

Name: Answer key

Date: \_\_\_\_\_

Period: \_\_\_\_\_

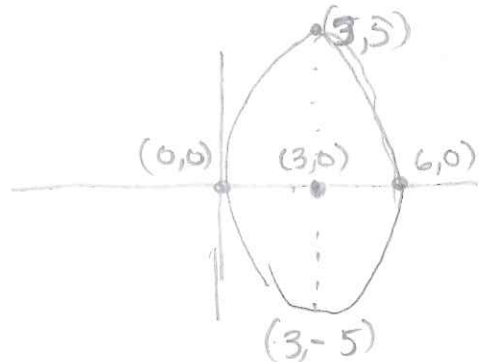
Pre-Calculus

### Writing Equations for Ellipses

1. Write the equation for an ellipse with center  $(3,0)$ , vertical major axis of length 10 and minor axis of length 6.

$$a=3$$
$$b=5$$

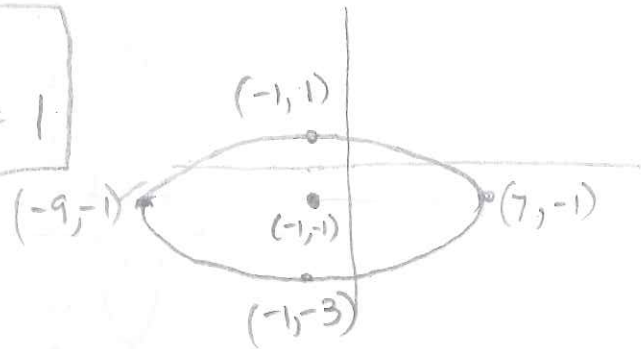
$$\frac{(x-3)^2}{9} + \frac{y^2}{25} = 1$$



2. Write the equation for an ellipse with center  $(-1, -1)$  with horizontal major axis of length 16 and minor axis of length 4.

$$a=8$$
$$b=2$$

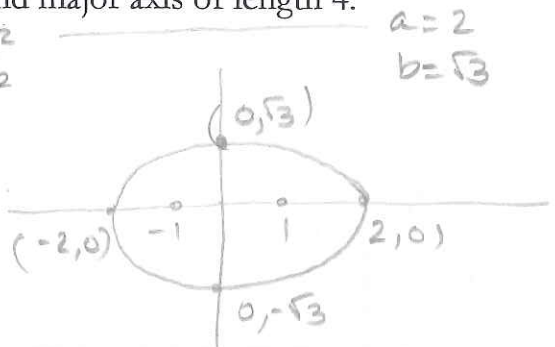
$$\frac{(x+1)^2}{64} + \frac{(y+1)^2}{4} = 1$$



3. Write the equation for an ellipse with foci  $(-1,0)$  and  $(1,0)$  and major axis of length 4.

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

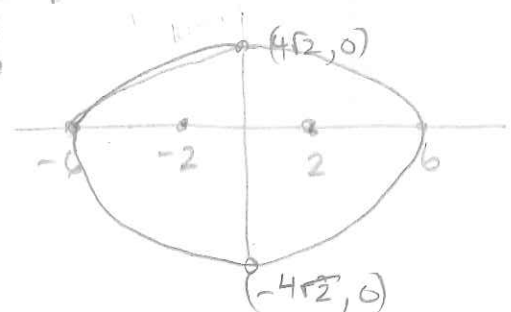
$$b^2 = a^2 - c^2$$
$$b^2 = 2^2 - 1^2$$
$$= 4 - 1$$
$$= 3$$
$$b = \sqrt{3}$$



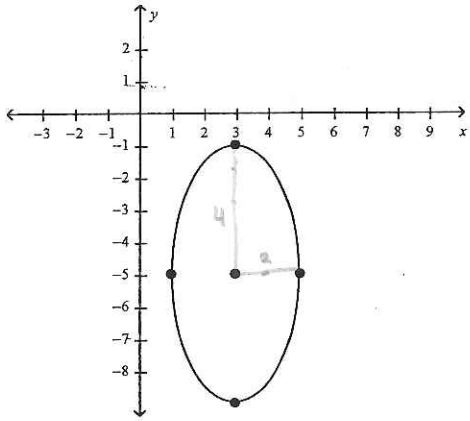
4. Write the equation for an ellipse with vertices at  $(6,0)$  and  $(-6,0)$  and foci at  $(2,0)$  and  $(-2,0)$

$$\frac{x^2}{36} + \frac{y^2}{32} = 1$$

$$b^2 = a^2 - c^2$$
$$b^2 = 36 - 4$$
$$\sqrt{b^2} = \sqrt{32}$$
$$b = 4\sqrt{2}$$

