Name: Answer key Per: _____ Date: _____

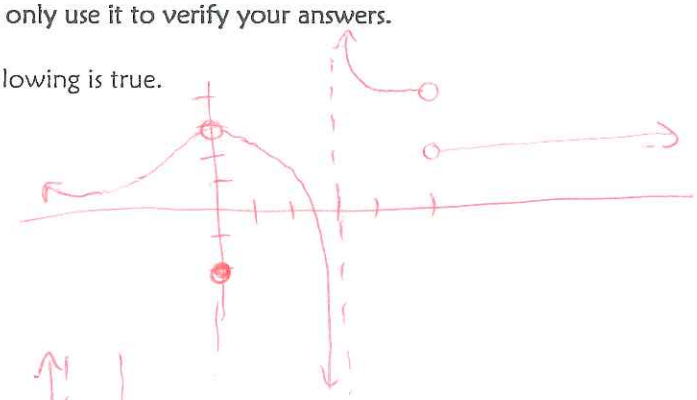
Serafino ▪ Precalculus

13–R Limits Review

Review

You will not have access to a calculator for this assessment so only use it to verify your answers.

1. To the right, sketch a function for which all of the following is true.

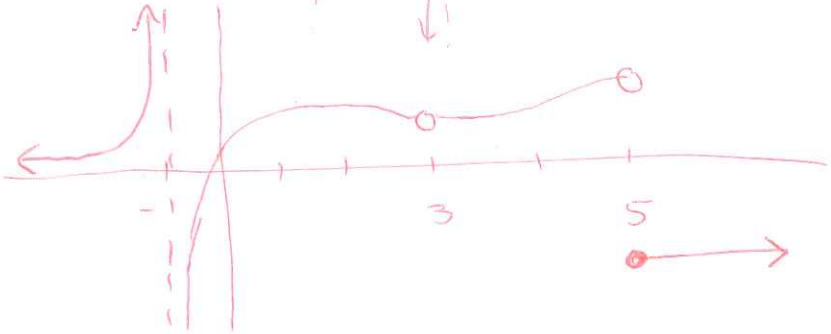


$\lim_{x\to 0} f(x) = 3$ $\quad f(0) = -2$

$\lim_{x\to 3^-} f(x) = -\infty$ $\quad \lim_{x\to 3^+} f(x) = \infty$

$\lim_{x\to 5^-} f(x) = 4$ $\quad \lim_{x\to 5^+} f(x) = 2$

2. To the right, sketch a function that has:



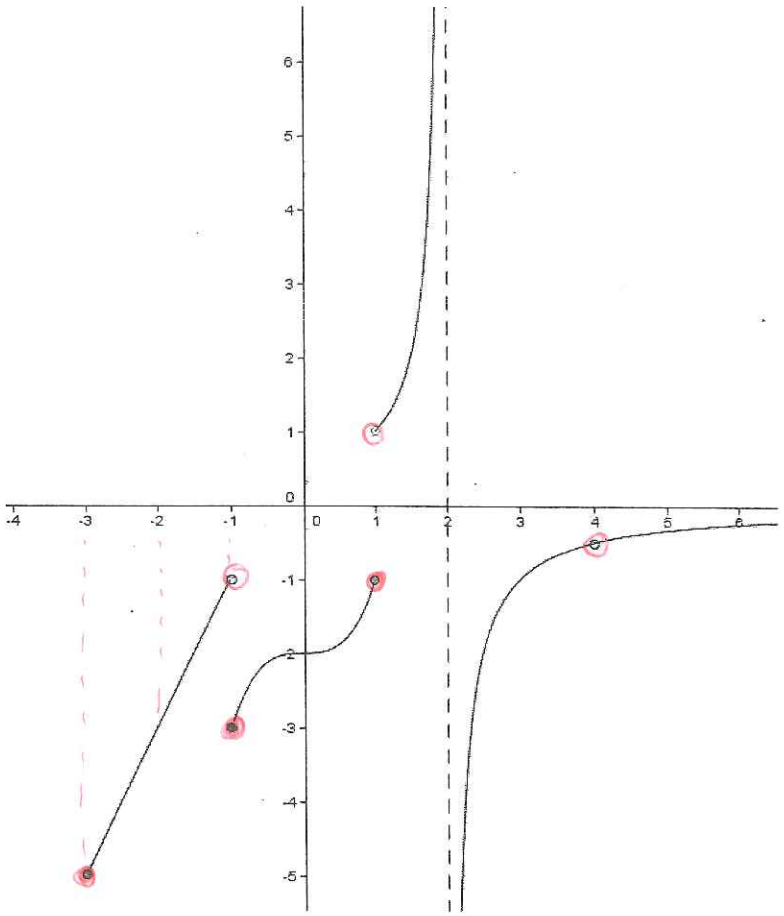
Has an infinite discontinuity at f(–1)

Has a removable discontinuity at f(3)

Has a jump discontinuity at f(5)

