

# ANSWER KEY

Day 3

## Special Segments Practice

I. Matching: Match the picture to the special segment. You will use the special segment more than once.

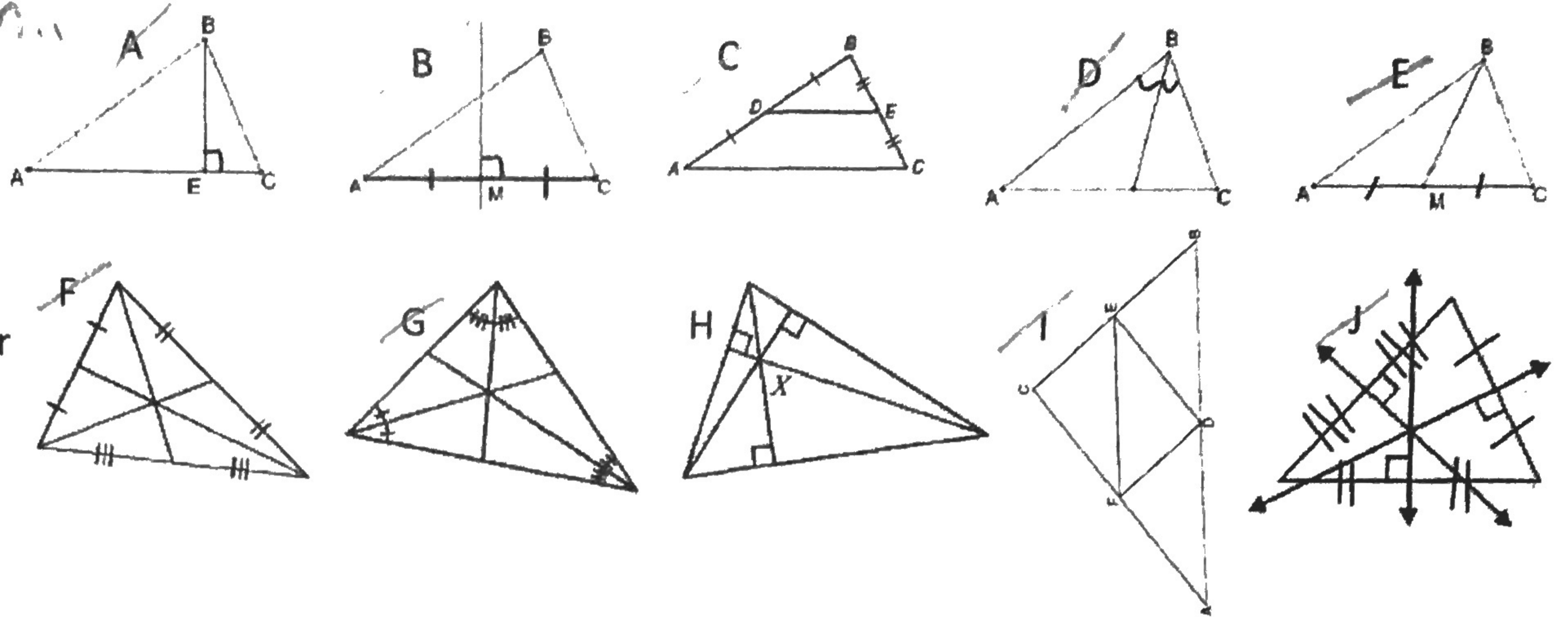
1) Midsegment

2) Altitude

3) Angle Bisector

4) Perpendicular Bisector

5) Median



II. Matching: Match the point of concurrency to the special segment and to the correct fact about location.

6) Orthocenter

7) Incenter

8) Circumcenter

9) Centroid

A) Medians

B) Altitudes

C) Angle Bisectors

D) Perpendicular Bisectors

i) Equidistant from vertices

ii) Equidistant from sides

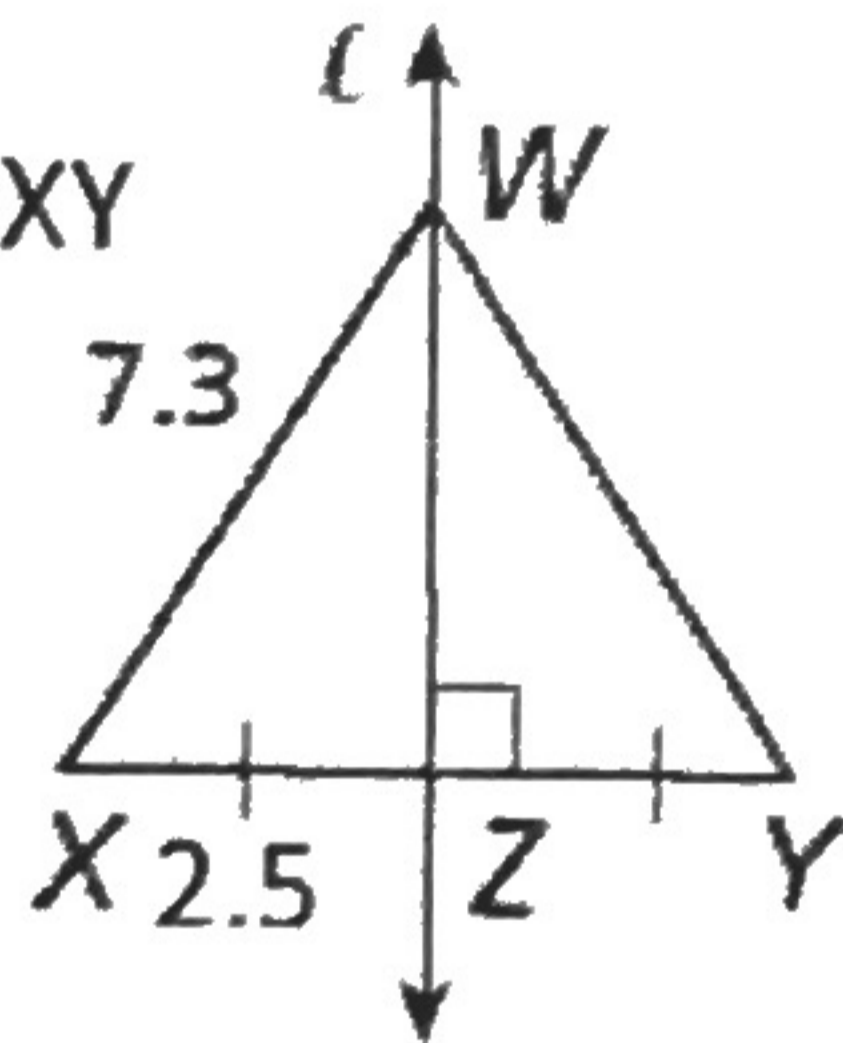
iii) 2 (small section) = (larger section)

