

Name: _____ Per: _____ Date: _____

Serafino • Precalculus

Limits Review**13-R**

Review

You will not have access to a calculator for this assessment so only use it to verify your answers.

1. To the right, sketch a function for which all of the following is true.

$$\lim_{x \rightarrow 0} f(x) = 3 \quad f(0) = -2$$

$$\lim_{x \rightarrow 3^-} f(x) = -\infty \quad \lim_{x \rightarrow 3^+} f(x) = \infty$$

$$\lim_{x \rightarrow 5^-} f(x) = 4 \quad \lim_{x \rightarrow 5^+} f(x) = 2$$

2. To the right, sketch a function that has:

Has an infinite discontinuity at $f(-1)$

Has a removable discontinuity at $f(3)$

Has a jump discontinuity at $f(5)$

Has a limit that is negative as $x \rightarrow \infty$

3. The graph of the function f is given to the right. Use it to determine the following limits.

a. $\lim_{x \rightarrow -3^-} f(x) =$ $\lim_{x \rightarrow -3^+} f(x) =$

b. $\lim_{x \rightarrow -1^-} f(x) =$ $\lim_{x \rightarrow -1^+} f(x) =$

c. $\lim_{x \rightarrow 0^-} f(x) =$ $\lim_{x \rightarrow 0^+} f(x) =$

d. $\lim_{x \rightarrow 1^-} f(x) =$ $\lim_{x \rightarrow 1^+} f(x) =$

e. $\lim_{x \rightarrow 2^-} f(x) =$ $\lim_{x \rightarrow 2^+} f(x) =$

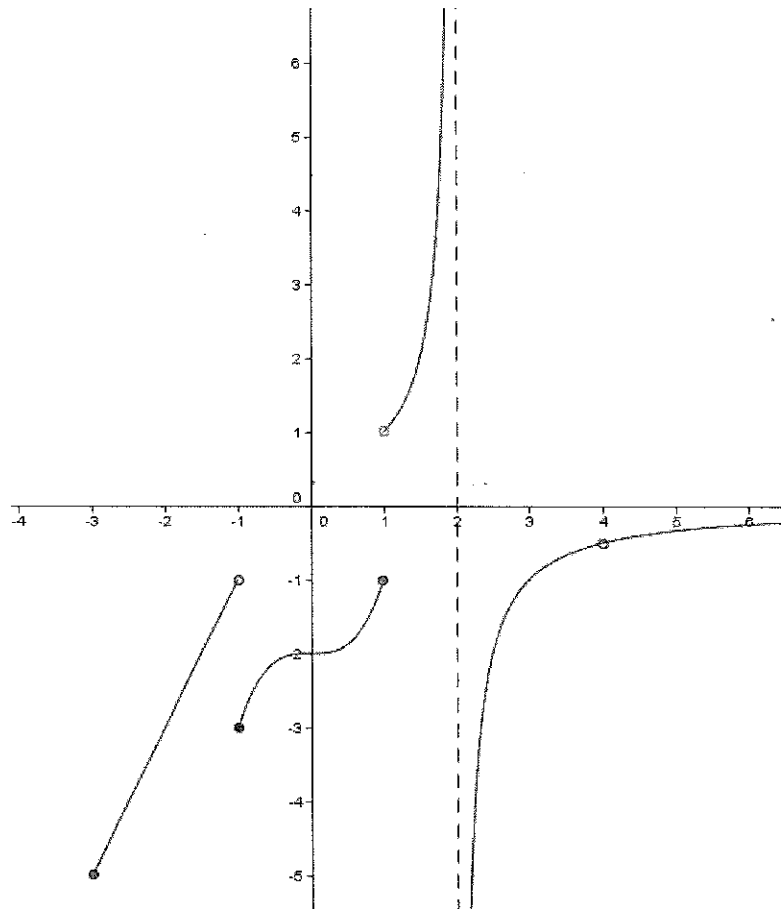
f. $\lim_{x \rightarrow 4^-} f(x) =$ $\lim_{x \rightarrow 4^+} f(x) =$

- g. Above, circle the letter(s) of the problems whose limits that exist

h. Name one removable discontinuity

i. Name one infinite discontinuity

j. Name one jump discontinuity



4. Find the limits of the piecewise function:

a. $\lim_{x \rightarrow 0^-} f(x) =$ $\lim_{x \rightarrow 0^+} f(x) =$

b. $\lim_{x \rightarrow -1^-} f(x) =$ $\lim_{x \rightarrow -1^+} f(x) =$

c. $\lim_{x \rightarrow 2^-} f(x) =$ $\lim_{x \rightarrow 2^+} f(x) =$

d. $\lim_{x \rightarrow 3^+} f(x) =$ $\lim_{x \rightarrow 3^-} f(x) =$

e. $\lim_{x \rightarrow 4^-} f(x) =$ $\lim_{x \rightarrow 4^+} f(x) =$

$$f(x) = \begin{cases} -x + 1, & 0 \leq x < 1 \\ 1, & 1 \leq x < 2 \\ 2, & x = 2 \\ x - 1, & 2 < x \leq 3 \\ -x + 5, & 3 < x \leq 4. \end{cases}$$

f. Put a star next to the letter whose x-value is right continuous.

