Name: _____ Date: _____

Period: _____ Pre-Calculus: ***Final Exam Review***

Directions: Answer each question. If you do not know how to do one, go to scevmath.org and practice problems/look at notes from that section.

I. Solving Triangles

1. Find all of the missing side lengths, angle measures and the **area** for the following triangles:

a. $A=33^\circ; B=72^\circ; a=4.2\,in$ — $C=75^\circ$ $b=7.0$ $c=7.1$

b. $A=35.2^\circ; B=67.5^\circ; c=12\,cm$ — $a=7.1$ $b=11.9$ $C=77.3^\circ$ $A=39.3$

c. $A=50^\circ; a=12\,in; b=42\,in$ — No possible △

d. $A=49^\circ; a=6.8\,ft; b=7.9\,ft$ — Two △s

△1	△2
$c=8.5$	$c=1.9$
$B=61.3^\circ$	$B=118.7^\circ$
$C=69.7^\circ$	$C=12.3^\circ$
$Area=25.2$	$A=5.7$

e. $C=120^\circ; a=10\,cm; c=25\,cm$ — $A=19.9$ $b=18.5$ $B=39.7^\circ$ $A=20.3^\circ$

f. $a=4\,km; b=11\,km; c=12\,km$ — $A=19.4^\circ$ $B=66^\circ$ $C=94.6^\circ$ $Area=21.9$

2. Know the cases of SSA (Ambiguous case).

Ga = Given Angle Sa = Side Adjacent to Ga

So = Side opposite GA

Angle Side Side → Obtuse / Acute

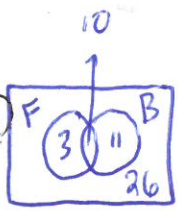
I So > Sa

II: So < Sa (Three cases)

So < h

So = h

So > h



I So > Sa

So < Sa

II. Combinatorics

3. In a survey of 50 people, 13 liked football and 21 liked baseball. Ten liked both. How many liked neither? 26

4. In how many ways can a president, vice president, and secretary be chosen from a group of 17 people? $nP_r(17,3) = {}_{17}P_3 = 17\cdot16\cdot15 = 4080$ order matters

5. In how many ways can a committee of three people be chosen from a group of 17 people? order does not matter ${}_{17}C_3 = nCr(17,3) = \frac{17!}{14!3!} = 680$

6. If you have three shirts, five skirts, and two pairs of shoes, how many outfits can you make? $3\cdot5\cdot2 = 30$

7. In how many ways can five people line up? ${}_5P_5 = 5! = 5\cdot4\cdot3\cdot2\cdot1 = 120$

8. What are the odds of choosing a club if you were randomly selecting a card from a shuffled standard deck of cards. $13:39 = 1:3$ → Clubs ; Non Clubs Not Clubs / Total Probability not odds

9. How many 3 card poker hands are possible? In 3 card poker, you get three distinct cards from standard 52 card deck. ${}_{52}C_3 = 22100$

10. Find the coefficient of the x^3y^6 term in the expansion $(3x-2y)^9$.

$nC_r(n,r)(a)^{n-r}(b)^r$

$(a+b)^4$

$nCr(9,0)(3x)^9(-2y)^0 + nCr(9,1)(3x)^8(-2y)^1 + \dots$

$\dots nCr(9,6)(3x)^3(-2y)^6$

$84(27x^3)(64y^6) = 145152x^3y^6$

11. Complete the first 8 rows of Pascal's Triangle.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

III. Probability

12. A coin is flipped and a die is tossed. List the sample space.

{1H, 2H, 3H, 4H, 5H, 6H, 1T, 2T, 3T, 4T, 5T, 6T}

13. An honest coin is flipped three times.

{HHH, HHT, HTH, THH, HTT, THT, HTT, TTT}

a. List the sample space.
b. What is the probability of getting 1 head and 2 tails?
c. What are the odds of getting 2 heads and 1 tail?

