# Chapter 5 Test

1. $$\begin{aligned}\sin\theta\sec\theta &= \sin\theta\cdot\frac{1}{\cos\theta} && \text{Reciprocal identity}\\ &= \frac{\sin\theta}{\cos\theta} && \text{Multiply}\\ &= \tan\theta && \text{Ratio identity}\end{aligned}$$

2. $$\begin{aligned}\frac{\cot\theta}{\csc\theta} &= \frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}} && \text{Ratio and reciprocal identities}\\ &= \cos\theta && \text{Divide}\end{aligned}$$

3. $$\begin{aligned}(\sec x-1)(\sec x+1) &= \sec^2 x-1 && \text{Multiply}\\ &= \tan^2 x && \text{Pythagorean identity}\end{aligned}$$

4. $$\begin{aligned}\tan\theta\sin\theta &= \frac{\sin\theta}{\cos\theta}\sin\theta && \text{Ratio identity}\\ &= \frac{\sin^2\theta}{\cos\theta} && \text{Multiply}\\ &= \frac{1-\cos^2\theta}{\cos\theta} && \text{Pythagorean identity}\\ &= \frac{1}{\cos\theta}-\frac{\cos^2\theta}{\cos\theta} && \text{Separate into 2 fractions}\\ &= \sec\theta-\cos\theta && \text{Reciprocal identity and reduce second fraction}\end{aligned}$$

5. $$\begin{aligned}\frac{\cos t}{1-\sin t} &= \frac{\cos t}{1-\sin t}\cdot\frac{1+\sin t}{1+\sin t} && \text{Multiply by a fraction equal to 1}\\ &= \frac{\cos t(1+\sin t)}{1-\sin^2 t} && \text{Multiply}\\ &= \frac{\cos t(1+\sin t)}{\cos^2 t} && \text{Pythagorean identity}\\ &= \frac{1+\sin t}{\cos t} && \text{Reduce}\end{aligned}$$

6. $$\frac{1}{1-\sin t}+\frac{1}{1+\sin t}=\frac{1(1+\sin t)+1(1-\sin t)}{(1-\sin t)(1+\sin t)}$$ Add fractions

$$=\frac{2}{1-\sin^2 t}$$ Simplify

$$=\frac{2}{\cos^2 t}$$ Pythagorean identity

$$=2\sec^2 t$$ Reciprocal identity

7. $$\sin(\theta-90^\circ)=\sin\theta\cos 90^\circ-\cos\theta\sin 90^\circ$$ Difference identity

$$=(\sin\theta)(0)-(\cos\theta)(1)$$ Substitute exact values

$$=-\cos\theta$$ Simplify

8. $$\cos\left(\frac{\pi}{2}+\theta\right)=\cos\frac{\pi}{2}\cos\theta-\sin\frac{\pi}{2}\sin\theta$$ Sum identity

$$=0(\cos\theta)-1(\sin\theta)$$ Substitute exact values

$$=-\sin\theta$$ Simplify

9. $$\cos^4 A-\sin^4 A=(\cos^2 A+\sin^2 A)(\cos^2 A-\sin^2 A)$$ Factor

$$=1(\cos^2 A-\sin^2 A)$$ Pythagorean identity

$$=\cos 2A$$ Double-angle identity

10. $$\frac{\sin 2A}{1-\cos 2A}=\frac{2\sin A\cos A}{1-(1-2\sin^2 A)}$$ Double-angle identities

$$=\frac{2\sin A\cos A}{2\sin^2 A}$$ Subtract

$$=\frac{\cos A}{\sin A}$$ Reduce

$$=\cot A$$ Ratio identity

11. $$\frac{\cos 2x}{\sin x\cos x}=\frac{\cos^2 x-\sin^2 x}{\sin x\cos x}$$ Double-angle identity

$$=\frac{\cos^2 x}{\sin x\cos x}-\frac{\sin^2 x}{\sin x\cos x}$$ Separate into 2 fractions

