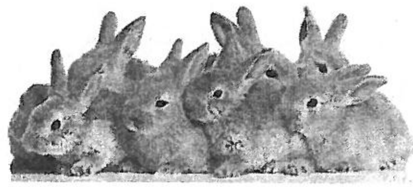
Name: Answer Key Per: ________ Date: ____________

Serafino ▪ Algebra 2E

# 7A Exponential Functions & Graphs

Notes Packet



We know "Exponent" means "to the power of";
**An exponential function has a function in the exponent.**

It is probably the most useful function in the real world.
Let's say I have 20 bunnies. The number of bunnies TRIPLES every year. How many bunnies will I have in 5 years?

Some math you might do is: $20 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 4{,}860$

We start with 20, then triple that. Then triple that. We are basically multiply each "new total" by 3… 5 times.
But what if I wanted to see how much you'd have over 10 years? 50 years? 100.34791 years? You're not gonna sit there doing x3x3x3x3x3x3x3 on your calculator. We can rewrite this function in a much more efficient way.

We write is as $y = 20(3)^5$ . So $y = 20(3)^x$ is the exponential model that reflects money (y) over time (x)

There is not really a "parent" function here, because as you see, we need TWO numbers in place for "a" and "b", leaving x and y free for input and output. Let's talk about what "a" and "b" mean.

$$f(x) = a \cdot b^x$$

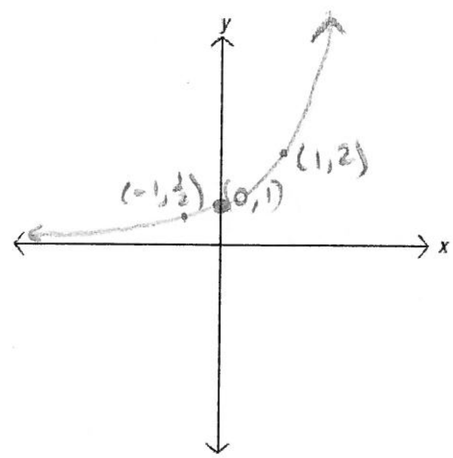
"a" → initial or starting value "b" → growth/decay rate "x" is your input → usually time

Let's start graphing to get a visual on what's going on here.

$y = 2^x$

| x | 1 | 2 | 3 |
|---|---|---|---|
| f(x) | 2 | 4 | 8 |

| x | 0 | –1 | –2 | –3 |
|---|---|---|---|---|
| f(x) | 1 | 1/2 | 1/4 | 1/8 |




Here a = 1 and b = 2

As x → –∞, f(x) → 0

As x → ∞, f(x) → ∞

**EXPONENTIAL GROWTH**

What can we notice? A few things.

1. When x = 0, the "b" term disappears because $b^0 = 1$.
   This means that if there is no "a", then the starting value is automatically 1.

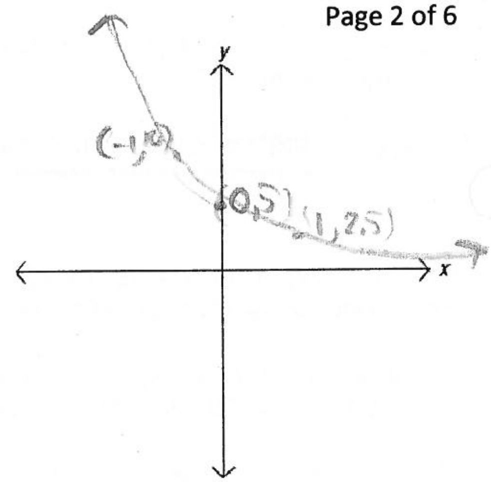
2. We also see some asymptotic behavior.

has asymptotes

Let's do another one. $y = 5\left(\frac{1}{2}\right)^x$

| x | 1 | 2 | 3 |
|---|---|---|---|
| f(x) | 5/2 | 5/4 | 5/8 |

| x | 0 | –1 | –2 | –3 |
|---|---|---|---|---|
| f(x) | 5 | 10 | 20 | 40 |




Here a = 5 and b = 1/2

As x → –∞, f(x) → ∞

As x → ∞, f(x) → 0

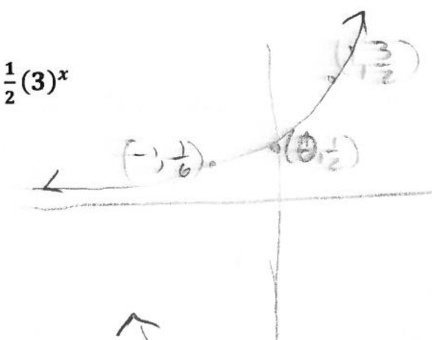
Now what can we notice as time goes on?