iv) No location fact

III. Solve: Use the properties of special segments to solve the following problems.

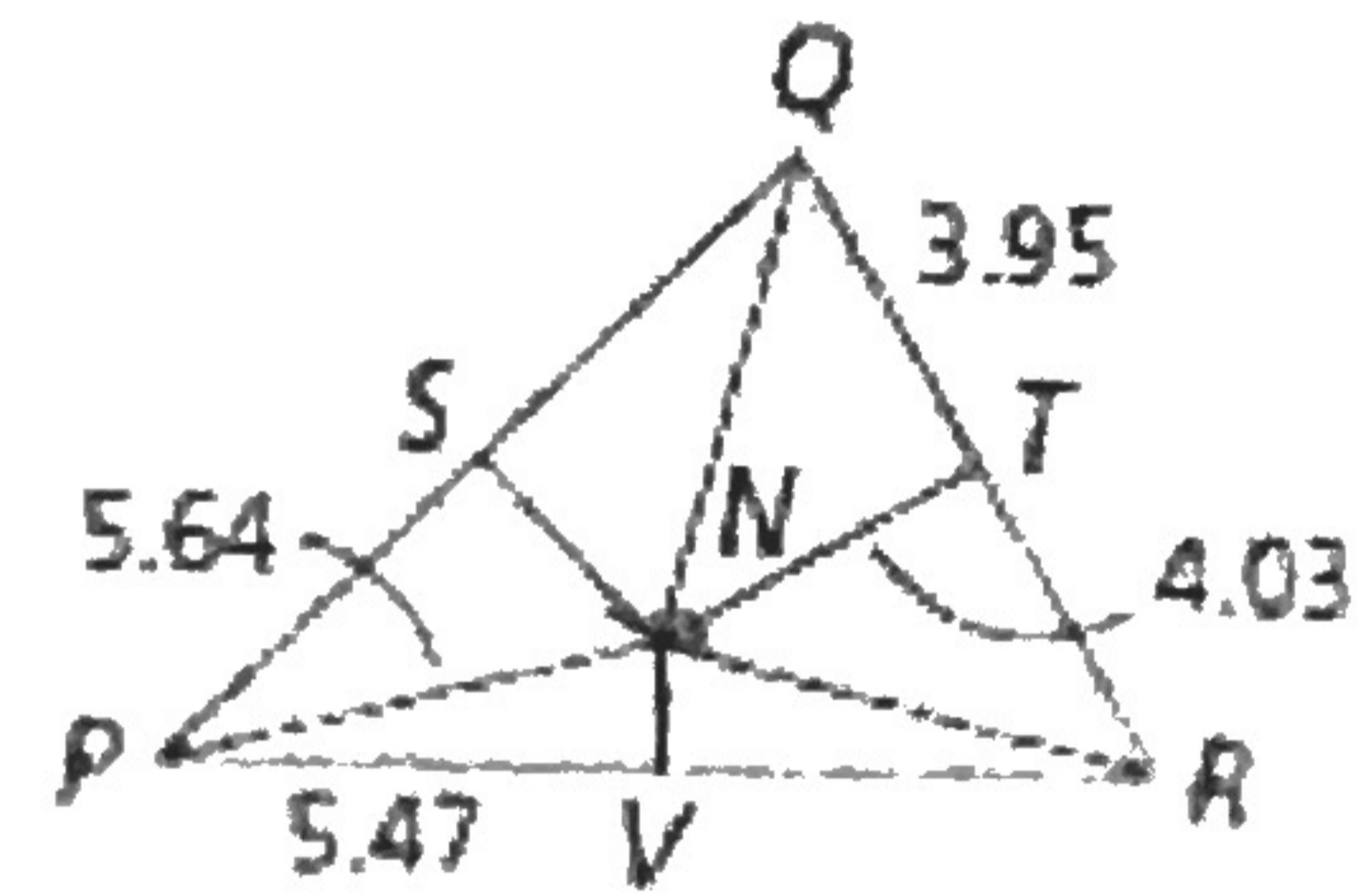
10) Find WY, ZY, and XY

$WY = 7.3$   
 $ZY = 2.5$   
 $XY = 5$



11) N is the circumcenter.  
Find QN, RN, QR

$QN = 5.64$   
 $RN = 5.64$   
 $QR = 7.98$



12) Use the picture at the right to answer the following questions.