Has a limit that is negative as x→ ∞

3. The graph of the function f is given to the right. Use it to determine the following limits.



a. $\lim_{x\to -3^-} f(x) =$ DNE $\lim_{x\to -3^+} f(x) = -5$

b. $\lim_{x\to -1^-} f(x) = -1$ $\lim_{x\to -1^+} f(x) = -3$

c. $\lim_{x\to 0^-} f(x) = -2$ $\lim_{x\to 0^+} f(x) = -2$

d. $\lim_{x\to 1^+} f(x) = -1$ $\lim_{x\to 1^+} f(x) = 1$

e. $\lim_{x\to 2^-} f(x) =$ DNE ∞ $\lim_{x\to 2^+} f(x) =$ DNE $-\infty$

f. $\lim_{x\to 4^-} f(x) = -\frac{1}{2}$ $\lim_{x\to 4^+} f(x) = -\frac{1}{2}$

g. Above, circle the letter(s) of the problems Whose limits that exist

h. Name one removable discontinuity $x = 4$

i. Name one infinite discontinuity $x = 2$

j. Name one jump discontinuity $x = -1, 1$

4. Find the limits of the piecewise function:

$$f(x) = \begin{cases} -x+1, & 0 \le x < 1 \\ 1, & 1 \le x < 2 \\ 2, & x = 2 \\ x-1, & 2 < x \le 3 \\ -x+5, & 3 < x \le 4. \end{cases}$$

a. $\lim_{x\to 0^-} f(x) =$ DNE $\lim_{x\to 0^+} f(x) =$ 1

b. $\lim_{x\to 1^-} f(x) =$ 0 $\lim_{x\to 1^+} f(x) =$ 1

c. $\lim_{x\to 2^-} f(x) =$ 1 $\lim_{x\to 2^+} f(x) =$ 1

d. $\lim_{x\to 3^-} f(x) =$ 2 $\lim_{x\to 3^+} f(x) =$ 2

e. $\lim_{x\to 4^-} f(x) =$ 1 $\lim_{x\to 4^+} f(x) =$ DNE

f. Put a star next to the letter whose x–value is right continuous.

g. Put a smiley face next to the letter whose value is left–continuous

Evaluate limits: Remember your options: direct evaluation, factoring & canceling, multiplying by the conjugate. It may help to remember your rules for horizontal asymptotes, and to recall the sum/diff of cubes:

$$x^3 + y^3 = (x+y)(x^2 - xy + y^3)$$
$$x^3 - y^3 = (x-y)(x^2 + xy + y^3)$$

5. $\lim_{x\to 3} \frac{x^3-27}{x-3}$ $\frac{(x-3)(x^2+3x+9)}{(x-3)}$ = 27

$(3x+7)(3x-6)$ 42 $6|7$

6. $\lim_{x\to 3} \frac{x^2-7x+12}{-x+3}$ $\frac{(x-4)(x-3)}{-(x-3)} = \frac{-1}{-1}$ = 1

7. $\lim_{x\to 2} \frac{3x^2-13x+14}{x^2-4}$ $\frac{(3x-7)(x-2)}{(x+2)(x-2)}$ = $-\frac{1}{4}$

8. $\lim_{x\to 1} 3x^3 - 2x^2 + 4$ = 5

9. $\lim_{x\to 2} \frac{x-2}{x^2-4}$ $\frac{x-2}{(x+2)(x-2)}$ $\frac{1}{x+2}$ = $\frac{1}{4}$

10. $\lim_{x\to 2} \frac{5}{x-4}$ = $-\frac{5}{2}$

11. $\lim_{x\to 4} \frac{x^2-5x+4}{x^2-2x-8}$ $\frac{(x-4)(x-1)}{(x-4)(x+2)}$ $\frac{x-1}{x+2}$ = $\frac{1}{2}$

-6 $-3|1$ -2 $(2x-6)(2x+1)$

12. $\lim_{x\to 2} \frac{3x^2-x-10}{x^2-4}$ $\frac{(3x+5)(x-2)}{(x-2)(x+2)}$ = $\frac{11}{4}$

13. $\lim_{x\to 2} \frac{x^4-81}{2x^2-5x-3}$ x^2-9 EV $\frac{(x^2+9)(x+3)(x-3)}{(2x+1)(x-3)}$ = 13

14. $\lim_{x\to 4} \frac{x-16}{\sqrt{x}-4} \cdot \frac{\sqrt{x}+4}{\sqrt{x}+4}$ $x-16$ = 8

15. $\lim_{x\to 2} x^3 - x^2$ = 4

16. $\lim_{x\to 3} \frac{2x+1}{x-3}$ = DNE (vertical asymptote)

$f(1) = -3/2$
$f(2) = -5$
$f(4) = 9$
$f(5) = 11/2$

$\lim_{x\to 3^-} f(x) = -\infty$

$x \to 3^+ \to \infty$

17. $\lim_{x \to 0} \frac{6x^2 + 3x}{x}$ $6x+3$ $=3$

18. $\lim_{x \to 0} \frac{7x}{x}$ $=7$

19. $\lim_{x \to 1} \frac{2-x}{(x-1)^2}$ $\frac{-(x-2)}{(x-1)(x-1)}$ $=$ DNE (∞)