It will never be zero!

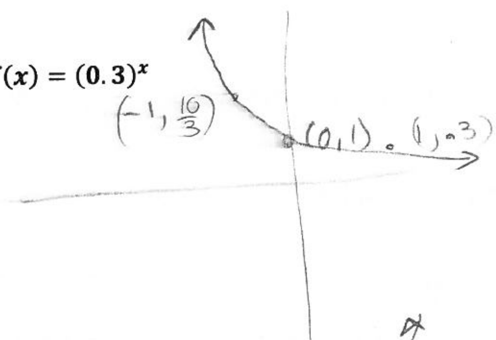
**EXPONENTIAL DECAY**

1. **Sketch each function, including the y–intercept, a point to the right and a point to the left.**

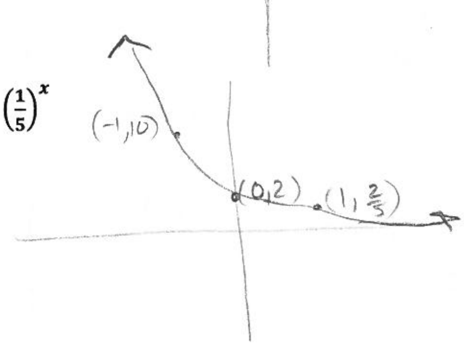
a. $f(x) = \frac{1}{2}(3)^x$



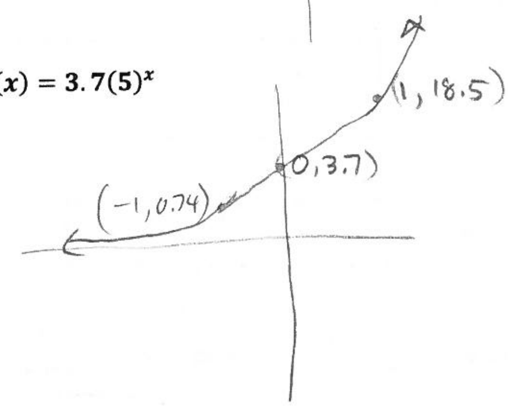

c. $f(x) = (0.3)^x$




b. $f(x) = 2\left(\frac{1}{5}\right)^x$




d. $f(x) = 3.7(5)^x$




RECOGNIZING EXPONENTIAL GROWTH VS. DECAY

When **b is > 1,** we have **GROWTH** → we are taking MORE THAN ONE of what we had.

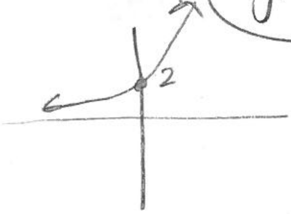
When **b is < 1,** we have **DECAY** → we have LESS THAN ONE of what we had.

When b = 1, we would end up with something like 5 · 1 · 1 · 1 · 1 .... so, that would end up being a straight horizontal line. That's just a constant. We can't really call that exponential growth or decay because it's not growing or decaying... so many textbooks make it clear that b ≠ 1, and a and b ≠ 0 (but that should be obvious).

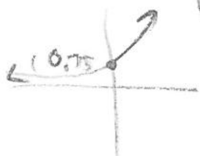

$y = ab^x$

**2. State whether the following functions model exponential growth or decay. Then give a very crude sketch, only including the y–intercept.**

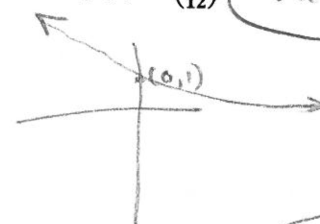
a. $f(x) = 2(1.35)^x$ growth



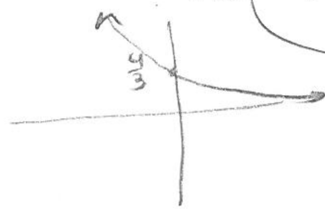

b. $f(x) = 0.75\left(\frac{11}{10}\right)^x$ growth




c. $f(x) = \left(\frac{5}{12}\right)^x$ decay




d. $f(x) = \frac{4}{3}\left(\frac{3}{4}\right)^x$ decay




**PERCENT INCREASE / DECREASE**

Most of the World's Exponential Models are given a Percent Increase or Decrease

Weight Gain: Sarah is 9 years old and is 67 lbs. Her doctor says she is going to gain 15% of her weight each year for the next 5 years. How much will Sarah weigh when she is 14?

a. Write the model first! Always start by writing the "naked" equation: $y = a \cdot b^x$. Fill in the information you have and leave x and y open for input and output.