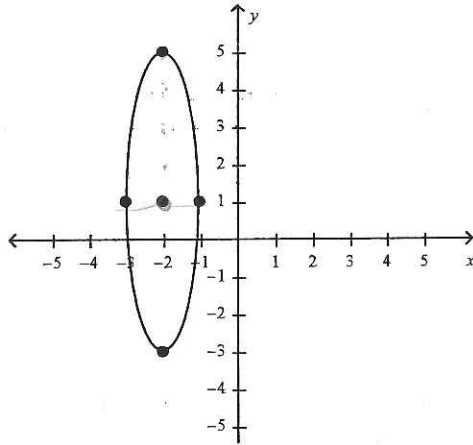


5.



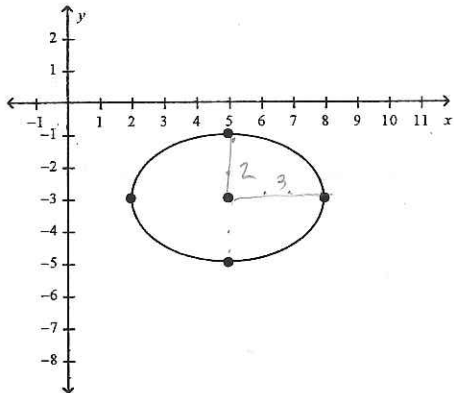
$$\frac{(x-3)^2}{4} + \frac{(y+5)^2}{16} = 1$$

6.



$$\frac{(x+2)^2}{4} + \frac{(y-1)^2}{16} = 1$$

7.



$$\frac{(x-5)^2}{9} + \frac{(y+3)^2}{4} = 1$$

*\* WORK IS ON NEXT PAGE*

1. Find the center, vertices, covertices, foci, length of major and minor axes, and sketch the graph.

a.  $\frac{(x-4)^2}{9} + \frac{(y+3)^2}{25} = 1$

center:  $(4, -3)$

major axis:  $10$

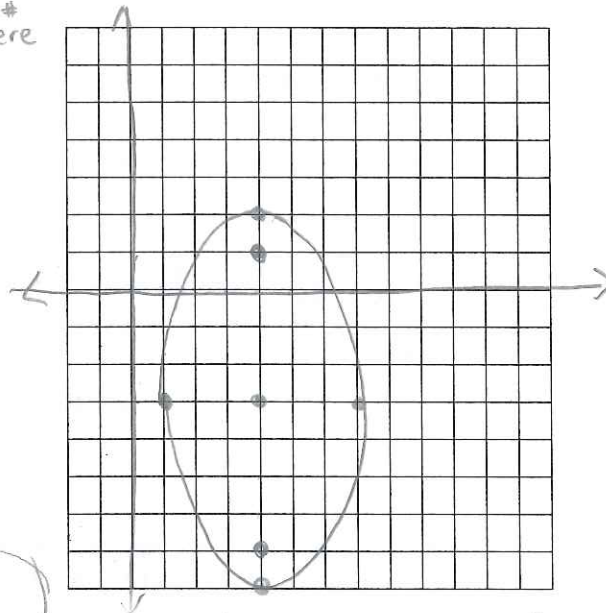
length (minor axis:  $6$ )

vertices:  $(4, 2)$   $(4, -8)$

covertices:  $(7, -3)$   $(1, -3)$

foci:  $(4, 1)$   $(4, -7)$

$c^2 = a^2 - b^2$   
 $c^2 = 25 - 9$   
 $c^2 = 16$   
 $c = 4$   
*put bigger # here*



