

## Chapter 6 Test

1.  $2\sin\theta - 1 = 0$   
 $2\sin\theta = 1$  Add 1 to both sides  
 $\sin\theta = \frac{1}{2}$  Divide both sides by 2  
 $\hat{\theta} = 30^\circ$  and  $\theta$  is in QI or QII  
 $\theta = 30^\circ$  or  $150^\circ$
2.  $\sqrt{3}\tan\theta + 1 = 0$   
 $\sqrt{3}\tan\theta = -1$  Subtract 1 from both sides  
 $\tan\theta = -\frac{1}{\sqrt{3}}$  Divide both sides by  $\sqrt{3}$   
 $\hat{\theta} = 30^\circ$  and  $\theta$  is in QII or QIV  
 $\theta = 150^\circ$  or  $330^\circ$
3.  $\cos\theta - 2\sin\theta\cos\theta = 0$   
 $\cos\theta(1 - 2\sin\theta) = 0$  Factor  
 $\cos\theta = 0$  or  $1 - 2\sin\theta = 0$  Set each factor = 0  
 $\theta = 90^\circ$  or  $270^\circ$   $-2\sin\theta = -1$  Solve each equation  
 $\sin\theta = \frac{1}{2}$   
 $\theta = 30^\circ$  or  $150^\circ$
4.  $\tan\theta - 2\cos\theta\tan\theta = 0$   
 $\tan\theta(1 - 2\cos\theta) = 0$  Factor  
 $\tan\theta = 0$  or  $1 - 2\cos\theta = 0$  Set each factor = 0  
 $\theta = 0^\circ$  or  $180^\circ$   $-2\cos\theta = -1$  Solve each equation  
 $\cos\theta = \frac{1}{2}$   
 $\theta = 60^\circ$  or  $300^\circ$
5.  $4\cos\theta - 2\sec\theta = 0$   
 $4\cos\theta - \frac{2}{\cos\theta} = 0$  Reciprocal identity  
 $4\cos^2\theta - 2 = 0$  Multiply both sides by  $\cos\theta$   
 $4\cos^2\theta = 2$  Add 2 to both sides

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$$\cos^2 \theta = \frac{1}{2}$$

$$\cos \theta = \pm \frac{1}{\sqrt{2}}$$

$$\theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ$$

Divide both sides by 4

Take square root of both sides

$$\hat{\theta} = 45^\circ$$

6.

$$2 \sin \theta - \csc \theta = 1$$

$$2 \sin \theta - \frac{1}{\sin \theta} = 1$$

$$2 \sin^2 \theta - 1 = \sin \theta$$

$$2 \sin^2 \theta - \sin \theta - 1 = 0$$

$$(2 \sin \theta + 1)(\sin \theta - 1) = 0$$

$$2 \sin \theta + 1 = 0 \quad \text{or} \quad \sin \theta - 1 = 0$$

$$2 \sin \theta = -1 \quad \sin \theta = 1$$

$$\sin \theta = -\frac{1}{2} \quad \theta = 90^\circ$$

$$\theta = 210^\circ \text{ or } 330^\circ$$

Reciprocal identity

Multiply both sides by  $\sin \theta$

Rewrite in standard form

Factor

Set each factor = 0

Solve each equation

7.

$$\sin \frac{\theta}{2} + \cos \theta = 0$$

$$\pm \sqrt{\frac{1 - \cos \theta}{2}} + \cos \theta = 0$$

$$\pm \sqrt{\frac{1 - \cos \theta}{2}} = -\cos \theta$$

$$\frac{1 - \cos \theta}{2} = \cos^2 \theta$$

$$1 - \cos \theta = 2 \cos^2 \theta$$

$$2 \cos^2 \theta + \cos \theta - 1 = 0$$

$$(2 \cos \theta - 1)(\cos \theta + 1) = 0$$

$$2 \cos \theta - 1 = 0 \quad \text{or} \quad \cos \theta + 1 = 0$$

$$2 \cos \theta = 1 \quad \cos \theta = -1$$

$$\cos \theta = \frac{1}{2} \quad \theta = 180^\circ$$

$$\theta = 60^\circ \text{ or } 300^\circ$$

Half-angle identity

Subtract  $\cos \theta$  from both sides

Square both sides

Multiply both sides by 2

Rewrite in standard form

Factor

Set each factor = 0

Solve each equation and check

Possible solutions

We check each possible solution and the only one that checks is  $180^\circ$ .

