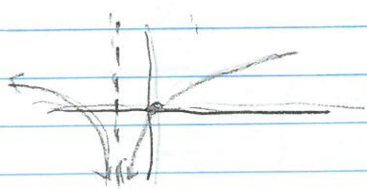
(s) $\dfrac{3x^3+x^2-2}{-2x^3+x^2+x+1}$ $y=-3/2$ $\boxed{\lim_{x\to\infty} f(x) = -3/2}$

(t) $\dfrac{x+5}{2x^2+1}$ $y=0$ $\boxed{\lim_{x\to\infty} f(x) = 0}$

(u) $\cos\left(\dfrac{x^5+1}{x^6+x^5+100}\right)$ $\rightarrow y=0$ $\cos(0)=1$ $\boxed{\lim_{x\to\infty} f(x) = 1}$

(v) $\dfrac{2x}{(x-2)(x+2)}$ 2 is a V.A. $\boxed{\lim_{x\to 2} f(x) = \text{DNE}}$

(w) $\dfrac{3x}{(x+1)(x+1)}$ $\boxed{\lim_{x\to -1} f(x) = \text{DNE} \quad (-\infty)}$



(x) $\dfrac{(x-\cancel{5})(x+5)}{(x-\cancel{5})(x+1)}$ $\uparrow$ VA $\boxed{\lim_{x\to -1} f(x) = \text{DNE}}$

(y) $$\frac{\sqrt{x^2-5}+2}{x-3}\cdot\frac{\left(\sqrt{x^2-5}-2\right)}{\left(\sqrt{x^2-5}-2\right)}$$

$$x^2-5-4 = x^2-9 = \frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}\left(\sqrt{x^2-5}-2\right)}$$

$$\frac{3+3}{\left(\sqrt{9-5}\right)-2} = \frac{6}{2-2=0}$$

ugh.

$$\boxed{\lim_{x\to 3} f(x) = \text{DNE}}$$

(B) $$\frac{2x^2-3x}{1}$$

EB: $y=2x^2$

$$\boxed{\lim_{x\to\infty} f(x) = \text{DNE } (\infty)}$$

(C) $$\frac{\sqrt{x+2}-\sqrt{2-x}}{x}\cdot\frac{\left(\sqrt{x+2}+\sqrt{2-x}\right)}{\left(\sqrt{x+2}+\sqrt{2-x}\right)} = \frac{(x+\cancel{2})-(\cancel{2}-x)}{x\left(\sqrt{x+2}+\sqrt{2-x}\right)}$$

$$\frac{2\cancel{x}}{\cancel{x}\left(\sqrt{x+2}+\sqrt{2-x}\right)} \quad \frac{2}{\sqrt{2}+\sqrt{2}} \quad \frac{\cancel{2}}{\cancel{2}\sqrt{2}} \quad \frac{1}{\sqrt{2}} \quad \boxed{\lim_{x\to 0} f(x) = \frac{\sqrt{2}}{2}}$$