$a = 10$   $b = 2\sqrt{5}$

b.  $x^2 + 5y^2 - 8x - 30y - 39 = 0$

center:  $(4, 3)$

major axis:  $20$

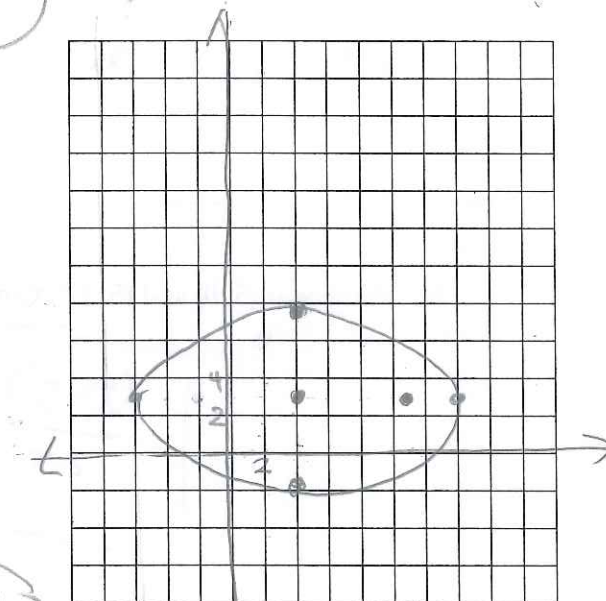
length (minor axis:  $4\sqrt{5}$ )

vertices:  $(14, 3)$   $(-6, 3)$

covertices:  $(4, 3 \pm 2\sqrt{5})$

foci:  $(4 \pm 4\sqrt{5}, 3)$

$c^2 = a^2 - b^2$   
 $c^2 = 100 - 20$   
 $c^2 = 80$   
 $c = \sqrt{80} = 4\sqrt{5}$



c.  $9x^2 + 4y^2 + 54x + 8y + 49 = 0$

center:  $(-3, -1)$

major axis:  $6$

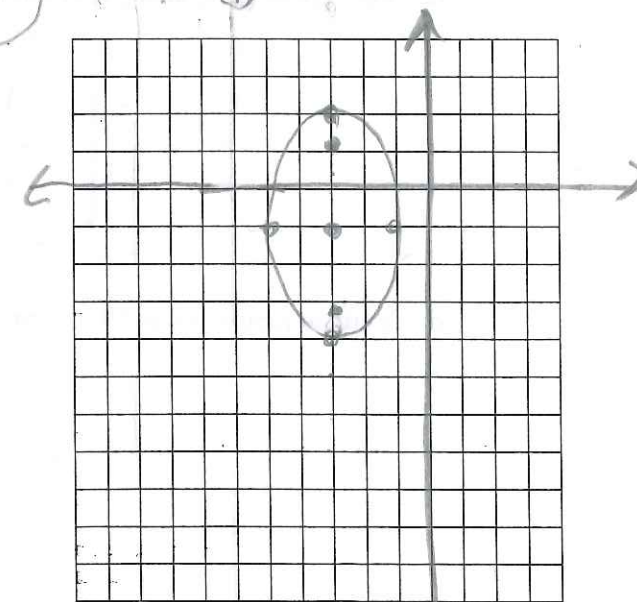
length (minor axis:  $4$ )

vertices:  $(-3, 2)$   $(-3, -4)$

covertices:  $(-1, -1)$   $(-5, -1)$

foci:  $(-3, -1 + \sqrt{5})$   $(-3, -1 - \sqrt{5})$   
 $\approx 1, 2, 3, 6$   
 $\approx -3, 2, 3, 6$

$a^2 - b^2 = c^2$   
 $3^2 - 2^2 = c^2$   
 $9 - 4 = c^2$   
 $5 = c^2$   
 $c = \sqrt{5}$