$ncr(3,1)\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^2 = 3\left(\frac{1}{8}\right) = \frac{3}{8}$

3 : 5

14. A card is drawn at random from a standard deck of 52 cards.
a. Find the probability that the card is a club. 13/52 = 1/4
b. Find the odds of not drawing a club. 39 : 13 = 3 : 1
c. Find the odds of drawing a black ace. 2 : 50 = 1 : 25
d. Find the probability that it is a jack of hearts, queen of hearts, or king of hearts.

$\frac{3}{52}$

15. A coin is flipped successively 4 times. Find the probability of getting exactly 1 head.

$_4C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^3 = \frac{1}{4}$

16. A die is tossed 6 times. What is the probability of rolling exactly 4 "2"s?

$ncr(6,4)\cdot\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^2 = \frac{125}{15552} \approx 0.0080$

17. A box contains 4 red and 4 green marbles.
a. If a marble is drawn and replaced 3 times successively, what is the probably that 2 green marbles are drawn? $ncr(3,2)\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^1 = \frac{3}{8} = 0.375$
b. If a marble is drawn 3 times without replacement, what is the probability that that 3 red marbles are drawn?

$\frac{4}{8}\cdot\frac{3}{7}\cdot\frac{2}{6} = \frac{1}{14}$

18. What is the probability that when rolling two die, the sum of the numbers is a perfect square.

only x^2 you can get are 4/9, 3 ways to get 4, 4 ways to get 9 → 7/36

19. What is the probability of rolling an even number on a die given you just rolled an odd number?

independent → 1/2

20. What is the probability of rolling a 6 given it is an even number?

Even # on a die → 1/3

21. Use the following table which are the results of surveying 590 people on who they voted for in the 2008 presidential election to answer the questions.

	Obama	McCain	Undecided/Other	Won't Vote	Totals
Males	84	31	33	25	173
Females	247	60	69	41	417
Totals	331	91	102	66	590

a. What is the probability someone voted Obama given they were a female? 247/417

b. What is the probability someone is a man given they were a McCain supporter? 31/91

c. What is the probability an undecided voter is a male? 33/102

IV. Matrices

* Calc, all others by hand.

$$A=\begin{bmatrix}5&2&0\\-2&1&-3\end{bmatrix} \quad B=\begin{bmatrix}-1&6&2\\0&-3&4\end{bmatrix} \quad C=\begin{bmatrix}3&-2\\6&1\end{bmatrix} \quad D=\begin{bmatrix}2&-1&0\\4&3&1\\5&-2&5\end{bmatrix} \quad E=\begin{bmatrix}4&2&-3\\1&0&-2\\-5&3&1\end{bmatrix}$$

22. Find $A+B$ $\begin{bmatrix}4&8&2\\-2&-2&1\end{bmatrix}$

23. Find A^TB $\begin{bmatrix}5&-2\\2&1\\0&-3\end{bmatrix}\begin{bmatrix}-1&6&2\\0&-3&4\end{bmatrix}=\begin{bmatrix}-5&24&2\\-2&9&8\\0&9&-12\end{bmatrix}$ $3\times2=2\times3$

24. Find $6B-\frac{1}{2}A$ $\begin{bmatrix}-6&36&12\\0&-18&24\end{bmatrix}-\begin{bmatrix}-\frac{1}{2}&3&1\\0&-\frac{3}{2}&2\end{bmatrix}=\begin{bmatrix}-5.5&33&11\\0&-16.5&22\end{bmatrix}$

25. Find C^{-1} $\frac{1}{3-(-12)}\begin{bmatrix}1&2\\-6&3\end{bmatrix}=\frac{1}{15}\begin{bmatrix}1&2\\-6&3\end{bmatrix}$

