# Problem Set 3.2

**1.** $\theta = \dfrac{s}{r}$ Definition of radian measure

$= \dfrac{9\,\text{cm}}{3\,\text{cm}}$ Substitute given values

$= 3$ Divide

**3.** $\theta = \dfrac{s}{r}$ Definition of radian measure

$= \dfrac{5\,\text{in}}{10\,\text{in}}$ Substitute given values

$= \dfrac{1}{2}$ Divide

**5.** $\theta = \dfrac{s}{r}$ Definition of radian measure

$= \dfrac{12\pi\ \text{inches}}{4\ \text{inches}}$ Substitute given values

$= 3\pi$ Divide

**7.** $\theta = \dfrac{s}{r}$ Definition of radian measure

$= \dfrac{\frac{1}{2}\,\text{cm}}{\frac{1}{4}\,\text{cm}}$ Substitute given values

$= 2$ Divide

**9.** $\theta = \dfrac{s}{r}$ Definition of radian measure

$= \dfrac{450}{4000}$ Substitute given values

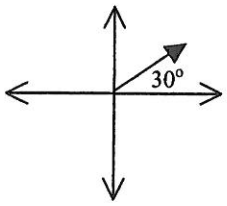
$= 0.1125$ Divide

**11.** (b) $30^\circ = 30\left(\dfrac{\pi}{180}\right)$

$= \dfrac{\pi}{6}$ radians

(c) Reference angle is itself :

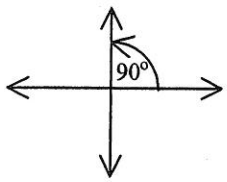
$30^\circ = \dfrac{\pi}{6}$ radians




**13.** (b) $90^\circ = 90\left(\dfrac{\pi}{180}\right)$

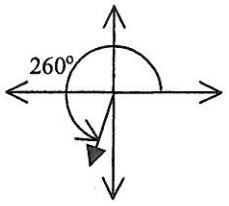
$= \dfrac{\pi}{2}$ radians

(c) Reference angle is undefined.



**15.** (b) $260^\circ = 260\left(\frac{\pi}{180}\right)$

$= \frac{13\pi}{9}$ radians




(c) $\hat{\theta} = 260^\circ - 180^\circ$

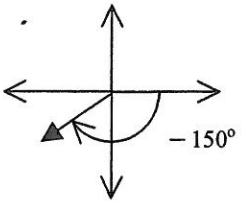
$= 80^\circ$

$80^\circ = 80\left(\frac{\pi}{180}\right)$

$= \frac{4\pi}{9}$ radians

**17.** (b) $-150^\circ = -150\left(\frac{\pi}{180}\right)$

$= -\frac{5\pi}{6}$




(c) $-150^\circ = -150 + 360^\circ$

$= 210^\circ$

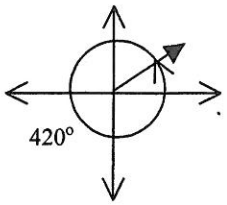
$\hat{\theta} = 210^\circ - 180^\circ = 30^\circ$

$30^\circ = 30\left(\frac{\pi}{180}\right)$

$= \frac{\pi}{6}$ radians

**19.** (b) $420^\circ = 420\left(\frac{\pi}{180}\right)$

$= \frac{7\pi}{3}$ radians



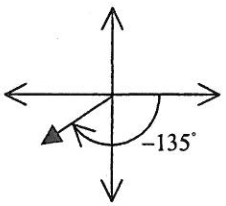

(c) $\hat{\theta} = 420^\circ - 360^\circ = 60^\circ$

$60^\circ = 60\left(\frac{\pi}{180}\right)$

$= \frac{\pi}{3}$ radians

**21.** (b) $-135^\circ = -135\left(\frac{\pi}{180}\right)$

$= -\frac{3\pi}{4}$ radians

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(c) $-135^\circ = -135^\circ + 360^\circ$