g. Put a smiley face next to the letter whose value is left-continuous

Evaluate limits: Remember your options: direct evaluation, factoring & canceling, multiplying by the conjugate. It may help to remember your rules for horizontal asymptotes, and to recall the sum/diff of cubes:

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

5. $\lim_{x \rightarrow 3} \frac{x^3 - 27}{x - 3}$

6. $\lim_{x \rightarrow 3} \frac{x^2 - 7x + 12}{-x + 3}$

7. $\lim_{x \rightarrow 2} \frac{3x^2 - 13x + 14}{x^2 - 4}$

8. $\lim_{x \rightarrow 1} 3x^3 - 2x^2 + 4$

9. $\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 4}$

10. $\lim_{x \rightarrow 2} \frac{5}{x - 4}$

11. $\lim_{x \rightarrow 4} \frac{x^2 - 5x + 4}{x^2 - 2x - 8}$

12. $\lim_{x \rightarrow 2} \frac{3x^2 - x - 10}{x^2 - 4}$

13. $\lim_{x \rightarrow 2} \frac{x^4 - 81}{2x^2 - 5x - 3}$

14. $\lim_{x \rightarrow 4} \frac{x - 16}{\sqrt{x} - 4}$

15. $\lim_{x \rightarrow 2} x^3 - x^2$

16. $\lim_{x \rightarrow 3} \frac{2x + 1}{x - 3}$

17.
$$\lim_{x \rightarrow 0} \frac{6x^2 + 3x}{x}$$

18.
$$\lim_{x \rightarrow 0} \frac{7x}{x}$$

19.
$$\lim_{x \rightarrow 1} \frac{2-x}{(x-1)^2}$$

21.
$$\lim_{x \rightarrow \pi} \cot x$$

22.
$$\lim_{x \rightarrow \infty} \frac{x^3 + 5x}{2x^3 - x^2 + 4}$$

23.
$$\lim_{x \rightarrow \infty} \frac{x^2 + 2}{x^3 + x^2 - 1}$$

24.
$$\lim_{x \rightarrow \infty} \frac{3x^3 + x}{4x^2 - x + 1}$$

25.
$$\lim_{x \rightarrow \infty} \frac{4x^2 - x + 1}{3x^3 + x}$$

26.
$$f(x) = \begin{cases} \frac{x^2 + 5x - 6}{x - 2} & \text{if } x \neq -2 \\ 0 & \text{if } x = -2 \end{cases}$$

$$\lim_{x \rightarrow 3} f(x)$$

27.
$$f(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ 3x + 2 & \text{if } x \geq 0 \end{cases}$$

$$\lim_{x \rightarrow 0} f(x) =$$

Is the following function continuous? If not, state the type of continuity that exists. Sketch to confirm.

28.
$$f(x) = \frac{x+3}{x^2 - 5x + 6}$$

29.
$$f(x) = \frac{x+7}{x^2 + 6x - 7}$$

30.
$$f(x) = \begin{cases} x^2 + 2 & x < 1 \\ 5x - 2 & x \geq 1 \end{cases}$$

31.
$$f(x) = \begin{cases} 2x + 3 & x \leq 0 \\ x^2 - 8 & x > 0 \end{cases}$$

More limits practice: On a separate sheet, write the limit notation and evaluate the following limits. If the limit does not exist, state why. You can sketch/graph the function to check your answer. You should be able to do these without a calc.

Function	Find limits at
Eg. $f(x) = -\frac{1}{2}x + 7$	-4, 8 $\lim_{x \rightarrow -4} f(x) = -\frac{1}{2}(-4) + 7 = 9$ $\lim_{x \rightarrow 8} f(x) = -\frac{1}{2}(8) + 7 = 3$
1. $f(x) = 2x^2 - 3x + 8$	0, 2
2. $g(x) = \frac{x^2 - 1}{x + 1}$	4, -1
3. $h(x) = \frac{x - 4}{\sqrt{x + 4} - 3}$	4, 5
4. $f(x) = \sin(x)$	$\pi, 3\pi/2$
5. $g(x) = \begin{cases} 3x - 5 & \text{if } x \geq 1 \\ 2 - x & \text{if } x < 1 \end{cases}$	-3, 3, 1
6. $h(x) = \frac{x - 4}{x^2 - x - 12}$	0, 4
7. $f(x) = e^x$	0, -2
8. $f(x) = \frac{x^4 - 81}{x^2 - 9}$	0, 3, -3
9. $g(x) = \frac{x^2 - 6x + 9}{x - 3}$	4, 3
10. $g(x) = \frac{x^3 - 8}{x - 2}$	0, 3, 2
11. $h(x) = \begin{cases} x^2 - 1 & \text{if } x \geq 2 \\ 5 - x & \text{if } x < 2 \end{cases}$	-4, 2, 5
12. $f(x) = \ln x$	1, e
13. $g(x) = \frac{x^3 + 27}{x + 3}$	3, -3