26. Find CC^{-1} $\begin{bmatrix}1&0\\0&1\end{bmatrix}$

27. Find $|D|$ $\det(D)=49$

28. Find DE $DE=\begin{bmatrix}7&4&-4\\24&11&-17\\43&25&-6\end{bmatrix}$

29. Use Cramer's Rule to solve the following systems of equations

a. $4x+3y=8$, $5x-2y=6$

$D=-23$ $D_X=-34$ $D_Y=-16$

$x=\frac{-34}{-23}=\frac{34}{23}$ $y=\frac{-16}{-23}=\frac{16}{23}$

b. $x-7y=10$, $3x+5y=14$

$D=26$ $D_X=148$ $D_Y=-16$

$x=\frac{148}{26}=\frac{74}{13}$ $y=\frac{-16}{26}=\frac{-8}{13}$

c. $-2x+3y=4$, $4x-6y=10$

$D=0$

No Solution

V. Conics

23. Identify the conic section.

a. $4x^2+y^2-8x+6y-23=0$ Ellipse

b. $9x^2-y^2-18x+4y-31=0$ Hyperbola

c. $8y+12-x^2+4x=0$ Parabola

d. $x^2+y^2+8x-8y+7=0$ Circle

e. $y^2=-3x$ Parabola

f. $5y^2=10-4x^2$ Ellipse

24-27 AK on next page.

24. Graph each of the following equations. Be sure to include the following:
    1. Label and state the coordinates of the vertex and focus.
    2. Plot the endpoints of the focal width.
    3. Show and state the equation for the directrix.

a. $(x+2)^2 = -8(y+3)$

b. $(y-1)^2 = 16x$

c. $x^2 = 4(y-4)$

d. $(y+6)^2 = -12(x-1)$

25. Graph each of the following equations. Be sure to include the following:
    1. State the center.
    2. Give the coordinates of vertices and covertices.
    3. State the length of the major and minor axes.
    4. Give the coordinates of the foci.

a. $\frac{(y+2)^2}{49} + \frac{(x+3)^2}{64} = 1$

b. $\frac{x^2}{16} + \frac{(y-1)^2}{25} = 1$

26. Graph each of the following equations. Be sure to include the following:
    1. State the center.
    2. Give the coordinates of the vertices.
    3. State the length of the conjugate and transverse axes.
    4. Give the equations for the asymptotes.

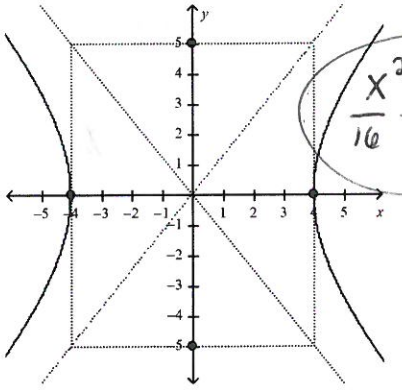
a. $\frac{(y-2)^2}{16} - \frac{(x+1)^2}{25} = 1$

b. $\frac{(x+3)^2}{9} - \frac{y^2}{64} = 1$

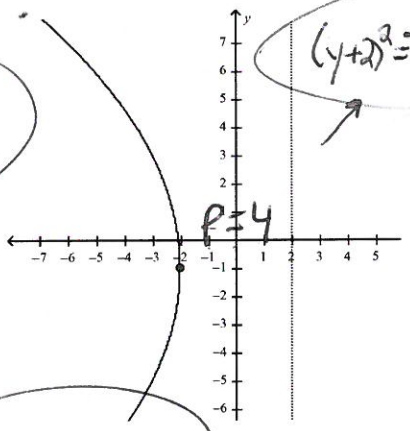
27. Given the following information, write the equation for the conic.
    a. Parabola with focus at (3,2) and directrix at y=-4.
    b. Ellipse with foci at (-5,2) and (5,2) and the major axis is 20.
    c. Ellipse with horizontal minor axis is 16, major axis is 18, and center is at (-3, 0).
    d. Circle with (-10, 3) and (-2,7) as endpoints of the diameter.
    e. Hyperbola with (-5,0) and (5,0) as vertices and foci (-7,0) and (7,0).

28. State the equation for each of the graphs below.

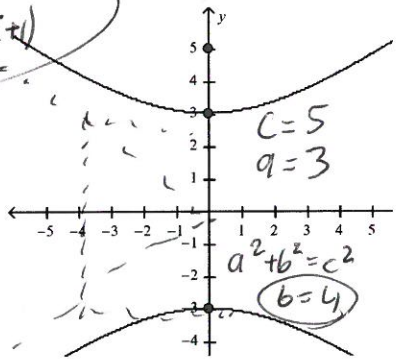
a.