$y = ab^x$

$y = \underline{67}\,(\underline{1 + 0.15})^x$

First of all, we need to put 15% into a number we can use. 15/100 = 0.15 so that is something we need.

Because it is GROWTH, we want 100% of the prior amount, plus an addition 15%.

100% + 15% = 1 + 0.15

1.15

You are free to write any exponential growth model with a 1 + or you can write a single number.
I prefer the 1+ (you'll see why later)

$y = a(1 + \%)^x$

$y = 67(1.15)^x$
$y = 67(1 + 0.15)^x$

So the model that represents Sarah's weight over the next few years is:

b. Use that model to figure out how much Sarah will weigh when she is 14.

f(5) = 134.76 lbs

c. Use the same model to figure out how much she will weigh when she is 19.

$67(1.15)^{10} = \boxed{271.052 \text{ lbs}}$

d. So why do you think her doctor specifically said, "over the next 5 years?"

She was going Through puberty!

Depreciation of Value: Your have a motorcycle that is worth \$9,000 now.
It depreciates in value by 5% every year.

a. Write a function to model the value of the motorcycle over time: $\boxed{y = 9000(1-.05)^x \quad 9{,}000(0.95)^x}$

$y = a \cdot b^x$

$y = a(1 - \%)^x$

We are LOSING value, so b must be less than 1. In fact, 5% less than 1.

Another way of thinking of this is... what % value are we KEEPING? 95%

b. How much will that motorcycle be worth in 7 years? 20 years? 50 years?

$\boxed{f(7) = \$6{,}285.04}$ $\boxed{f(20) = \$3{,}226.37}$ $\boxed{f(50) = \$692.5}$

**3. Writing and evaluating exponential growth/decay functions.**

a. A 3–ft tree grows 3.2% per year. How tall will the ~~three~~ tree be in 4 years?

$y = 3(1+.032)^4 = \boxed{3.4028 \text{ ft}}$

b. The butterfly population decreases by 9.2% each year due to toxic emissions. There are 100,000 butterflies now. How many will there be in 50 years?

$y = 100{,}000(1-.092)^{50} = \boxed{802 \text{ butterflies}}$

c. You deposit \$500 in a bank that pays an interest rate of 4% per year. How much will you have in 10 years?

$y = 500(1+.04)^{10} = \boxed{\$740.12}$

d. You deposit \$500 into a bank that pays 4% interest every 6 MONTHS. How much will you have in 10 years?

$500(1+.04)^{20} = \boxed{\$1095.56}$

e. The price of a new home is \$350,000. The value appreciates 2% every year.
If you can spend \$450,000 on a house, will you be able to afford it in 10 years? 15 years?

$350{,}000(1+.02)^{10} = \boxed{426{,}648 \text{ yes}}$

$350{,}000(1+.02)^{15} = \boxed{471{,}053 \text{ NO}}$

f. A population of endangered birds is decreasing at a rate of 0.75% per year. There are currently 200,000 of them. How many will there be in 100 years?

$200{,}000(1-.0075)^{100} = \boxed{94{,}206 \text{ birds!}}$

## SOLVING FOR "A"

You win a pony! You inherit a prize–winning pony from a rich aunt. Your aunt bought the pony 20 years ago, when it was worth the most money, and it has depreciated in value by 6.2% every year since then. Your aunt states the pony is now worth $2,000 now. How much was the pony worth when your aunt bought it?

$y = ab^x$

$2000 = a(1-.062)^{20}$

$(.938)^{20}$

$\frac{2000}{.278} = \frac{a(0.278)}{0.278}$

$a = \boxed{\$7,194.24}$

Planning ahead: You want to be able to buy a $20,000 boat with some friends in 15 years. If your bank pays 2.7% interest, how much do you have to put in now to afford the boat in 15 years?

$y = ab^x$

$20,000 = a(1+.027)^{15}$

$20,000 = a \cdot 1.49127128$

$\boxed{a = \$13,411.38}$

## SOLVING FOR "B"

The Iberian Lynx is decreasing in population in a forest. In 2003, there were 250. In 2006, there were 175.

a) What is the decay rate of the lynx? To do this, you can USE an exponential function to create the exponential function.