21. $\lim_{x \to \pi} \cot x$ $=$ DNE

$\frac{1}{\tan \pi} = \frac{1}{0}$

22. $\lim_{x \to \infty} \frac{x^3+5x}{2x^3-x^2+4}$ same $=\frac{1}{2}$

23. $\lim_{x \to -\infty} \frac{x^2+2}{x^3+x^2-1}$ bigger $=0$

24. $\lim_{x \to \infty} \frac{3x^3+x}{4x^2-x+1}$ wins $=\infty$

25. $\lim_{x \to \infty} \frac{4x^2-x+1}{3x^3+x}$ bigger. $=0$

26. $f(x) = \begin{cases} \frac{x^2+5x-6}{x-2} & \text{if } x \neq -2 \\ 0 & \text{if } x = -2 \end{cases}$

$\lim_{x \to 3} f(x)$ $=18$:)

just for fun. $\lim_{x \to 2} f(x) = \frac{(x+6)(x-1)}{(x-2)} =$ DNE

27. $f(x) = \begin{cases} x+2 & \text{if } x < 0 \\ 3x+2 & \text{if } x \geq 0 \end{cases}$

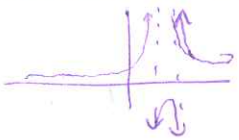
$\lim_{x \to 0^-} f(x) = 2$ $\lim_{x \to 0^+} f(x) = 2$ $f(0) = 2$

$\lim_{x \to 0} f(x) = 2$

Is the following function continuous? If not, state the type of continuity that exists. Sketch to confirm.

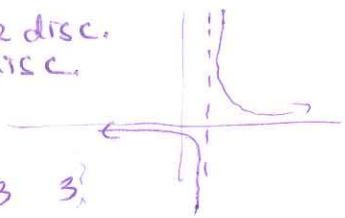
28. $f(x) = \frac{x+3}{x^2-5x+6}$ $\frac{x+3}{(x-2)(x-3)}$

No: Infinite dis. at $x=2$, $x=3$



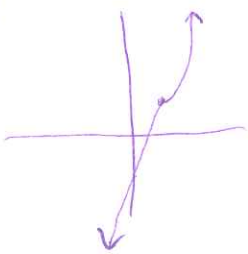
29. $f(x) = \frac{x+7}{x^2+6x-7}$ $\frac{x+7}{(x+7)(x-1)} = \frac{1}{x-1}$

No: $x=-7$ removeable disc.
$x=1$ infinite disc.



30. $f(x) = \begin{cases} x^2+2 & x < 1 \\ 5x-2 & x \geq 1 \end{cases}$ 3 3 $f(1)=3$

Yes!



31. $f(x) = \begin{cases} 2x+3 & x \leq 0 \\ x^2-8 & x > 0 \end{cases}$ 3 -8

No; left continuous

$x=0$ is jump discontinuity

More limits practice: On a separate sheet, write the limit notation and evaluate the following limits. If the limit does not exist, state why. You can sketch/graph the function to check your answer. You should be able to do these without a calc.

Function	Find limits at
Eg. $f(x)=-\frac{1}{2}x+7$	-4,8 $\lim_{x\to -4} f(x)=-\frac{1}{2}(-4)+7=9$ $\lim_{x\to 8} f(x)=-\frac{1}{2}(8)+7=3$
1. $f(x)=2x^2-3x+8$	0, 2 8 10
2. $g(x)=\dfrac{x^2-1}{x+1}$	4, -1 3 -2
3. $h(x)=\dfrac{x-5}{\sqrt{x+4}-3}$	4, 5 $\sqrt{8}+3$ 6
4. $f(x)=\sin(x)$	π, 3π/2 0 -1
5. $g(x)=\begin{cases}3x-5 & \text{if } x\geq 1\\ 2-x & \text{if } x<1\end{cases}$	–3, 3, 1 5 4 DNE
6. $h(x)=\dfrac{x-4}{x^2-x-12}$	0, 4 $\frac{1}{3}$ $\frac{1}{7}$
7. $f(x)=e^x$	0, –2 1 $\frac{1}{e^2}$
8. $f(x)=\dfrac{x^4-81}{x^2-9}$	0, 3, –3 9 18 18
9. $g(x)=\dfrac{x^2-6x+9}{x-3}$	4, 3 1 0
10. $g(x)=\dfrac{x^3-8}{x-2}$	0, 3, 2 4 19 12
11. $h(x)=\begin{cases}x^2-1 & \text{if } x\geq 2\\ 5-x & \text{if } x<2\end{cases}$	–4, 2, 5 9 3 24
12. $f(x)=\ln x$	1, e 0 1
13. $g(x)=\dfrac{x^3+27}{x+3}$	3, –3 9 9