8.  $\cos \frac{\theta}{2} - \cos \theta = 0$

$\pm \sqrt{\frac{1 + \cos \theta}{2}} - \cos \theta = 0$       Half-angle identity

$\pm \sqrt{\frac{1 + \cos \theta}{2}} = \cos \theta$       Add  $\cos \theta$  to both sides

$\frac{1 + \cos \theta}{2} = \cos^2 \theta$       Square both sides

$1 + \cos \theta = 2 \cos^2 \theta$       Multiply both sides by 2

$2 \cos^2 \theta - \cos \theta - 1 = 0$       Rewrite in standard form

$(2 \cos \theta + 1)(\cos \theta - 1) = 0$       Factor

$2 \cos \theta + 1 = 0$  or  $\cos \theta - 1 = 0$       Set each factor = 0

$2 \cos \theta = -1$        $\cos \theta = 1$       Solve each equation and check

$\cos \theta = -\frac{1}{2}$        $\theta = 0^\circ$       Possible solutions

$\theta = 120^\circ$  or  $240^\circ$

We check all the possible solutions and  $0^\circ$  and  $240^\circ$  check.

9.  $4 \cos 2\theta + 2 \sin \theta = 1$

$4(1 - 2 \sin^2 \theta) + 2 \sin \theta = 1$       Double-angle identity

$4 - 8 \sin^2 \theta + 2 \sin \theta = 1$       Simplify

$-8 \sin^2 \theta + 2 \sin \theta + 3 = 0$       Subtract 1 from both sides

$8 \sin^2 \theta - 2 \sin \theta - 3 = 0$       Multiply both sides by -1

$(4 \sin \theta - 3)(2 \sin \theta + 1) = 0$       Factor

$4 \sin \theta - 3 = 0$  or  $2 \sin \theta + 1 = 0$       Set each factor = 0

$4 \sin \theta = 3$        $2 \sin \theta = -1$       Solve each equation

$\sin \theta = \frac{3}{4}$        $\sin \theta = -\frac{1}{2}$

$\theta = 48.6^\circ$  or  $131.4^\circ$        $\theta = 210^\circ$  or  $330^\circ$

10.  $\sin(3\theta - 45^\circ) = -\frac{\sqrt{3}}{2}$       (The reference angle is  $60^\circ$ )

$3\theta - 45^\circ = 240^\circ + 360^\circ k$       or       $3\theta - 45^\circ = 300^\circ + 360^\circ k$

$3\theta = 285^\circ + 360^\circ k$        $3\theta = 345^\circ + 360^\circ k$

$\theta = 95^\circ + 120^\circ k$        $\theta = 115^\circ + 120^\circ k$

Let  $k = 0, 1,$  and  $2,$  then

$\theta = 95^\circ$        $\theta = 115^\circ$

$\theta = 215^\circ$        $\theta = 235^\circ$

$\theta = 335^\circ$        $\theta = 355^\circ$

11.  $\sin \theta + \cos \theta = 1$  Subtract  $\cos \theta$  from both sides  
 $\sin \theta = 1 - \cos \theta$  Square both sides  
 $\sin^2 \theta = 1 - 2\cos \theta + 2\cos^2 \theta$  Pythagorean identity  
 $1 - \cos^2 \theta = 1 - 2\cos \theta + \cos^2 \theta$  Rewrite in standard form  
 $2\cos^2 \theta - 2\cos \theta = 0$  Factor  
 $2\cos \theta(\cos \theta - 1) = 0$  Set each factor = 0  
 $\cos \theta = 0$  or  $\cos \theta - 1 = 0$  Solve each equation and check  
 $\theta = 90^\circ$  or  $270^\circ$   $\cos \theta = 1$   
 $\theta = 0^\circ$

We check the possible solutions and  $0^\circ$  and  $90^\circ$  check.

12.  $\sin \theta - \cos \theta = 1$  Add  $\cos \theta$  to both sides  
 $\sin \theta = 1 + \cos \theta$  Square both sides  
 $\sin^2 \theta = 1 + 2\cos \theta + \cos^2 \theta$  Pythagorean identity  
 $1 - \cos^2 \theta = 1 + 2\cos \theta + \cos^2 \theta$  Rewrite in standard form  
 $2\cos^2 \theta + 2\cos \theta = 0$  Factor  
 $2\cos \theta(\cos \theta + 1) = 0$  Set each factor = 0  
 $\cos \theta = 0$  or  $\cos \theta + 1 = 0$  Solve each equation and check  
 $\theta = 90^\circ$  or  $270^\circ$   $\cos \theta = -1$   
 $\theta = 180^\circ$

We check the possible solutions and  $90^\circ$  and  $180^\circ$  check.