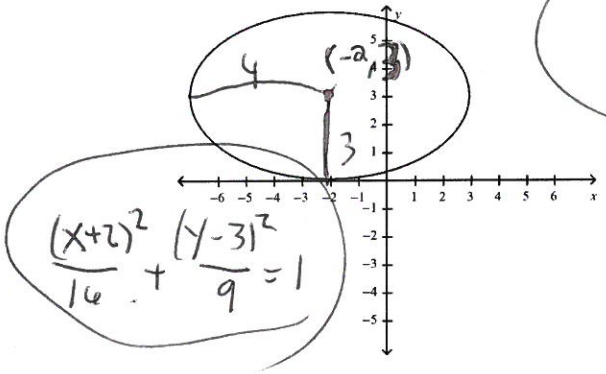
b.



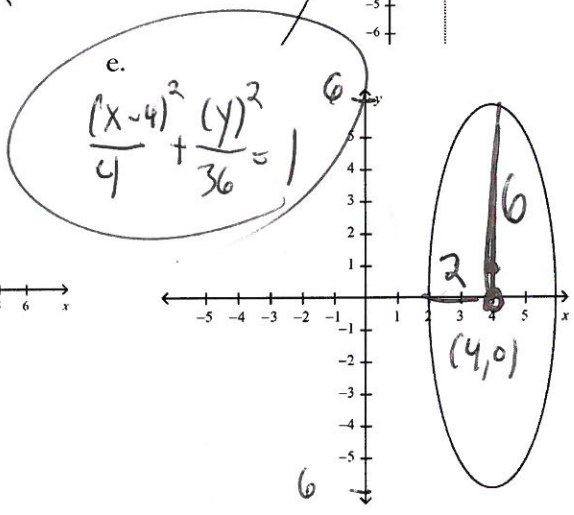
c.



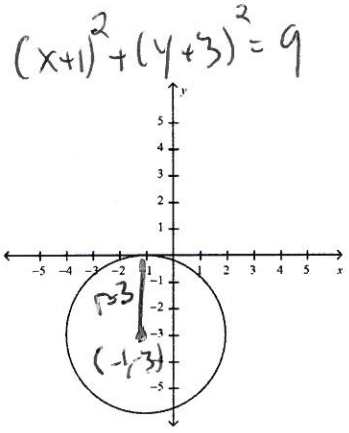
d.



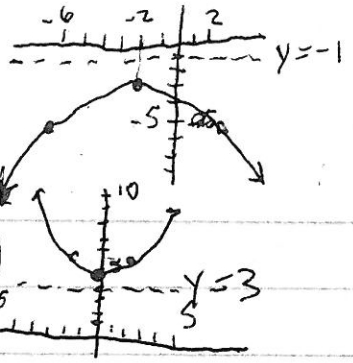
e.



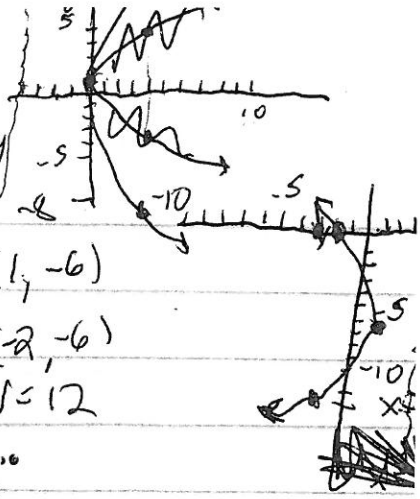
f.