$$=\frac{\cos x}{\sin x}-\frac{\sin x}{\cos x}$$ Reduce

$$=\cot x-\tan x$$ Ratio identities

**12.**

$$\frac{\tan x}{\sec x+1}=\frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}+1}$$ Ratio and reciprocal identities

$$=\frac{\sin x}{1+\cos x}$$ Multiply numerator and denominator by $\cos x$

$$=\tan\frac{x}{2}$$ Half-angle identity

**19.**

$A$ is in QIV. Then, $\cos A=\sqrt{1-\sin^2 A}$

$$=\sqrt{1-\frac{9}{25}}$$
$$=\sqrt{\frac{16}{25}}$$
$$=\frac{4}{5}$$

$B$ is in QII. Then, $\cos B=-\sqrt{1-\sin^2 A}$

$$=-\sqrt{1-\frac{144}{169}}$$
$$=-\sqrt{\frac{25}{169}}$$
$$=-\frac{5}{13}$$

We have $\sin A=-\frac{3}{5}$ $\quad$ $\sin B=\frac{12}{13}$

$\cos A=\frac{4}{5}$ $\quad$ $\cos B=-\frac{5}{13}$

Therefore, $\sin(A+B)=\sin A\cos B+\cos A\sin B$

$$=-\frac{3}{5}\left(-\frac{5}{13}\right)+\frac{4}{5}\left(\frac{12}{13}\right)$$
$$=\frac{15}{65}+\frac{48}{65}$$
$$=\frac{63}{65}$$

**20.**

$$\cos(A-B)=\cos A\cos B+\sin A\sin B$$
$$=\frac{4}{5}\left(-\frac{5}{13}\right)+\left(-\frac{3}{5}\right)\left(\frac{12}{13}\right)$$ (See Problem 19)
$$=-\frac{20}{65}-\frac{36}{65}$$
$$=-\frac{56}{65}$$

**21.**

$$\cos 2B = 1-2\sin^2 B$$
$$= 1-2\left(\frac{12}{13}\right)^2$$
$$= 1-2\left(\frac{144}{169}\right)$$
$$= 1-\frac{288}{169}$$
$$= -\frac{119}{169}$$

**22.**

$$\sin 2B = 2\sin B\cos B$$
$$= 2\left(\frac{12}{13}\right)\left(-\frac{5}{13}\right) \quad \text{(See Problem 19)}$$
$$= -\frac{120}{169}$$

**23.** Using the information from Problem 19:

If $270^\circ \le A \le 360^\circ$, then $135^\circ \le \frac{A}{2} \le 180^\circ$.

Therefore, $\frac{A}{2}$ is in QII.

$$\sin\frac{A}{2} = \sqrt{\frac{1-\cos A}{2}}$$
$$= \sqrt{\frac{1-\frac{4}{5}}{2}}$$
$$= \sqrt{\frac{1}{10}}$$
$$= \frac{1}{\sqrt{10}}$$

**24.**

$$\cos\frac{A}{2} = -\sqrt{\frac{1+\cos A}{2}} \quad \text{(See Problem 23)}$$
$$= -\sqrt{\frac{1+\frac{4}{5}}{2}}$$
$$= -\sqrt{\frac{9}{10}}$$
$$= -\frac{3}{\sqrt{10}}$$

**25.**

$$\sin 75^\circ = \sin\left(30^\circ + 45^\circ\right)$$
$$= \sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ$$
$$= \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right)$$
$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$
$$= \frac{\sqrt{2}+\sqrt{6}}{4}$$

**26.**

$$\cos 15^\circ = \cos\left(45^\circ - 30^\circ\right)$$
$$= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$$
$$= \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) + \frac{\sqrt{2}}{2}\left(\frac{1}{2}\right)$$
$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$$
$$= \frac{\sqrt{6}+\sqrt{2}}{4}$$