13.  $\cos 3\theta = -\frac{1}{2}$  (Reference angle is  $60^\circ$ )  
 $3\theta = 120^\circ + 360^\circ k$  or  $3\theta = 240^\circ + 360^\circ k$   
 $\theta = 40^\circ + 120^\circ k$   $\theta = 80^\circ + 120^\circ k$   
We let  $k = 0, 1,$  and  $2$ :  
 $\theta = 40^\circ$   $\theta = 80^\circ$   
 $\theta = 160^\circ$   $\theta = 200^\circ$   
 $\theta = 280^\circ$   $\theta = 320^\circ$

14.  $\tan 2\theta = 1$  (Reference angle is  $45^\circ$ )  
 $2\theta = 45^\circ + 360^\circ k$  or  $2\theta = 225^\circ + 360^\circ k$   
 $\theta = 22.5^\circ + 180^\circ k$   $\theta = 112.5^\circ + 180^\circ k$   
We let  $k = 0$  and  $1$ :  
 $\theta = 22.5^\circ$   $\theta = 112.5^\circ$   
 $\theta = 202.5^\circ$   $\theta = 292.5^\circ$

15.  $\cos 2x - 3\cos x = -2$   
 $2\cos^2 x - 1 - 3\cos x = -2$  Double-angle identity  
 $2\cos^2 x - 3\cos x + 1 = 0$  Add 2 to both sides  
 $(2\cos x - 1)(\cos x - 1) = 0$  Factor  
 $2\cos x - 1 = 0$  or  $\cos x - 1 = 0$  Set each factor = 0  
 $2\cos x = 1$   $\cos x = 1$  Solve each equation  
 $x = 0 + 2k\pi$   
 $\cos x = \frac{1}{2}$   $x = 2k\pi$   
 $x = \frac{\pi}{3} + 2k\pi$  or  
 $x = \frac{5\pi}{3} + 2k\pi$
16.  $\sqrt{3}\sin x - \cos x = 0$   
 $\sqrt{3}\sin x = \cos x$  Add  $\cos x$  to both sides  
 $\sqrt{3}\frac{\sin x}{\cos x} = 1$  Divide both sides by  $\cos x$  ( $\cos x \neq 0$ )  
 $\sqrt{3}\tan x = 1$  Ratio identity  
 $\tan x = \frac{1}{\sqrt{3}}$  Divide both sides by  $\sqrt{3}$ .  
 $x = \frac{\pi}{6} + k\pi$  (This answer can also be written as  $\frac{\pi}{6} + 2k\pi$  or  $\frac{7\pi}{6} + 2k\pi$ )
17.  $\sin 2x \cos x + \cos 2x \sin x = -1$   
 $\sin(2x + x) = -1$  Sum identity  
 $\sin 3x = -1$  Simplify  
 $3x = \frac{3\pi}{2} + 2k\pi$   
 $x = \frac{\pi}{2} + \frac{2k\pi}{3}$
18.  $\sin^3 4x = 1$   
 $\sin 4x = 1$  Take cube root of both sides  
 $4x = \frac{\pi}{2} + 2k\pi$   
 $x = \frac{\pi}{8} + \frac{k\pi}{2}$

19.  $5\sin^2\theta - 3\sin\theta = 2$   
 $5\sin^2\theta - 3\sin\theta - 2 = 0$   
 $(5\sin\theta + 2)(\sin\theta - 1) = 0$   
 $5\sin\theta + 2 = 0$  or  $\sin\theta - 1 = 0$   
 $5\sin\theta = -2$        $\sin\theta = 1$   
 $\sin\theta = -\frac{2}{5}$        $\theta = 90^\circ$   
 $\sin\theta = -0.4$   
 $\hat{\theta} = 23.6^\circ$   
 $\theta = 203.6^\circ$  or  $336.4^\circ$

Subtract 2 from both sides  
Factor  
Set each factor = 0  
Solve each equation

20.  $4\cos^2\theta - 4\cos\theta = 2$   
 $4\cos^2\theta - 4\cos\theta - 2 = 0$   
 $2\cos^2\theta - 2\cos\theta - 1 = 0$   
 $\cos\theta = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-1)}}{2(2)}$        $a = 2, b = -2, c = -1$   
 $= \frac{2 \pm \sqrt{12}}{4}$   
 $\cos\theta = \frac{2 \pm 3.4641}{4}$   
 $\cos\theta = 1.3660$       or       $\cos\theta = -0.3660$   
No solution       $\theta = 111.5^\circ$  or  $248.5^\circ$

Subtract 2 from both sides  
Divide both sides by 2

21. Graph  $y_1 = 3\sin(x) - 2$  on your graphing calculator. Use the zero or root finder to locate the zeros of the function. The solution is 0.7297 or 2.4419.

22. Graph  $y_1 = \cos(x) - 4\sin(x) + 3$  on your graphing calculator. Use the zero or root finder to locate the zeros of the function. The solution is 1.0598 or 2.5717.

23. Graph  $y_1 = (\sin(x))^2 + 3\sin(x) - 1$  on your graphing calculator. Use the zero or root finder to locate the zeros of the function. The solution is 0.3076 or 2.8340.

24. Graph  $y_1 = \sin(2x) - \frac{3}{5}$  on your graphing calculator. Use the zero or root finder to locate the zeros of the function. The solution is 0.3218, 1.2490, 3.4633, or 4.3906.