24 a. 1) V=(-4,-7)
F=(-2,-5)
FW=8



b. V=(0,1)
F=(4,1)
FW=16



c. 3) V=(0,4) FW=4
F=(0,5)

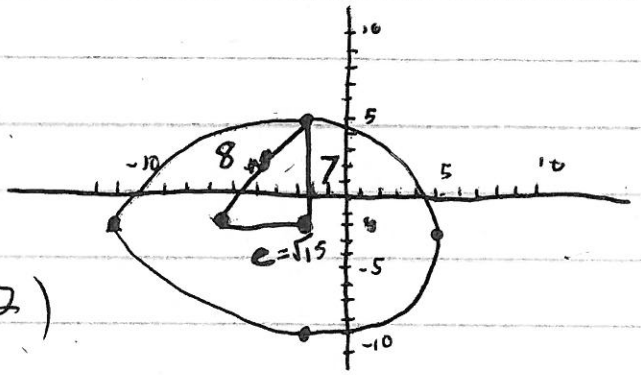
d. 4) V=(1,-6)
F=(-2,-6)
FW=12

25 a. 1) (-3,-2)

2) (-11,-2) & (5,-2) Vertices
(-3,5) & (-3,-9) Covertices

3) Major = 16
Minor = 14

4) $(-3 \pm \sqrt{15}, -2)$

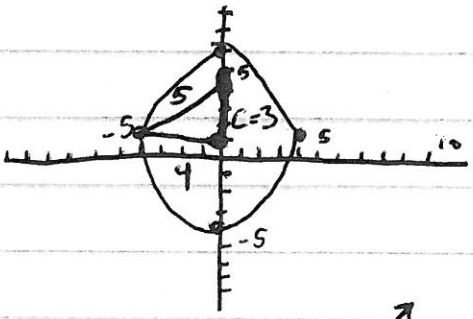


b. 1) (0,1)

2) (0,6) & (0,-4) Vertices
(-4,1) & (4,1) covertices

3) Major = 10 Minor = 8

4) $(0, 1 \pm 3)$

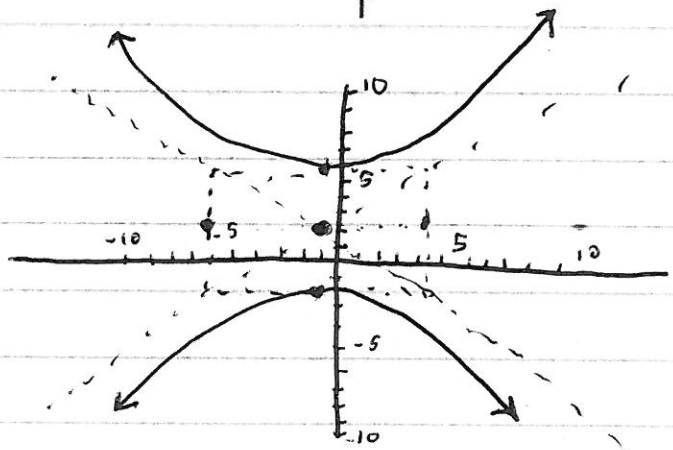


26 a. 1) (-1,2)

2) (-1,6) & (-1,-2)

3) Trans → 8
Conj → 10

4) $y - 2 = \pm \frac{4}{5}(x+1)$

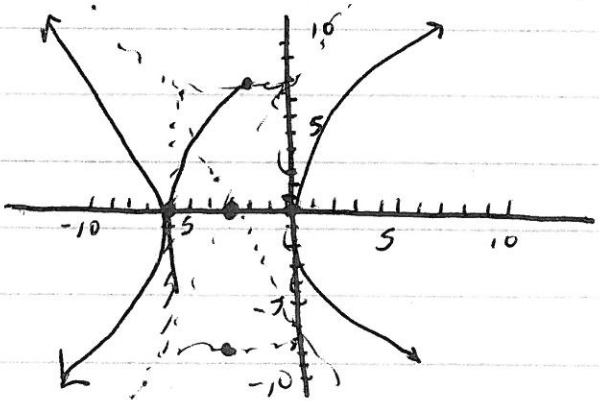


b. 1) (-3,0)

2) (-6,0) & (0,0)