a) A segment parallel to  $\overline{AC}$

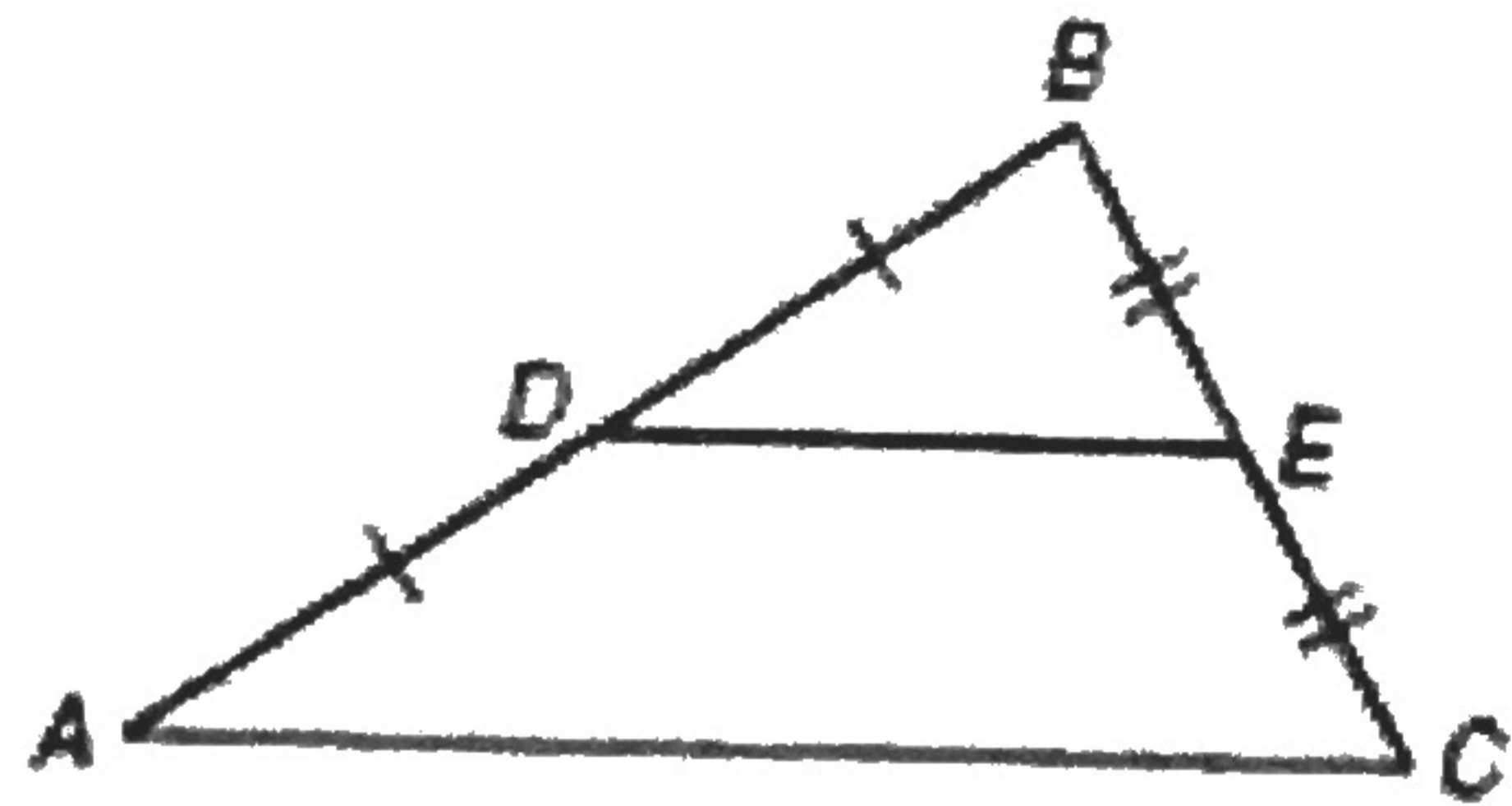
$\overline{DE}$

b) A segment that has half the length of  $\overline{AC}$

$\overline{DE}$

c) A segment that has twice the length of  $\overline{EC}$

$\overline{BC}$

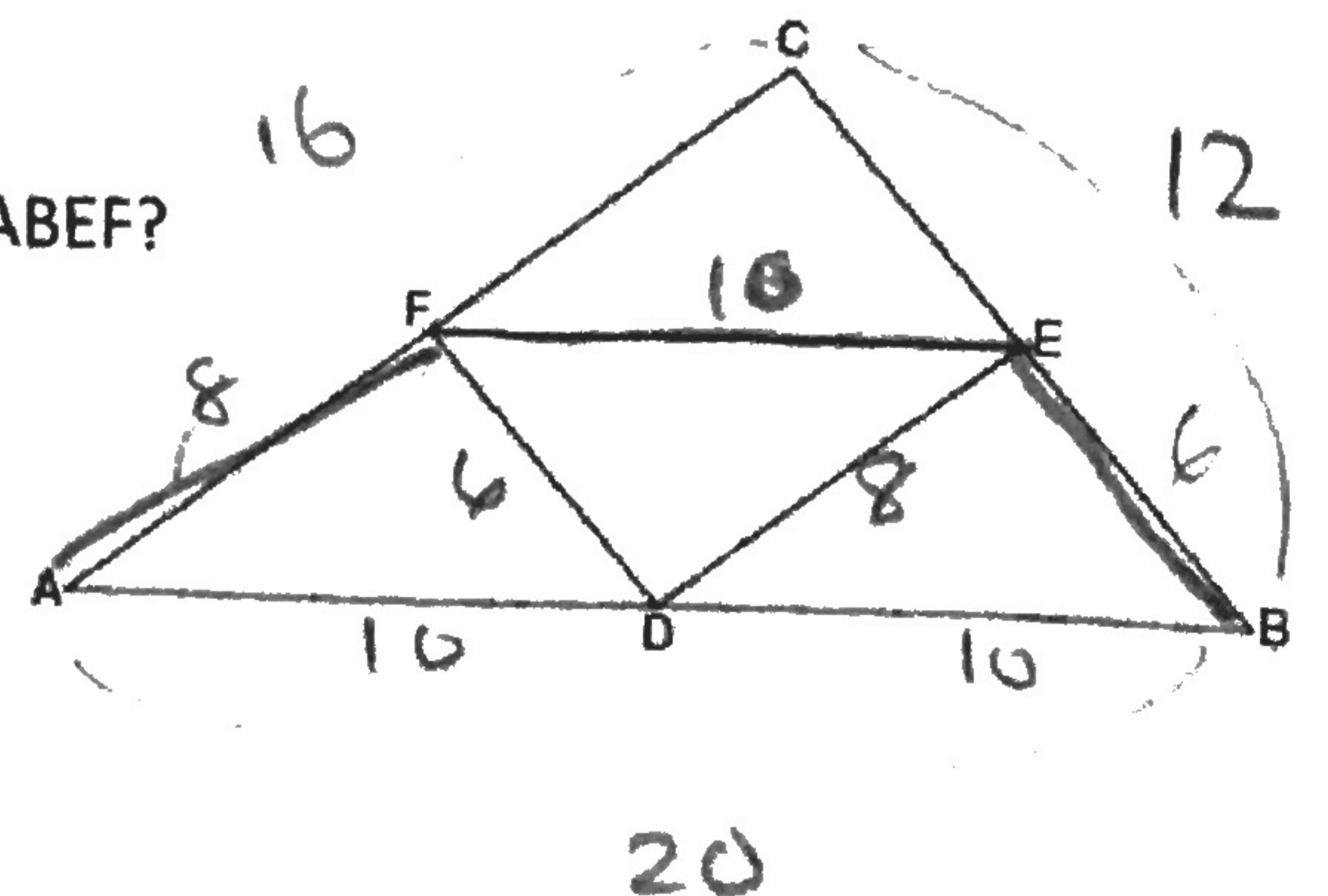


13) In the diagram of  $\triangle ABC$  shown below, D is the midpoint of  $\overline{AB}$ , E is the midpoint of  $\overline{BC}$ , and F is the midpoint of  $\overline{AC}$ .

If  $AB = 20$ ,  $BC = 12$ , and  $AC = 16$ , what is the perimeter of trapezoid ABEF?

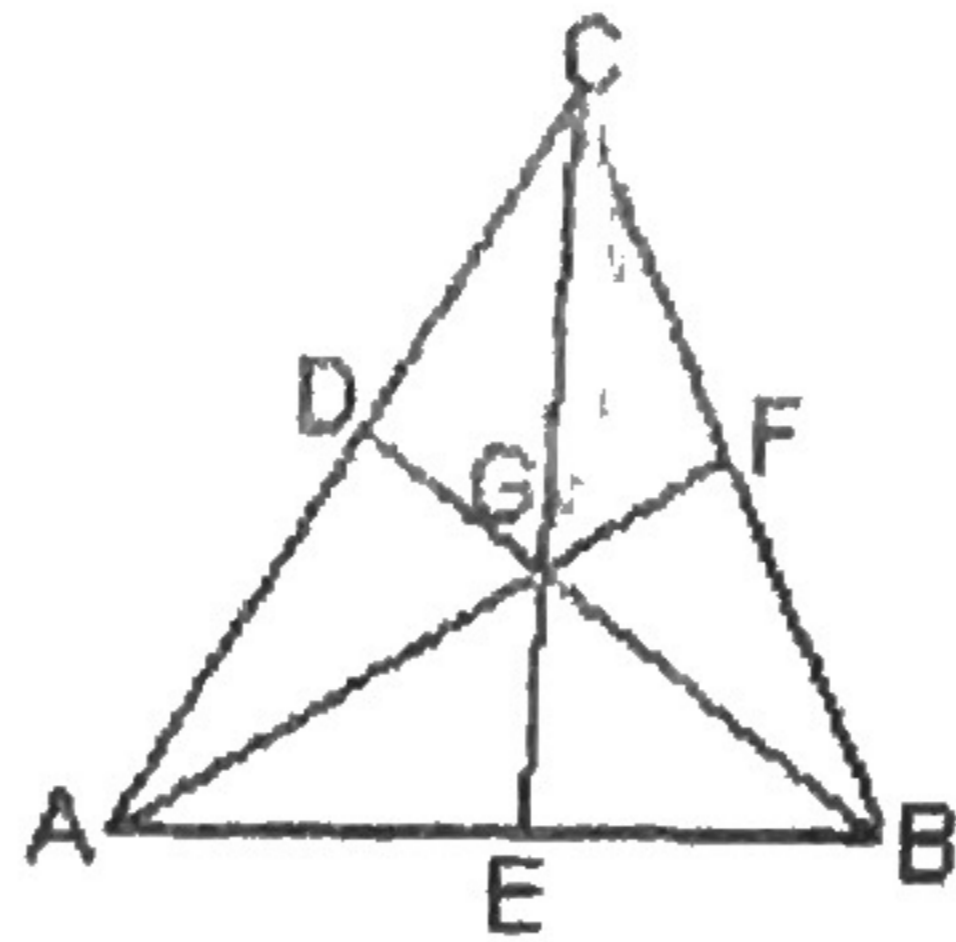
A 24   B 36   C 40   **D 44**

$10 + 10 + 8 + 10 + 6$



14) G is the centroid. If  $CG = 20$ , find  $GE$  and  $CE$ ?