$= 225^\circ$

$\hat{\theta} = 225^\circ - 180^\circ = 45^\circ$

$45^\circ = 45\left(\frac{\pi}{180}\right)$

$= \frac{\pi}{4}$ radians

**23.** $120^\circ 40' = \left(120 + \frac{40}{60}\right)^\circ = 120.67^\circ$

$120.67^\circ = 120.67\left(\frac{\pi}{180}\right)$

$= 2.11$

**25.** 1 minute $= \frac{1}{60}$ degree

$= \frac{1}{60}\left(\frac{\pi}{180}\right)$

$= \frac{\pi}{10,800}$

$= 0.000291$

**27.** From problem 25, we know that $1' = \frac{\pi}{10,800}$ radians.

$\theta = \frac{s}{r}$

$s = r\theta$

$= 4,000\left(\frac{\pi}{10,800}\right)$

$= 1.16$ miles

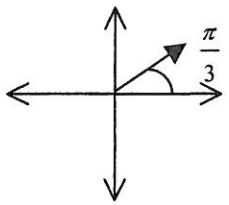
**29.** $\theta = \frac{5 \text{ min}}{60 \text{ min}} \cdot 2\pi$

$= \frac{1}{12} \cdot 2\pi$

$= \frac{\pi}{6}$

**31.** (a) $\frac{\pi}{3} = \frac{\pi}{3}\left(\frac{180}{\pi}\right)^\circ$

$= 60^\circ$




(c) Reference angle is itself:

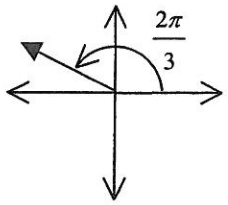
$\frac{\pi}{3} = 60^\circ$

**33.** (a) $\frac{2\pi}{3} = \frac{2\pi}{3}\left(\frac{180}{\pi}\right)^\circ$

$= 120^\circ$

(c) $\hat{\theta} = 180^\circ - 120^\circ = 60^\circ$

$60^\circ = \frac{\pi}{3}$




**35.** (a) $-\frac{7\pi}{6} = -\frac{7\pi}{6}\left(\frac{180}{\pi}\right)^\circ$

$= -210^\circ$

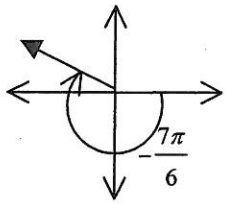
(c) $-210^\circ = -210^\circ + 360^\circ$

$= 150^\circ$

$\hat{\theta} = 180^\circ - 150^\circ = 30^\circ$

$30^\circ = 30\left(\frac{\pi}{180}\right)$

$= \frac{\pi}{6}$

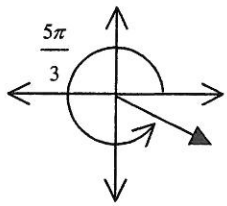



**37.** (a) $\frac{5\pi}{3} = \frac{5\pi}{3}\left(\frac{180}{\pi}\right)^\circ$

$= 300^\circ$

(c) $\hat{\theta} = 360^\circ - 300^\circ = 60^\circ$

$60^\circ = \frac{\pi}{3}$




**39.** (a) $4\pi = 4\pi\left(\frac{180}{\pi}\right)^\circ$

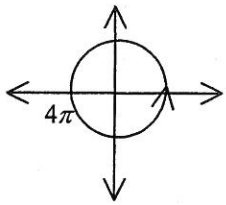
$= 720^\circ$

(c) $720^\circ = 720^\circ - 360^\circ = 360^\circ$

$360^\circ = 360^\circ - 360^\circ$

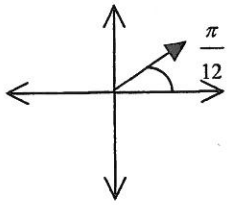
$= 0^\circ$

Reference angle is undefined.