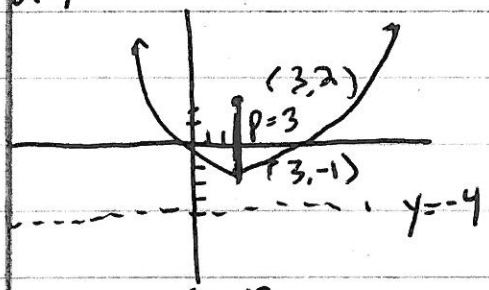
3) Trans → 6
Conj → 16

4) $y = -\frac{8}{3}(x+3)$



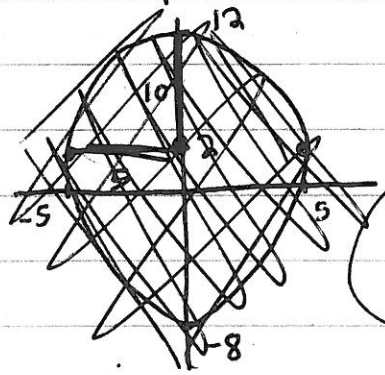
27

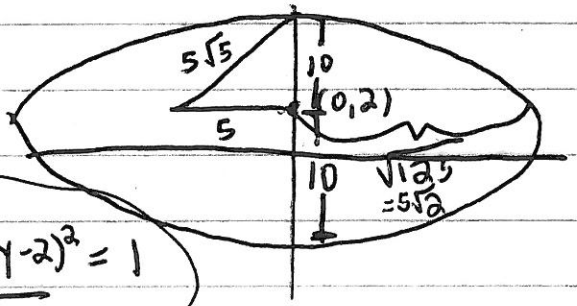
a.



$(y+1)^2 = 12(x-3)$

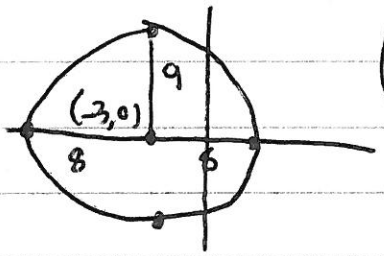
b.





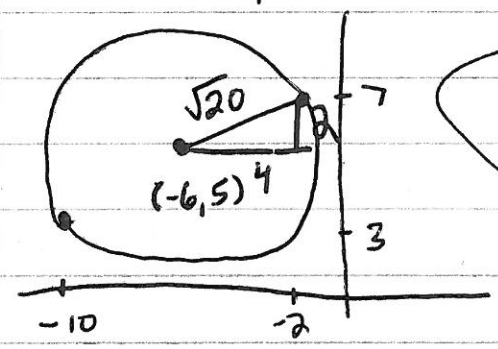
$\frac{x^2}{125} + \frac{(y-2)^2}{100} = 1$

c.



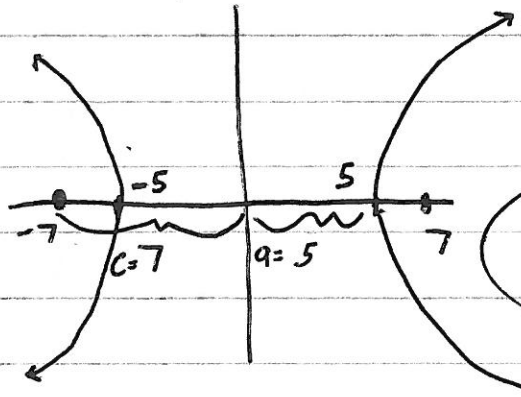
$\frac{(x+3)^2}{64} + \frac{y^2}{81} = 1$

d.



$(x+6)^2 + (y-5)^2 = 20$

e.



$a^2 + b^2 = c^2$

$5^2 + b^2 = 7^2 \rightarrow b^2 = 24$

$\frac{x^2}{25} - \frac{y^2}{24} = 1$

VI. Parametrics

NO CALCULATOR

29. Find two sets of parametric equations for the rectangular equation $y = x^2 - 8$ using the parameters $t = x$ and $t = 1 - x$.