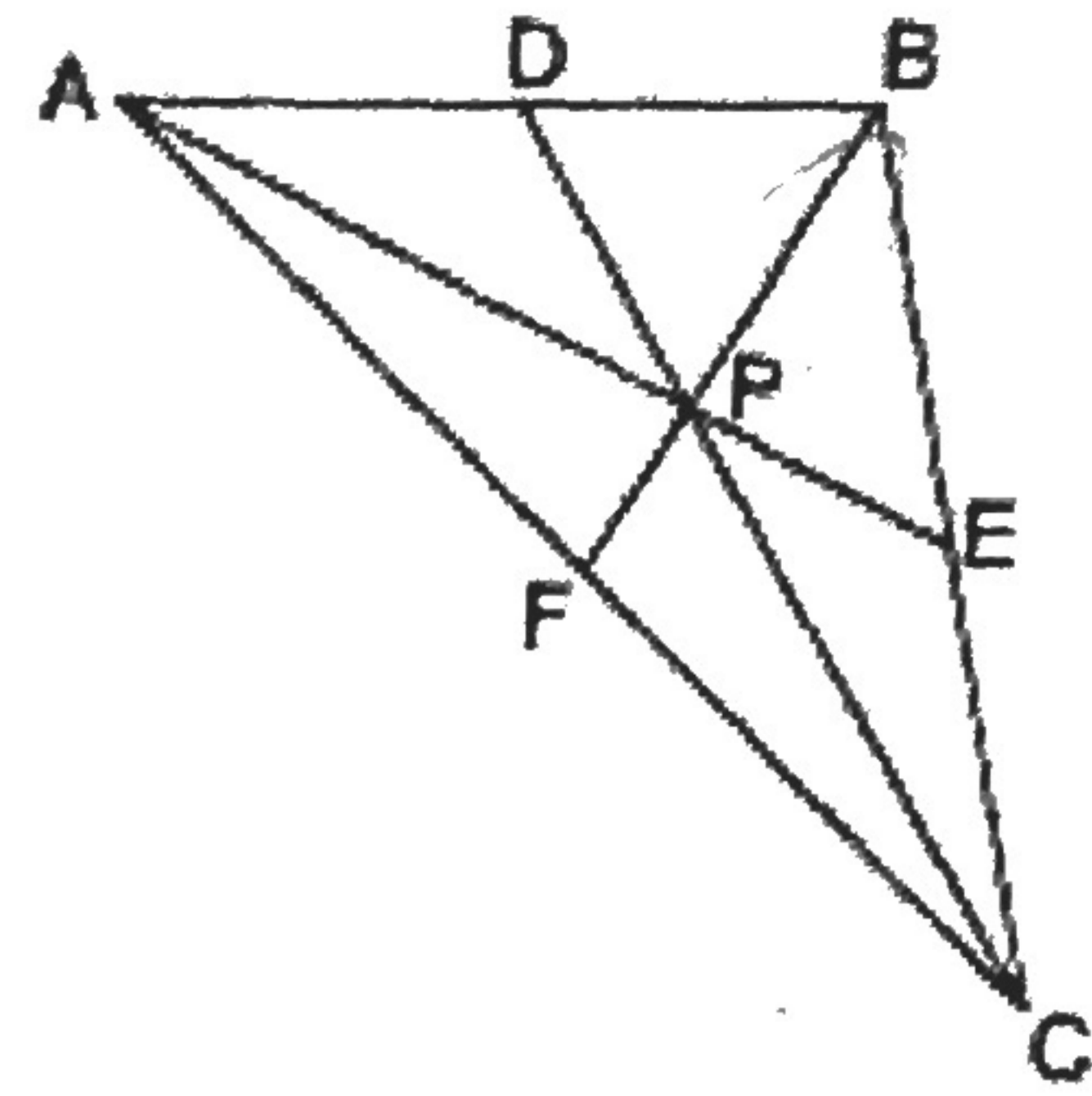
$$\begin{aligned} GE &= 10 \\ CE &= 30 \end{aligned}$$



15) In  $\triangle ABC$  shown below, P is the centroid and  $BF = 18$ . What is the length of  $BP$ ?

- A 6    B 9    C 3    **D 12**

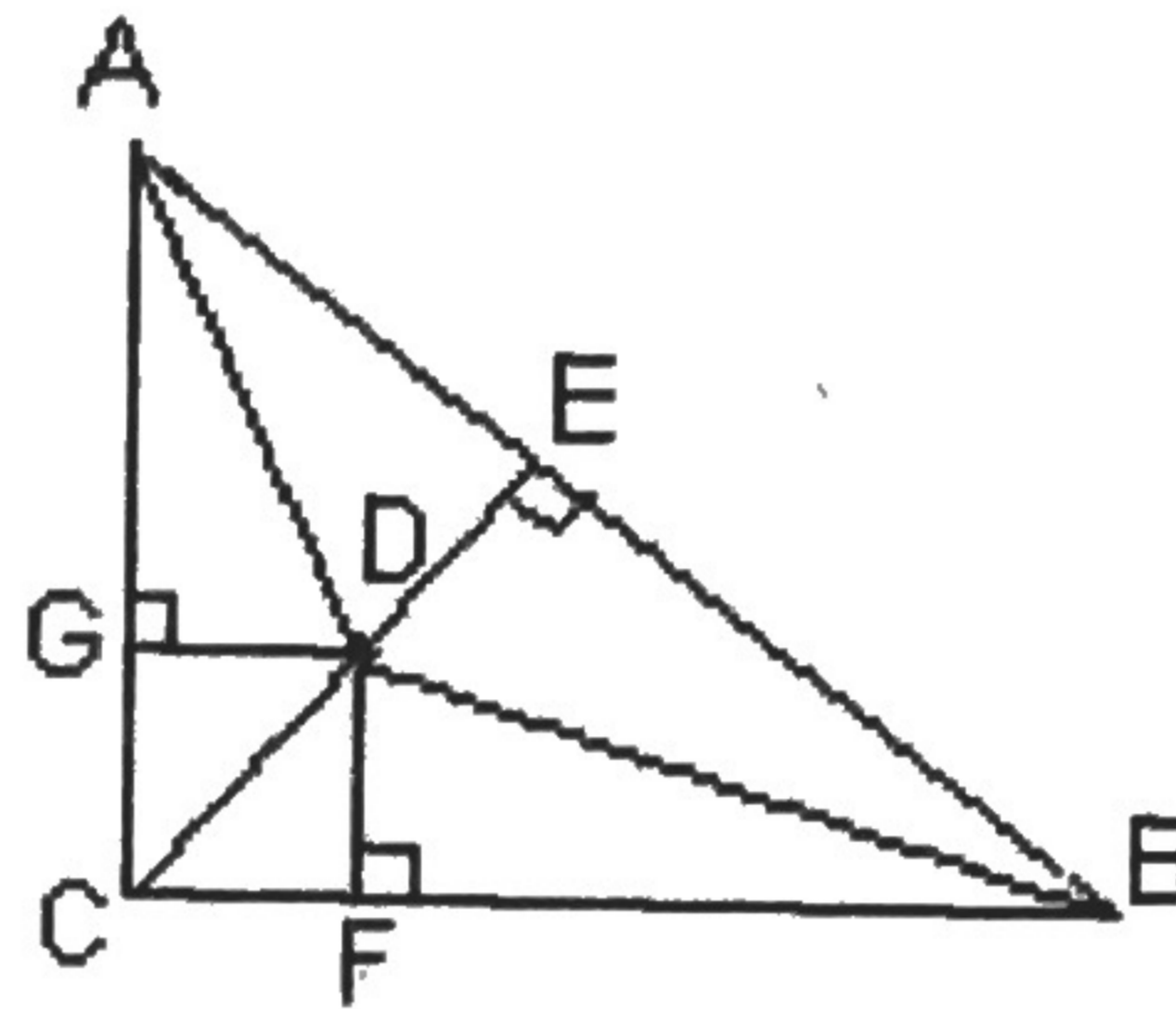
$$\frac{18}{3} = 6 \times 2 = 12$$



Use the picture to the right for 16 and 17.