**41.** (a) $\frac{\pi}{12}=\frac{\pi}{12}\left(\frac{180}{\pi}\right)^\circ=15^\circ$

(c) $\hat{\theta}=15^\circ=\frac{\pi}{12}$




**43.** $1=1\left(\frac{180}{\pi}\right)^\circ$

$=\left(\frac{180}{\pi}\right)^\circ$

$=57.3^\circ$

**45.** $1.3=1.3\left(\frac{180}{\pi}\right)^\circ$

$=74.5^\circ$

**47.** $0.75=0.75\left(\frac{180}{\pi}\right)^\circ$

$=\left(\frac{135}{\pi}\right)^\circ$

$=43.0^\circ$

**49.** $5=5\left(\frac{180}{\pi}\right)^\circ$

$=286.5^\circ$

**51.** Since $\frac{4\pi}{3}$ terminates in QIII, its sine will be negative.

$\hat{\theta}=\frac{4\pi}{3}-\pi$

$=\frac{\pi}{3}$

$\sin\frac{4\pi}{3}=-\sin\frac{\pi}{3}\qquad\left(\frac{\pi}{3}=60^\circ\right)$

$=-\frac{\sqrt{3}}{2}$

**53.** $\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}$

**55.** Since $\frac{2\pi}{3}$ terminates in QII, its secant will be negative.

$\hat{\theta}=\pi-\frac{2\pi}{3}=\frac{\pi}{3}$

$\sec\frac{2\pi}{3}=-\sec\frac{\pi}{3}$

$=-2$

**57.** Since $\frac{5\pi}{6}$ terminates in QII, its cosecant is positive.

$\hat{\theta}=\pi-\frac{5\pi}{6}=\frac{\pi}{6}$

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$$\csc\frac{5\pi}{6} = \csc\frac{\pi}{6}$$
$$= \frac{1}{\sin\frac{\pi}{6}}$$
$$= \frac{1}{\frac{1}{2}} = 2$$

**59.** Since $-\frac{\pi}{4}$ terminates in QIV, its sine will be negative.

$$\theta = -\frac{\pi}{4} + 2\pi = \frac{7\pi}{4}$$
$$\hat{\theta} = 2\pi - \frac{7\pi}{4} = \frac{\pi}{4}$$
$$4\sin\left(-\frac{\pi}{4}\right) = -4\sin\frac{\pi}{4}$$
$$= -4\left(\frac{\sqrt{2}}{2}\right)$$
$$= -2\sqrt{2}$$

**61.** $-\sin\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$

**63.** $2\cos\frac{\pi}{6} = 2\left(\frac{\sqrt{3}}{2}\right)$
$$= \sqrt{3}$$

**65.** $\sin 2x = \sin 2\left(\frac{\pi}{6}\right)$
$$= \sin\frac{\pi}{3}$$
$$= \frac{\sqrt{3}}{2}$$

**67.** $6\cos 3\left(\frac{\pi}{6}\right) = 6\cos\frac{\pi}{2}$
$$= 6(0)$$
$$= 0$$

**69.** $\sin\left(x + \frac{\pi}{2}\right) = \sin\left(\frac{\pi}{6} + \frac{\pi}{2}\right)$
$$= \sin\frac{2\pi}{3}$$

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Since $\frac{2\pi}{3}$ terminates in QII, its sine is positive.

$$\hat{\theta} = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$$

$$\sin\frac{2\pi}{3} = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

**71.** $4\cos\left[2\left(\frac{\pi}{6}\right)+\frac{\pi}{3}\right] = 4\cos\left(\frac{\pi}{3}+\frac{\pi}{3}\right) = 4\cos\frac{2\pi}{3}$

Since $\frac{2\pi}{3}$ terminates in QII, its cosine will be negative.

$$\hat{\theta} = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$$

$$4\cos\frac{2\pi}{3} = -4\cos\frac{\pi}{3}$$

$$= -4\left(\frac{1}{2}\right) = -2$$

**73.** For $x = 0$, $y = \sin 0$

$$= 0 \qquad (x, y) = (0, 0)$$

For $x = \frac{\pi}{4}$, $y = \sin\frac{\pi}{4}$

$$= \frac{1}{\sqrt{2}} \qquad (x, y) = \left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$$

For $x = \frac{\pi}{2}$, $y = \sin\frac{\pi}{2}$

$$= 1 \qquad (x, y) = \left(\frac{\pi}{2}, 1\right)$$

For $x = \frac{3\pi}{4}$, $y = \sin\frac{3\pi}{4}$

$$= \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} \qquad (x, y) = \left(\frac{3\pi}{4}, \frac{1}{\sqrt{2}}\right)$$