We lose 11.21% each year.

$\frac{250(b)^3}{250} = \frac{175}{250}$

$\sqrt[3]{b^3} = \sqrt[3]{.7}$

$\boxed{b = 0.8879}$

b. How many are there this year?

$y = 250(.8879)^{12} = 60$ !!

Beautiful Bride: Liza is getting married. She weighs 186 lbs, and wants to weigh 140 lbs in 6 months.

a) If she loses 4% of her body weight each month, how much will she weigh by her wedding?

$186(1-.04)^6 = \boxed{145.59 \text{ lbs}}$

b) She wants to get on track. What percent of her body weight must she lose each month to be 140 lbs in 6 months?

$\frac{186b^6}{186} = \frac{140}{186}$ $b = .9538$

$b^6 = .7526$

$\rightarrow$ $\boxed{\text{She must lose } 4.625\%}$

c) She wants to know how much she should weigh on each month to hit her goal. What are those weights?

Now = 186

1 mont = 177.4068

2 Month = 169.21

3 months, 161.39

4 months: 153.94

5 month: 146.82

6 month: 140.04

7A

# [redacted]-Exponential Growth and Decay Functions – Homework

Evaluate each expression for (a) x = -2 and (b) x = 3.

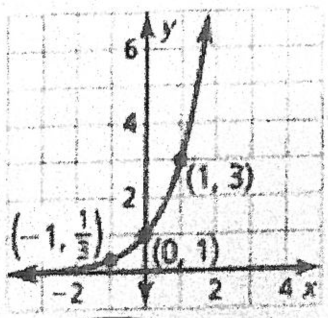
1) $2^x$ a) $\frac{1}{4}$ b) 8  2) $8 \cdot 3^x$ a) $\frac{8}{9}$ b) 216  3) $5+4^x$ a) $\frac{81}{16}$ b) 69

Tell whether the function represents *exponential growth* or *exponential decay*.

4) $y=\left(\frac{4}{3}\right)^x$ growth  5) $y=(1.2)^x$ growth  6) $y=\left(\frac{2}{5}\right)^x$ decay

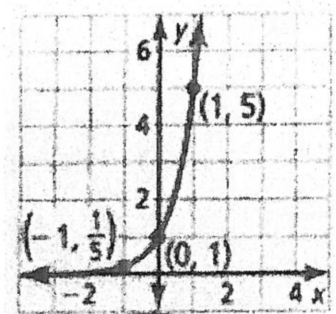
[redacted] write the function.

7)




$y = 3^x$

8)




$y = 5^x$

9) The value of a mountain bike $y$ (in dollars) can be approximated by the model $y=200(0.75)^t$, where $t$ is the number of years since the bike was new.

a. Tell whether the model represents exponential growth or exponential decay. decay
b. Identify the annual percent increase or decrease in the value of the bike. 25% decrease
c. Estimate when the value of the bike will be $50.

guess & check 4.5 years?

10) You take a 325 milligram dosage of ibuprofen. During each subsequent hour, the amount of medication in your bloodstream decreases by about 29% each hour.

a. Write an exponential model giving the amount $y$ (in milligrams) of ibuprofen in your bloodstream $t$ hours after the initial dose. $y = 325(0.71)^x$
~~b.~~ Estimate how long it takes for you to have 100 milligrams of ibuprofen in your bloodstream. 3.5 hours?
~~c.~~ Write an interval that represents the domain of the function. $0 < x < 24$

11) When a plant or animal dies, it stops acquiring carbon-14 from the atmosphere. The amount $y$ (in grams) of carbon-14 in the body of an organism after $t$ years is $y=a(0.5)^{t/5730}$, where $a$ is the initial amount (in grams). What percent of the carbon-14 is released each year? 50%

12) A website recorded the number $y$ of referrals it received from social media websites over a 10-year period. The results can be modeled by $y=2500(1.50)^t$, where $t$ is the year and $0 \le t \le 9$. Interpret the values of $a$ and $b$ in this situation. What is the annual percent increase? 50%

13) The population $p$ of a small town after $x$ years can be modeled by the function $p=6850(1.03)^x$. What is the average rate of change in the population over the first 6 years?

0, 6850
6, 8179.25

$\frac{\Delta y}{\Delta x}$ $\frac{1329.26}{6}$ = 221.5 pop per year