16) If  $DF = 13$ , what is  $DE$ ?

$$DE = 13$$



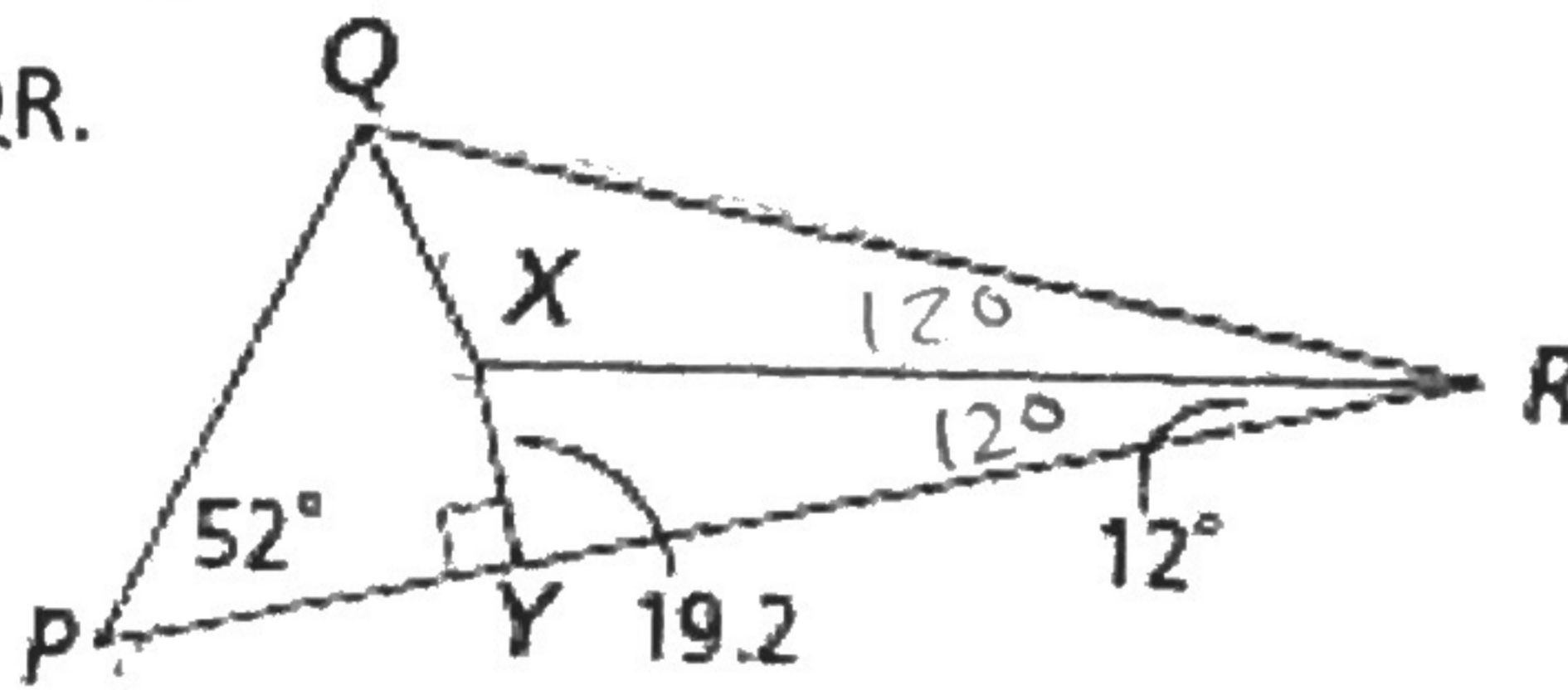
D is the incenter.

17)  $m\angle EAD = 15^\circ$  Find  $m\angle DAG$  and  $m\angle GAE$

$$\begin{aligned} m\angle DAG &= 15^\circ \\ m\angle GAE &= 30^\circ \end{aligned}$$

18) Find  $m\angle XRQ$ ,  $m\angle PRQ$ , and  $m\angle PQR$ .

$$\begin{aligned} m\angle XRQ &= 120 \\ m\angle PRQ &= 240 \\ m\angle PQR &= 106 \end{aligned}$$