For $x = \pi$, $y = \sin\pi$

$$= 0 \qquad (x, y) = (\pi, 0)$$

**75.** For $x = 0$ $\quad y = 2\sin 0$

$$= 2(0)$$

$$= 0 \qquad (x, y) = (0, 0)$$

For $x = \frac{\pi}{2}$, $\quad y = 2\sin\frac{\pi}{2}$

$$= 2(1) = 2 \qquad (x, y) = \left(\frac{\pi}{2}, 2\right)$$

For $x = \pi$, $\quad y = 2\sin\pi$

$$= 2(0) = 0 \qquad (x, y) = (\pi, 0)$$

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For $x=\frac{3\pi}{2}$, $y=2\sin\frac{3\pi}{2}$

$=2(-1)=-2$ $\quad (x,y)=\left(\frac{3\pi}{2},-2\right)$

For $x=2\pi$, $y=2\sin 2\pi$

$=2(0)=0$ $\quad (x,y)=(2\pi,0)$

**77.** For $x=0$, $y=\sin 2(0)=\sin 0=0$ $\quad (x,y)=(0,0)$

For $x=\frac{\pi}{4}$, $y=\sin 2\left(\frac{\pi}{4}\right)$ $\quad (x,y)=\left(\frac{\pi}{4},1\right)$

$=\sin\frac{\pi}{2}=1$

For $x=\frac{\pi}{2}$, $y=\sin 2\left(\frac{\pi}{2}\right)$ $\quad (x,y)=\left(\frac{\pi}{2},0\right)$

$=\sin\pi=0$

For $x=\frac{3\pi}{4}$, $y=\sin 2\left(\frac{3\pi}{4}\right)$ $\quad (x,y)=\left(\frac{3\pi}{4},-1\right)$

$=\sin\frac{3\pi}{2}=-1$

For $x=\pi$, $y=\sin 2(\pi)=0$ $\quad (x,y)=(\pi,0)$

**79.** For $x=\frac{\pi}{2}$, $y=\sin\left(\frac{\pi}{2}-\frac{\pi}{2}\right)$ $\quad (x,y)=\left(\frac{\pi}{2},0\right)$

$=\sin 0=0$

For $x=\pi$, $y=\sin\left(\pi-\frac{\pi}{2}\right)$ $\quad (x,y)=(\pi,1)$

$=\sin\frac{\pi}{2}=1$

For $x=\frac{3\pi}{2}$, $y=\sin\left(\frac{3\pi}{2}-\frac{\pi}{2}\right)$ $\quad (x,y)=\left(\frac{3\pi}{2},0\right)$

$=\sin\pi=0$

For $x=2\pi$, $y=\sin\left(2\pi-\frac{\pi}{2}\right)$ $\quad (x,y)=(2\pi,-1)$

$=\sin\frac{3\pi}{2}=-1$

For $x=\frac{5\pi}{2}$, $y=\sin\left(\frac{5\pi}{2}-\frac{\pi}{2}\right)$ $\quad (x,y)=\left(\frac{5\pi}{2},0\right)$

$=\sin 2\pi=0$

**81.** For $x=-\frac{\pi}{4}$, $y=3\sin\left[2\left(-\frac{\pi}{4}\right)+\frac{\pi}{2}\right]$ $\qquad (x, y)=\left(-\frac{\pi}{4},0\right)$

$$=3\sin\left(-\frac{\pi}{2}+\frac{\pi}{2}\right)$$

$$=3\sin 0=3\cdot 0=0$$

For $x=0$, $y=3\sin\left[2(0)+\frac{\pi}{2}\right]$ $\qquad (x, y)=(0,3)$

$$=3\sin\frac{\pi}{2}=3(1)=3$$

For $x=\frac{\pi}{4}$, $y=3\sin\left[2\left(\frac{\pi}{4}\right)+\frac{\pi}{2}\right]$ $\qquad (x, y)=\left(\frac{\pi}{4},0\right)$