**27.** $\tan\frac{\pi}{12}=\tan\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$

$$=\frac{\tan\frac{\pi}{3}-\tan\frac{\pi}{4}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{4}}$$

$$=\frac{\sqrt{3}-1}{1+\left(\sqrt{3}\right)(1)}=\frac{\sqrt{3}-1}{\sqrt{3}+1}$$

**28.** $\cot\frac{\pi}{12}=\frac{1}{\tan\frac{\pi}{12}}$

$$=\frac{1}{\left[\frac{\sqrt{3}-1}{\sqrt{3}+1}\right]}$$ (See Problem 27)

$$=\frac{\sqrt{3}+1}{\sqrt{3}-1}$$

**29.** $\cos 4x\cos 5x-\sin 4x\sin 5x=\cos(4x+5x)=\cos 9x$

**30.** $\sin 15^\circ\cos 75^\circ+\cos 15^\circ\sin 75^\circ=\sin\left(15^\circ+75^\circ\right)=\sin 90^\circ=1$

**31.** $\cos 2A=1-2\sin^2 A$

$$=1-2\left(-\frac{1}{\sqrt{5}}\right)^2$$

$$=1-\frac{2}{5}$$

$$=\frac{3}{5}$$

For $\cos\frac{A}{2}$ we must find $\cos A$.

$\cos A=-\sqrt{1-\sin^2 A}$ ($A$ is in QIII)

$$=-\sqrt{1-\frac{1}{5}}=-\sqrt{\frac{4}{5}}=-\frac{2}{\sqrt{5}}=-\frac{2\sqrt{5}}{5}$$

If $180^\circ\le A\le 270^\circ$, then $90^\circ\le\frac{A}{2}\le 135^\circ$, and $\frac{A}{2}$ is in QII.

$$\cos\frac{A}{2}=-\sqrt{\frac{1+\cos A}{2}}$$

$$=-\sqrt{\frac{1+\left(-\frac{2\sqrt{5}}{5}\right)}{2}}$$

$$=\sqrt{\frac{5-2\sqrt{5}}{10}}$$

32. If $\sec A = \sqrt{10}$, then $\cos A = \frac{1}{\sqrt{10}}$.

$$\sin A = \sqrt{1-\cos^2 A}$$
$$= \sqrt{1-\frac{1}{10}} = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}}$$

Then, $\sin 2A = 2\sin A \cos A$

$$= 2\left(\frac{3}{\sqrt{10}}\right)\left(\frac{1}{\sqrt{10}}\right) = \frac{6}{10} = \frac{3}{5}$$

Also, $\sin\frac{A}{2} = \sqrt{\frac{1-\cos A}{2}}$

$$= \sqrt{\frac{1-\frac{1}{\sqrt{10}}}{2}}$$
$$= \sqrt{\frac{\sqrt{10}-1}{2\sqrt{10}}} \text{ or } \sqrt{\frac{10-\sqrt{10}}{20}}$$

33. $\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A \tan B}$

$$3 = \frac{\tan A + \frac{1}{2}}{1-(\tan A)\left(\frac{1}{2}\right)}$$
$$3 = \frac{2\tan A + 1}{2-\tan A}$$
$$3(2-\tan A) = 2\tan A + 1$$
$$6 - 3\tan A = 2\tan A + 1$$
$$-5\tan A = -5$$
$$\tan A = 1$$

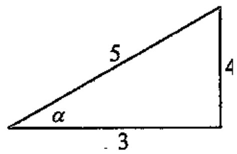
34.