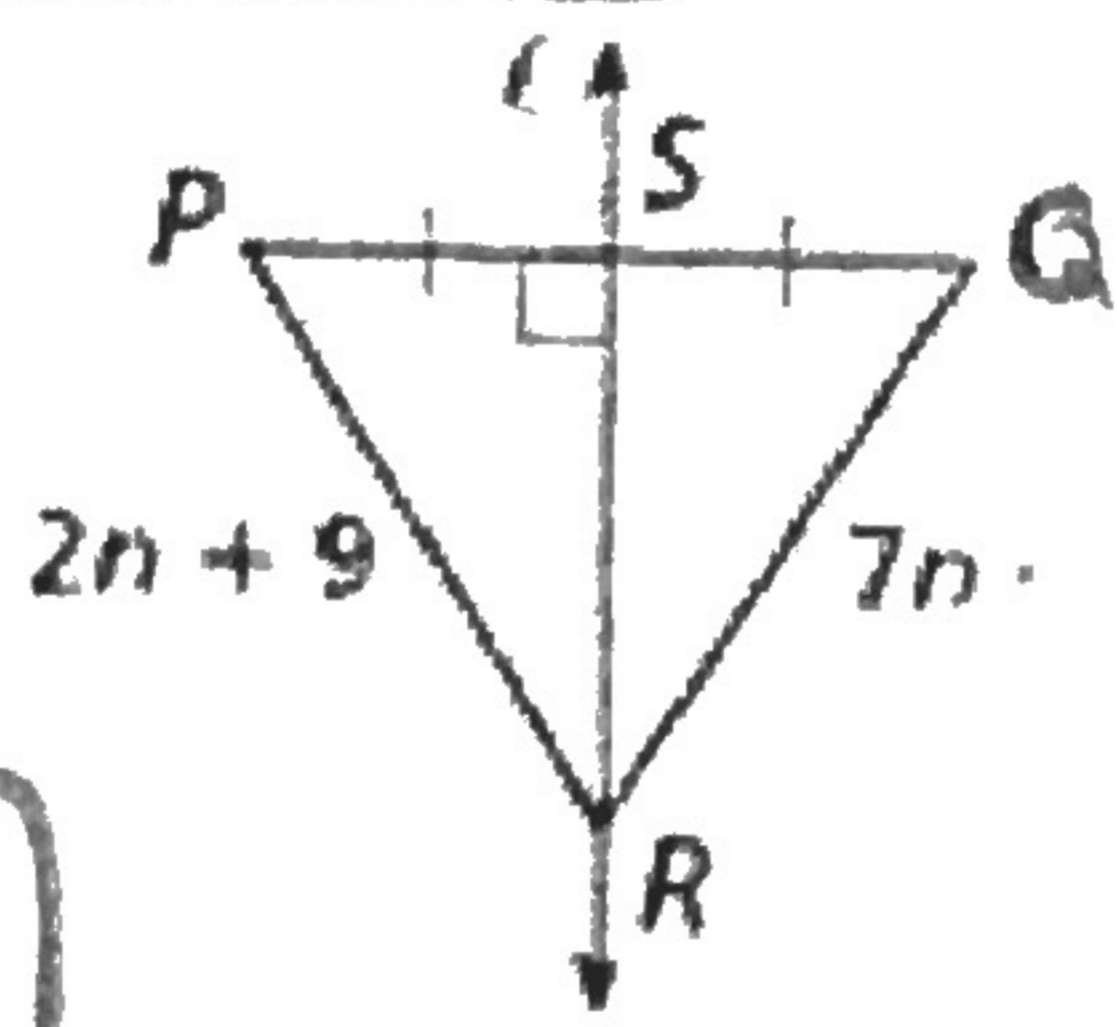


$$\begin{array}{r} 170 \\ 180 \\ -52 \\ -24 \\ \hline 106 \end{array}$$

19) Find  $n$

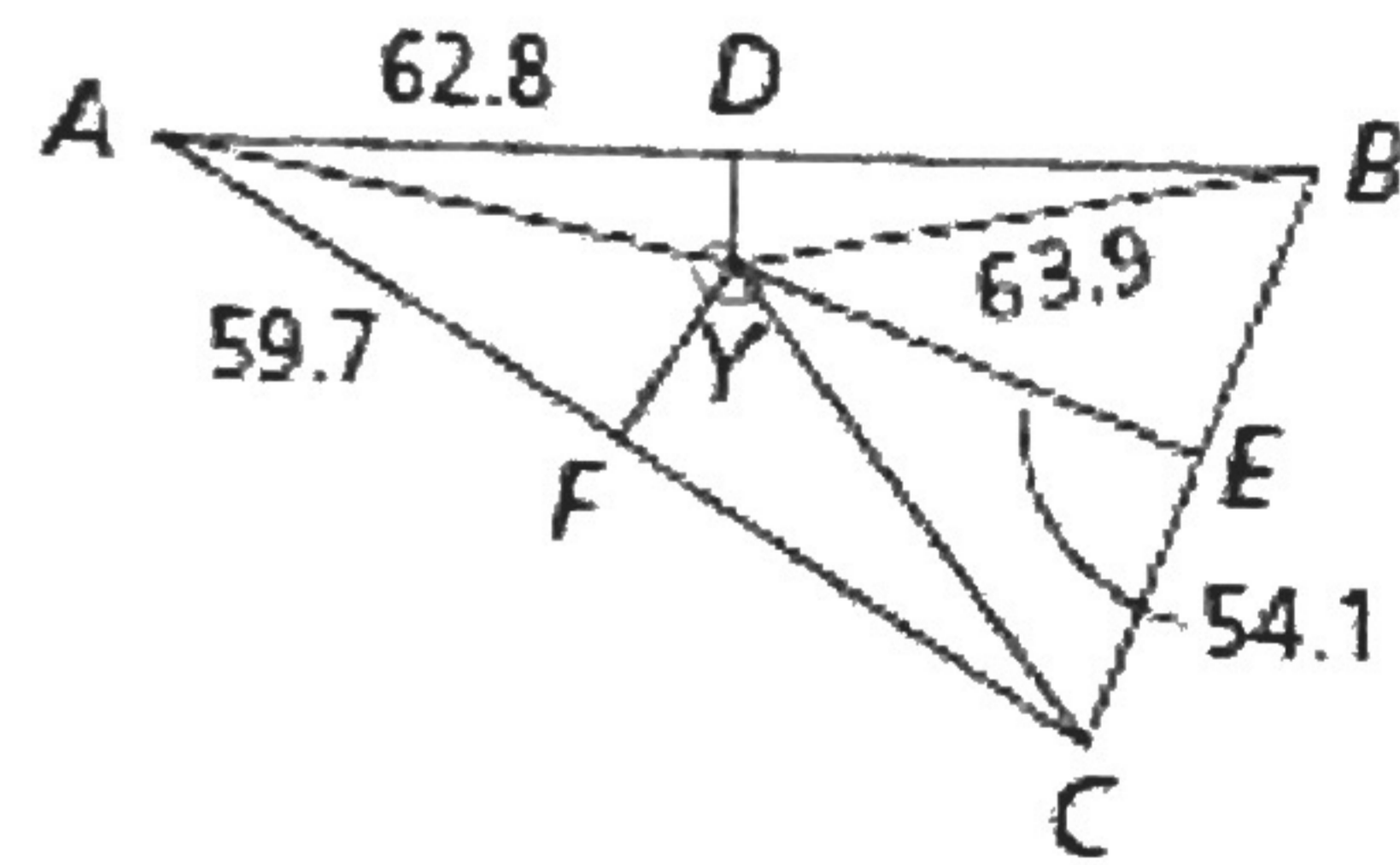
$$\begin{aligned} 2n + 9 &= 7n \\ 9 &= 5n \end{aligned}$$

$$n = \frac{9}{5}$$



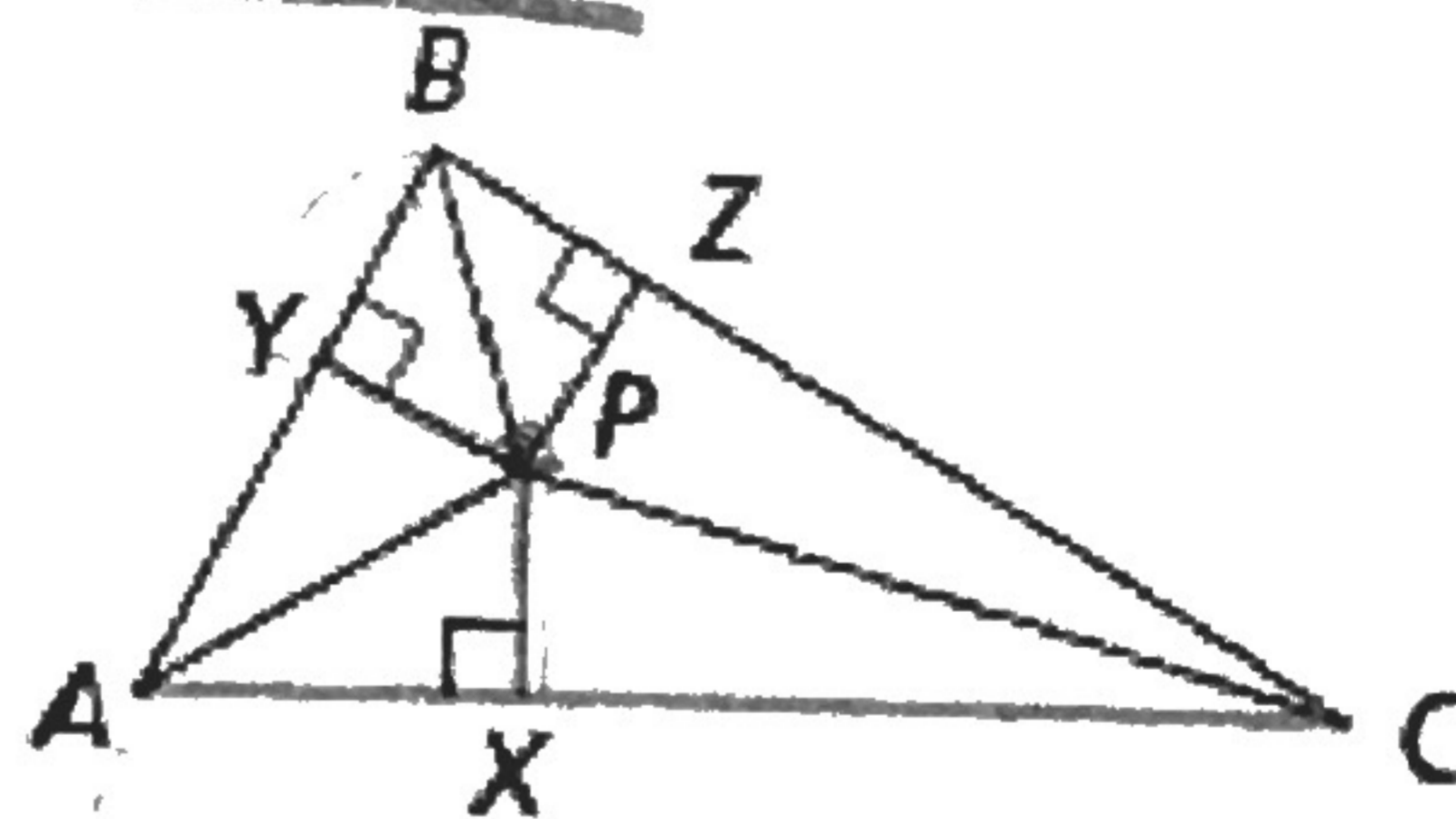
20) Y is the circumcenter. Find  $YC$  and  $AB$ .