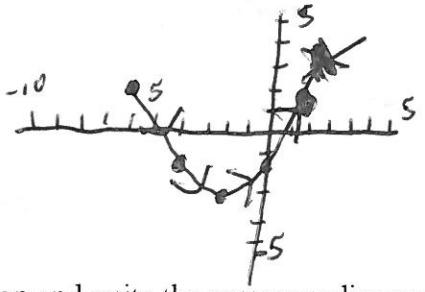
$\begin{cases} x = t \\ y = t^2 - 8 \end{cases}$ and $\begin{cases} x = 1 - t \\ y = (1-t)^2 - 8 \end{cases}$

$1 - 2t + t^2 - 8$

$t^2 - 2t - 9$

30. Sketch the graph represented by the parametric equations $x = 2t - 2$, $y = t^2 - 3$ for $-2 \le t \le 2$. Be sure to show the correct orientation.

t	-2	-1	0	1	2
x	-6	-4	-2	0	2
y	1	-2	-3	-2	1



31. Now eliminate the parameter from the previous quesiton and write the corresponding rectangular equation whose graph represents the curve.

$x = 2t - 2$

$\frac{x+2}{2} = \frac{2t}{2} \rightarrow t = \frac{x+2}{2} \rightarrow y = \left(\frac{x+2}{2}\right)^2 - 3$

$y = \frac{x^2 - 2}{2}$

32. Write one equation for each set of parametric equations in terms of only x and y. $\begin{cases} x = 2t - 5 \\ y = t + 2 \end{cases}$

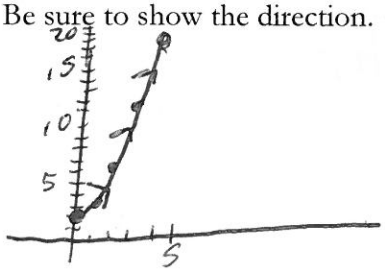
$x + 5 = 2t$

$\frac{x+5}{2} = t \rightarrow y = \frac{x+5}{2} + 2 = y = \frac{x+9}{2}$

33. Draw a graph to represent each set of parametric equations. Be sure to show the direction.

$\begin{cases} x = \sqrt{t} \\ y = t + 2 \end{cases}$ $0 \le t \le 16$

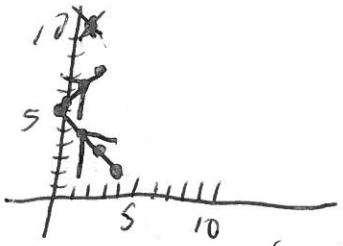
t	0	1	4	9	16
x	0	1	2	3	4
y	2	3	6	11	18



34. Sketch the curve represented by the parametric equations (indicate the direction of the curve.) Then eliminate the parameter and write the corresponding rectangular (x and y only) equation whose graph represents the curve.

$\begin{cases} x = |t - 2| \\ y = t + 3 \end{cases}$ $-2 \le t \le 4$

t	-2	-1	0	1	2	3	4
x	4	3	2	1	0	1	2
y	1	2	3	4	5	6	7



35. Eliminate the Parameter:

a) $\begin{cases} x = 3\cos t \\ y = 2\sin t \end{cases}$

$\frac{x^2}{9} + \frac{y^2}{4} = 1$

b) $\begin{cases} x = 3\sec t \\ y = 4\tan t \end{cases}$

$\frac{x^2}{9} - \frac{y^2}{16} = 1$

c) $\begin{cases} x = 2\sin t \\ y = 2\cos t \end{cases}$

$x^2 + y^2 = 4$

$x = 5\cos\theta + 3$
$y = 5\sin\theta - 4$

$x = 5\tan\theta - 1$
$y = 4\sec\theta + 3$

36. Write a set of Parametric Equations for the following:

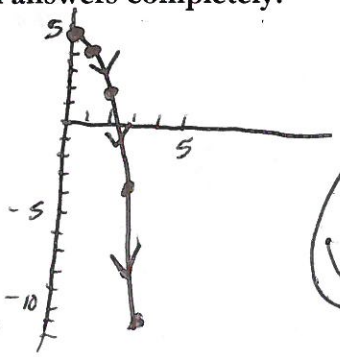
a) $(x-3)^2 + (y+4)^2 = 25$ b) $\dfrac{(y-3)^2}{16} - \dfrac{(x+1)^2}{25} = 1$

Parametric Review CALC. Permitted Show all work and simplify all answers completely.