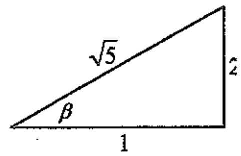
| | |
|---|---|
| $\cos 2x = 2\cos^2 x - 1$ | Double-angle identity |
| $\frac{1}{2} = 2\cos^2 x - 1$ | Substitute given value |
| $\frac{3}{2} = 2\cos^2 x$ | Subtract 1 from both sides |
| $\cos^2 x = \frac{3}{4}$ | Multiply both sides by $\frac{1}{2}$ |
| $\cos x = \pm\frac{\sqrt{3}}{2}$ | Take square root of both sides |

**35.** Let $\alpha = \arcsin\frac{4}{5}$ and $\beta = \arctan 2$.

Then $\sin\alpha = \frac{4}{5}$ and $\tan\beta = \frac{2}{1}$.

We can draw 2 triangles and label the sides accordingly:




From the figures, we have

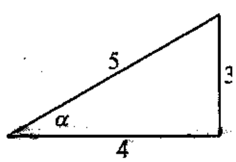
$\sin\alpha = \frac{4}{5}$ $\quad$ $\sin\beta = \frac{2}{\sqrt{5}}$

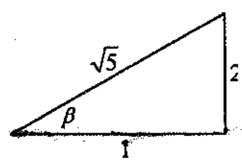
$\cos\alpha = \frac{3}{5}$ $\quad$ $\cos\beta = \frac{1}{\sqrt{5}}$

Therefore,
$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$= \frac{3}{5}\left(\frac{1}{\sqrt{5}}\right) + \frac{4}{5}\left(\frac{2}{\sqrt{5}}\right)$$
$$= \frac{3}{5\sqrt{5}} + \frac{8}{5\sqrt{5}}$$
$$= \frac{11}{5\sqrt{5}}$$

**36.** Let $\alpha = \arccos\frac{4}{5}$ and $\beta = \arctan 2$. Then $\cos\alpha = \frac{4}{5}$ and $\tan\beta = \frac{2}{1}$.

We can draw 2 triangles and label the sides accordingly:




From the figures, we have: $\quad$ $\sin\alpha = \frac{3}{5}$ $\quad$ $\sin\beta = \frac{2}{\sqrt{5}}$

$\cos\alpha = \frac{4}{5}$ $\quad$ $\cos\beta = \frac{1}{\sqrt{5}}$

This problem is continued on the next page

Therefore, $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

$$=\frac{3}{5}\left(\frac{1}{\sqrt{5}}\right)+\frac{4}{5}\left(\frac{2}{\sqrt{5}}\right)$$

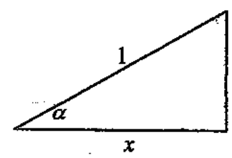
$$=\frac{3}{5\sqrt{5}}+\frac{8}{5\sqrt{5}}$$

$$=\frac{11}{5\sqrt{5}}$$

**37.** Let $\alpha=\sin^{-1}x$. Then $\sin\alpha=x$

$$\cos 2\alpha=1-2\sin^2\alpha$$
$$=1-2x^2$$

**38.** Let $\alpha=\cos^{-1}x$. Then $\cos\alpha=\frac{x}{1}$. We can draw a triangle and use the Pythagorean Theorem to find the opposite side:




opposite side $=\sqrt{1^2-x^2}=\sqrt{1-x^2}$

Therefore, $\sin\alpha=\frac{\sqrt{1-x^2}}{1}=\sqrt{1-x^2}$

$$\sin 2\alpha=2\sin\alpha\cos\alpha$$
$$=2\sqrt{1-x^2}\,(x)=2x\sqrt{1-x^2}$$

**39.** $\sin 6x\sin 4x=\frac{1}{2}\left[\cos(6x-4x)-\cos(6x+4x)\right]$

$$=\frac{1}{2}(\cos 2x-\cos 10x)$$

**40.** $\cos 15^\circ+\cos 75^\circ=2\cos\frac{15^\circ+75^\circ}{2}\cos\frac{15^\circ-75^\circ}{2}$

$$=2\cos 45^\circ\cos(-30^\circ)$$
$$=2\cos 45^\circ\cos 30^\circ$$
$$=2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$
$$=\frac{\sqrt{6}}{2}$$