d:  $25x^2 + y^2 - 300x + 8y + 891 = 0$

center:  $(6, 4)$

major axis:  $10$

length

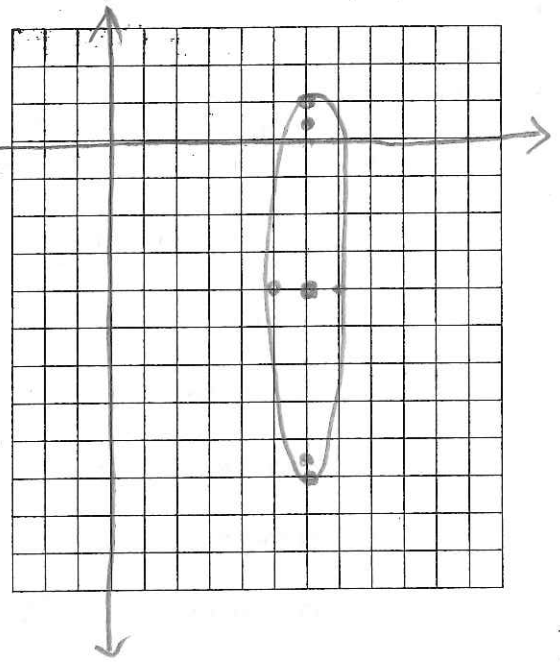
minor axis:  $2$

vertices:  $(6, 1)$   $(6, 9)$

covertices:  $(7, 4)$   $(5, 4)$

foci:  $(6, -4 \pm 2\sqrt{6})$   
 $\approx -8.89$   
 $\approx 0.89$

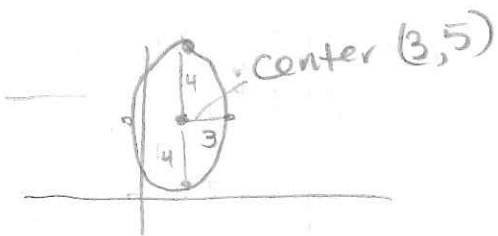
$$\frac{(x-6)^2}{1} + \frac{(y+4)^2}{25} = 1$$



2. Write the equation of an ellipse given the provided information.

a. Vertices at  $(3, 1)$  and  $(3, 9)$ . Minor axis of length 6.

$a=8$   $b=6$



$$\frac{(x-3)^2}{9} + \frac{(y-5)^2}{16} = 1$$

b. Vertices at  $(5, 0)$  and  $(5, 12)$ . Covertices at  $(0, 6)$  and  $(10, 6)$

$$\frac{(x-5)^2}{25} + \frac{(y-6)^2}{36} = 1$$

c. Foci are at  $(1, 0)$  and  $(-1, 0)$  and the length of the major axis is 4.

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

d. Vertical major axis is 10, minor axis is 6, and center is  $(3, 0)$ .

$$\frac{(x-3)^2}{25} + \frac{y^2}{9} = 1$$

Work for packet pages 3-4.

1. a) on packet

$$x^2 + 5y^2 - 8x - 30y - 39 = 0$$

$$(x^2 - 8x \quad \_) + (5y^2 - 30y \quad \_) = 39$$

$$(x^2 - 8x \quad \textcircled{+16}) + 5(y^2 - 6y \quad \_) = 39$$

$\boxed{+9}$ 
 $\textcircled{+16}$

$\rightarrow$ 
 $\boxed{+45}$

original

move & group

pull out what you can.  
Complete the square  
(keep balanced on right side)

b)  $(x-4)^2 + 5(y-3)^2 = 100$

$$\boxed{\frac{(x-4)^2}{100} + \frac{(y-3)^2}{20} = 1}$$

divide to make = 1

c)  $9x^2 + 4y^2 + 54x + 8y + 44 = 0$

$$9x^2 + 54x + 4y^2 + 8y = -44$$

$$9(x^2 + 6x + 9) + 4(y^2 + 2y + 1) = -44$$

$\textcircled{+81}$

$$\frac{9(x+3)^2}{36} + \frac{4(y+1)^2}{36} = \frac{36}{36}$$

$\boxed{+4}$

$$\boxed{\frac{(x+3)^2}{4} + \frac{(y+1)^2}{9} = 1}$$

a)  $25x^2 + y^2 - 300x + 8y + 891 = 0$

$$25x^2 - 300x + y^2 + 8y = -891$$

$$25(x^2 - 12x \quad \textcircled{+36}) + (y^2 + 8y \quad \textcircled{+16}) = -891$$

$\textcircled{+900}$ 
 $\textcircled{+16}$

$$\underline{\underline{(x-6)^2}} + \underline{\underline{(y+4)^2}} = 25$$

$$\rightarrow \boxed{\frac{(x-6)^2}{1} + \frac{(y+4)^2}{25} = 1}$$

Example 3

Let  $f(x) = \begin{cases} ax^2 + 6x & x \leq 2 \\ bx^3 - 3 & x > 2 \end{cases}$ . Find the values of  $a$  and  $b$  such that  $f(x)$  is differentiable at  $x = 2$ .

Example 4

Let  $f(x) = \begin{cases} bx^2 + 6x & x \leq 2 \\ ax^3 & x > 2 \end{cases}$ . Find the values of  $a$  and  $b$  such that  $f(x)$  is differentiable at  $x = 2$ .

hi ☺