$$\begin{aligned} YC &= 63.9 \\ AB &= 125.6 \end{aligned}$$



21) P is the incenter of  $\triangle ABC$ . Which must be true?

- A  $PA = PB$        C  $YA = YB$   
 B  $PX = PY$        D  $AX = BZ$

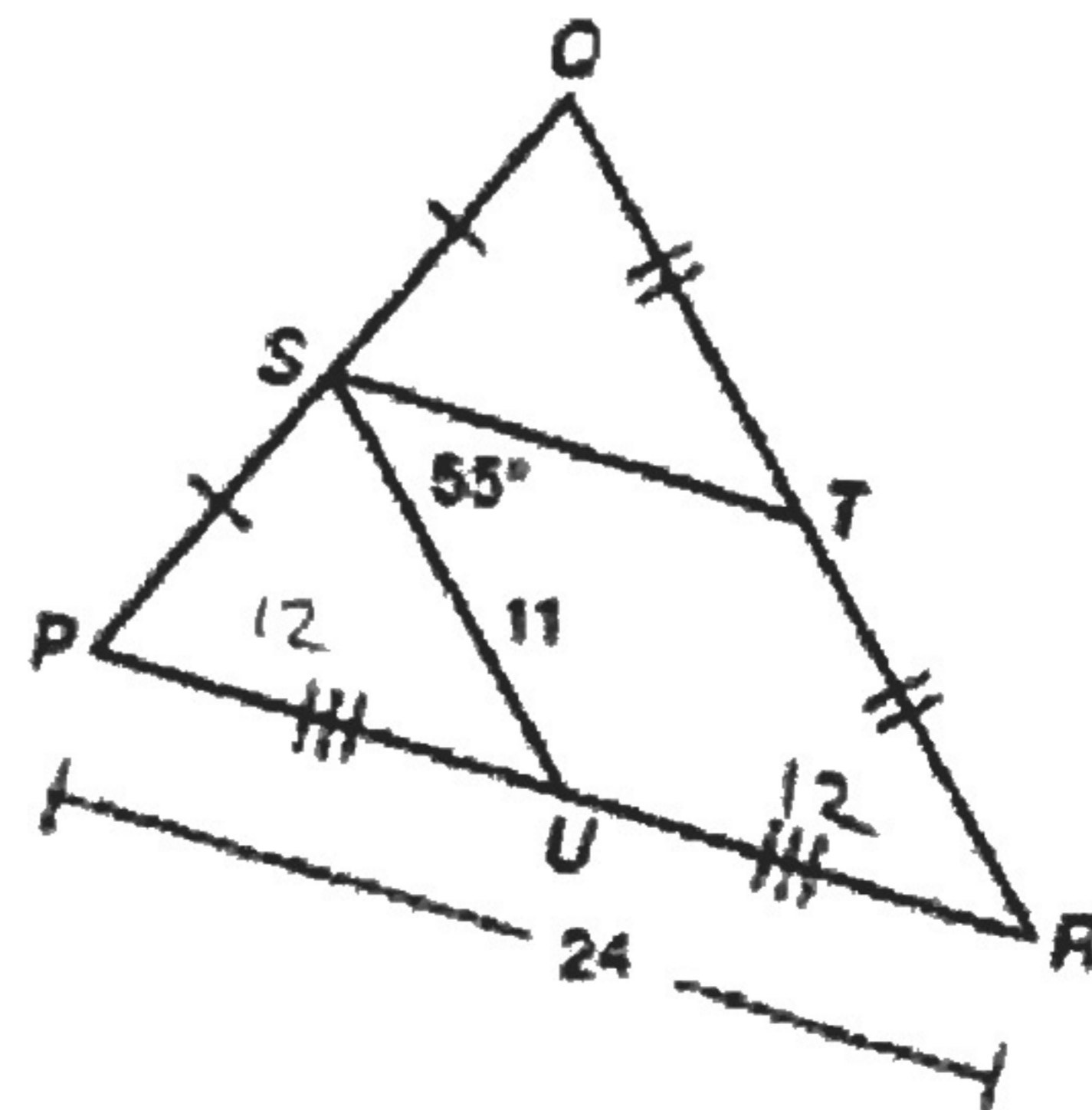


22) The circumcenter of the triangle is equidistant from the vertices of the triangle.

23) The incenter is important because it is equidistant from the sides of the triangle.

24) Use the picture at the right to answer the following questions.

- a)  $ST = 12$       d)  $QR = 22$   
 b)  $PU = 12$       e)  $m\angle SUP = 55^\circ$   
 c)  $m\angle SUR = 125^\circ$       f)  $m\angle PRQ = 55^\circ$