$$=3\sin\left(\frac{\pi}{2}+\frac{\pi}{2}\right)$$

$$=3\sin\pi=3(0)=0$$

For $x=\frac{\pi}{2}$, $y=3\sin\left[2\left(\frac{\pi}{2}\right)+\frac{\pi}{2}\right]$ $\qquad (x, y)=\left(\frac{\pi}{2},-3\right)$

$$=3\sin\left(\pi+\frac{\pi}{2}\right)$$

$$=3\sin\frac{3\pi}{2}=3(-1)=-3$$

For $x=\frac{3\pi}{4}$, $y=3\sin\left[2\left(\frac{3\pi}{4}\right)+\frac{\pi}{2}\right]$ $\qquad (x, y)=\left(\frac{3\pi}{4},0\right)$

$$=3\sin\left(\frac{3\pi}{2}+\frac{\pi}{2}\right)$$

$$=3\sin 2\pi=3(0)=0$$

**83.** $\theta=\frac{2\pi}{8}=\frac{\pi}{4}$ radians

**85.** $(x,y)=(1,-3)$

$r=\sqrt{x^2+y^2}=\sqrt{(1)^2+(-3)^2}=\sqrt{1+9}=\sqrt{10}$

$\sin\theta=\frac{y}{r}=\frac{-3}{\sqrt{10}}$ $\qquad \cot\theta=\frac{x}{y}=\frac{1}{-3}$

$\cos\theta=\frac{x}{r}=\frac{1}{\sqrt{10}}$ $\qquad \sec\theta=\frac{r}{x}=\frac{\sqrt{10}}{1}=\sqrt{10}$

$\tan\theta=\frac{y}{x}=\frac{-3}{1}=-3$ $\qquad \csc\theta=\frac{r}{y}=\frac{\sqrt{10}}{-3}$

**87.** $(x,y)=(m,n)$

$$r=\sqrt{x^2+y^2}=\sqrt{m^2+n^2}$$

$$\sin\theta=\frac{y}{r}=\frac{n}{\sqrt{m^2+n^2}} \qquad \cot\theta=\frac{x}{y}=\frac{m}{n}$$

$$\cos\theta=\frac{x}{r}=\frac{m}{\sqrt{m^2+n^2}} \qquad \sec\theta=\frac{r}{x}=\frac{\sqrt{m^2+n^2}}{m}$$

$$\tan\theta=\frac{y}{x}=\frac{n}{m} \qquad \csc\theta=\frac{r}{y}=\frac{\sqrt{m^2+n^2}}{n}$$

**89.** $\sin\theta=\frac{1}{2}$ and $\theta$ terminates in QII.

$\cos\theta=-\sqrt{1-\sin^2\theta}$ because $\cos\theta$ is negative in QII

$$=-\sqrt{1-\left(\frac{1}{2}\right)^2}$$

$$=-\sqrt{1-\frac{1}{4}}$$

$$=-\sqrt{\frac{3}{4}}=-\frac{\sqrt{3}}{2}$$

$$\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{1/2}{-\sqrt{3}/2}=-\frac{1}{\sqrt{3}} \qquad \cot\theta=\frac{1}{\tan\theta}=\frac{1}{-1/\sqrt{3}}=-\sqrt{3}$$

$$\sec\theta=\frac{1}{\cos\theta}=\frac{1}{-\sqrt{3}/2}=-\frac{2}{\sqrt{3}} \qquad \csc\theta=\frac{1}{\sin\theta}=\frac{1}{1/2}=2$$

**91.** A point on the line $y=2x$ in QI is (1, 2).

$(x,y)=(1,2)$

$$r=\sqrt{1^2+2^2}=\sqrt{1+4}=\sqrt{5}$$

$$\sin\theta=\frac{y}{r}=\frac{2}{\sqrt{5}} \qquad \cot\theta=\frac{x}{y}=\frac{1}{2}$$

$$\cos\theta=\frac{x}{r}=\frac{1}{\sqrt{5}} \qquad \sec\theta=\frac{r}{x}=\frac{\sqrt{5}}{1}=\sqrt{5}$$

$$\tan\theta=\frac{y}{x}=\frac{2}{1}=2 \qquad \csc\theta=\frac{r}{y}=\frac{\sqrt{5}}{2}$$