38. Consider the parametric equations $\begin{cases} x = \sqrt{t} \\ y = 5 - t \end{cases}$.

a. Complete the table

T	0	1	4	9	16
X	0	1	2	3	4
Y	5	4	1	-4	-11



b. Plot the points (x,y) generated by the table and sketch the graph of the parametric equations.

c. Using a graphing calculator, graph the curve represented by the parametric equations.

$t = x^2 \rightarrow y = 5 - x^2$

d. Find the rectangular (x and y) equation by eliminating the parameter. Sketch its graph. How does the graph differ from those in parts b and c?

No arrows, larger domain

ONE VA @ $x = -\frac{1}{2}$

$\dfrac{2(-\frac{1}{2}) - 1}{(2 + \frac{1}{(-\frac{1}{2})})} = \dfrac{-1}{0}$

VII. Limits

HA @ $y = 1$

39. $\lim\limits_{n\to\infty} \dfrac{(n^2-1)}{n^2} = 1$

40. $\lim\limits_{n\to\infty} \dfrac{(n-1)}{n^2} = 0$

HA @ $y = 0$

41. $\lim\limits_{n\to\infty} \dfrac{(n^2-1)}{n} = \infty$

SA. at $y = n$

$\dfrac{n}{n^2 - 1}$

$-n^2 \downarrow$

$0 \quad -1$

42. $\lim\limits_{n\to\infty} .9^n = 0$

HA @ $y = 0$

43. $\lim\limits_{x\to -4} -5x + 6$

$-5(-4) + 6$

$= 26$

44. $\lim\limits_{x\to 2} \dfrac{x-2}{x^2-4} = \dfrac{x-2}{(x-2)(x+2)} = \dfrac{1}{2+2} = \dfrac{1}{4}$

45. $\lim\limits_{x\to 0} \dfrac{2-\sqrt{4-x}}{x} = \dfrac{1}{4}$

(Graph)

46. $\lim\limits_{x\to -\frac{1}{2}} \dfrac{2x^2}{2x+1} = \dfrac{2(-\frac{1}{2})^2}{2(-\frac{1}{2})+1} = \dfrac{\frac{1}{2}}{0}$

$\dfrac{2x}{2 + \frac{1}{x}} = \dfrac{\frac{2x^2}{x}}{\frac{2x}{x} + \frac{1}{x}}$

a Hole.

47. List an interval with a range of 1 that includes at least one zero of f(x).

x	-3	-2	-1	0	1	2
f(x)	5	3	-2	-2.5	-3	-10

(-2, -1)

48. Identify the interval where the function $f(x) = x^2 + 1$ is increasing.

$(0, \infty)$

49. Identify the domain and range of $g(x) = \dfrac{2x+1}{x+2}$

D: $x \neq -2$

R: $y \neq 2$

$x \neq -4$

$y = 1$

$-\frac{1}{2}$

50. Graph $f(x) = \dfrac{x-3}{x+4}$. State the domain, range and points and types of discontinuity.

VA @ $x = -4$

$x \neq -4$ $y \neq 1$

Infinite @ $x = -4$

HA @ $y = 1$

51. Graph $g(x) = \dfrac{-2}{x-3}$. State the domain, range and points and types of discontinuity.

$x = 3$

$y = 0$

$x \neq 3$ $y \neq 0$

Infinite @ $x = 3$

D: $x \neq 3$

R: $y \neq 0$

Infinite discontinuity at $x = 3$.

52. Graph $h(x) = \dfrac{x^2+5x+4}{x+1}$. State the domain, range and points and types of discontinuity.

$h(x) = \dfrac{(x+4)(x+1)}{(x+1)}$

h(x)

D: $x \neq -1$

R: $y \neq 4$

Removable discontinuity @ $x = -1$