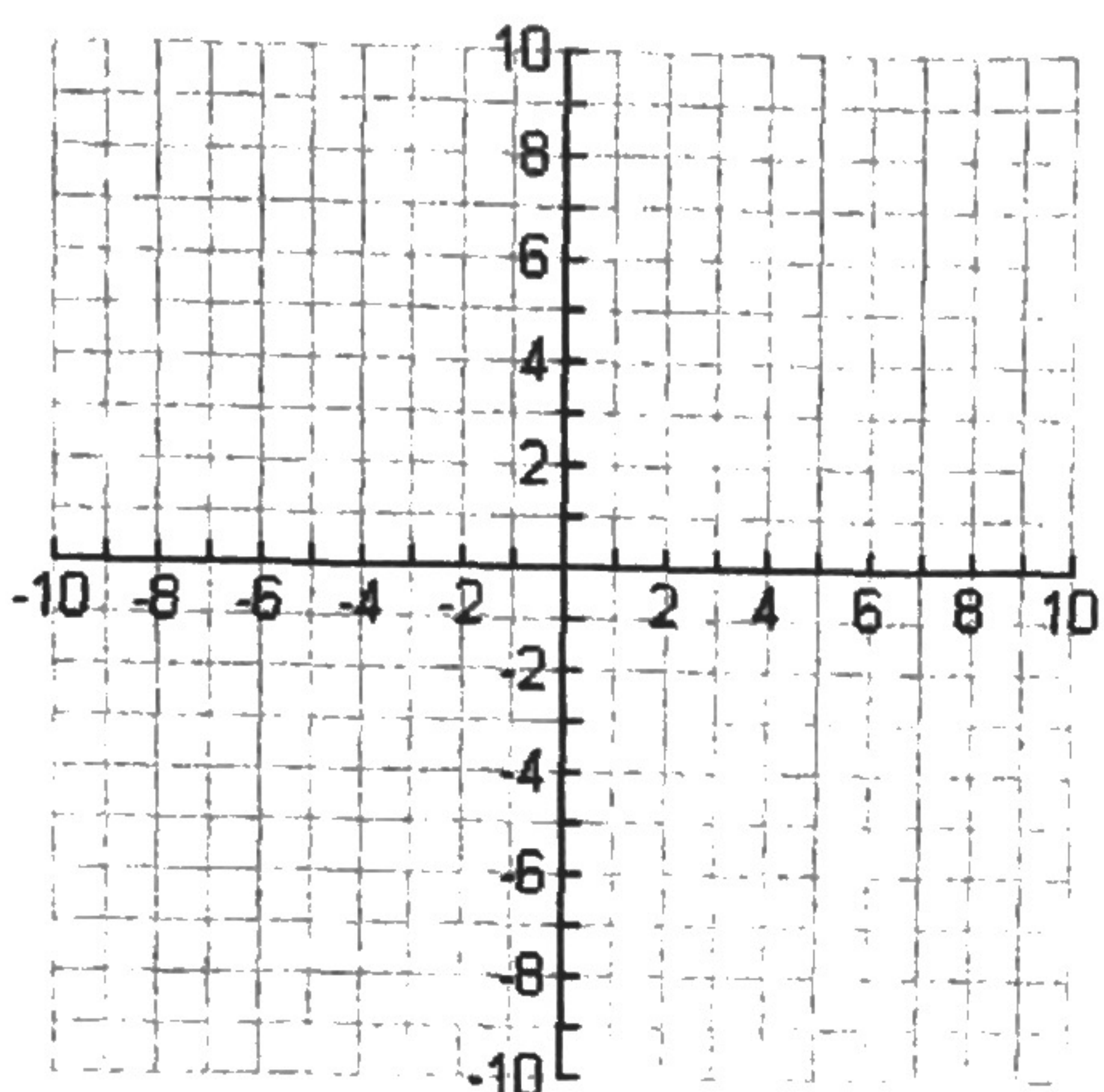


Day 4

Triangles on the Coordinate Plane Examples

Ex. 1 - Classify by angles and sides.

Together: D(1, 0) E(-3, -2) W(-1, 4)



$$DE = \sqrt{2^2 + 4^2} = 2\sqrt{5} = \sqrt{20}$$

$$EW = \sqrt{6^2 + 2^2} = 2\sqrt{10} = \sqrt{40}$$

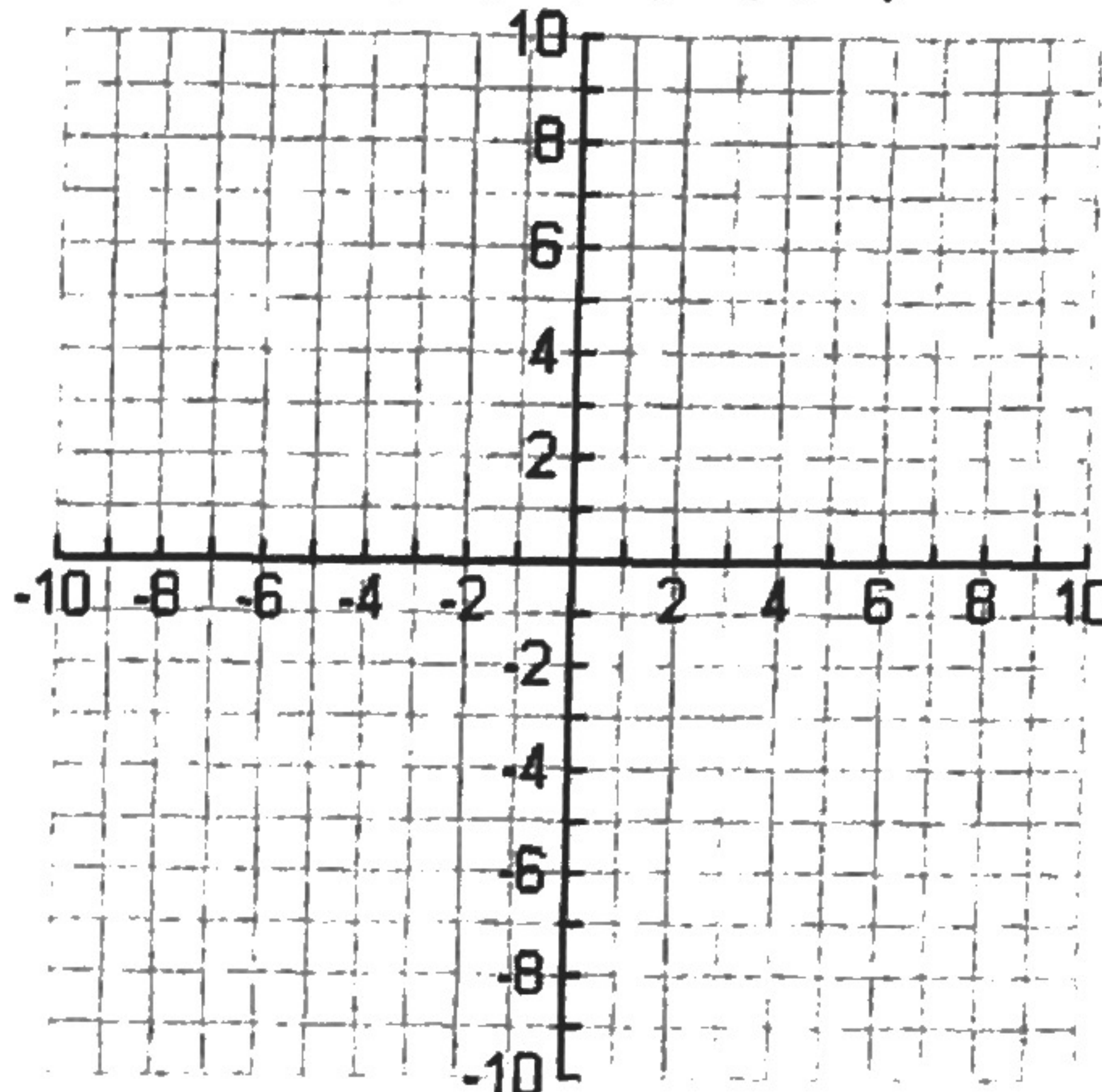
$$WD = \sqrt{4^2 + 2^2} = 2\sqrt{5} = \sqrt{20}$$

Right, isosceles

$$\sqrt{20^2 + 20^2} = \sqrt{40^2}$$

$$20 + 20 = 40$$

You try: F(-2, 1) O(-1, 5) G(2, 5)



$$FO = \sqrt{4^2 + 1^2} = \sqrt{17}$$

$$OG = \sqrt{0^2 + 5^2} = 5$$

$$GF = \sqrt{4^2 + 4^2} = 4\sqrt{2} = \sqrt{32}$$

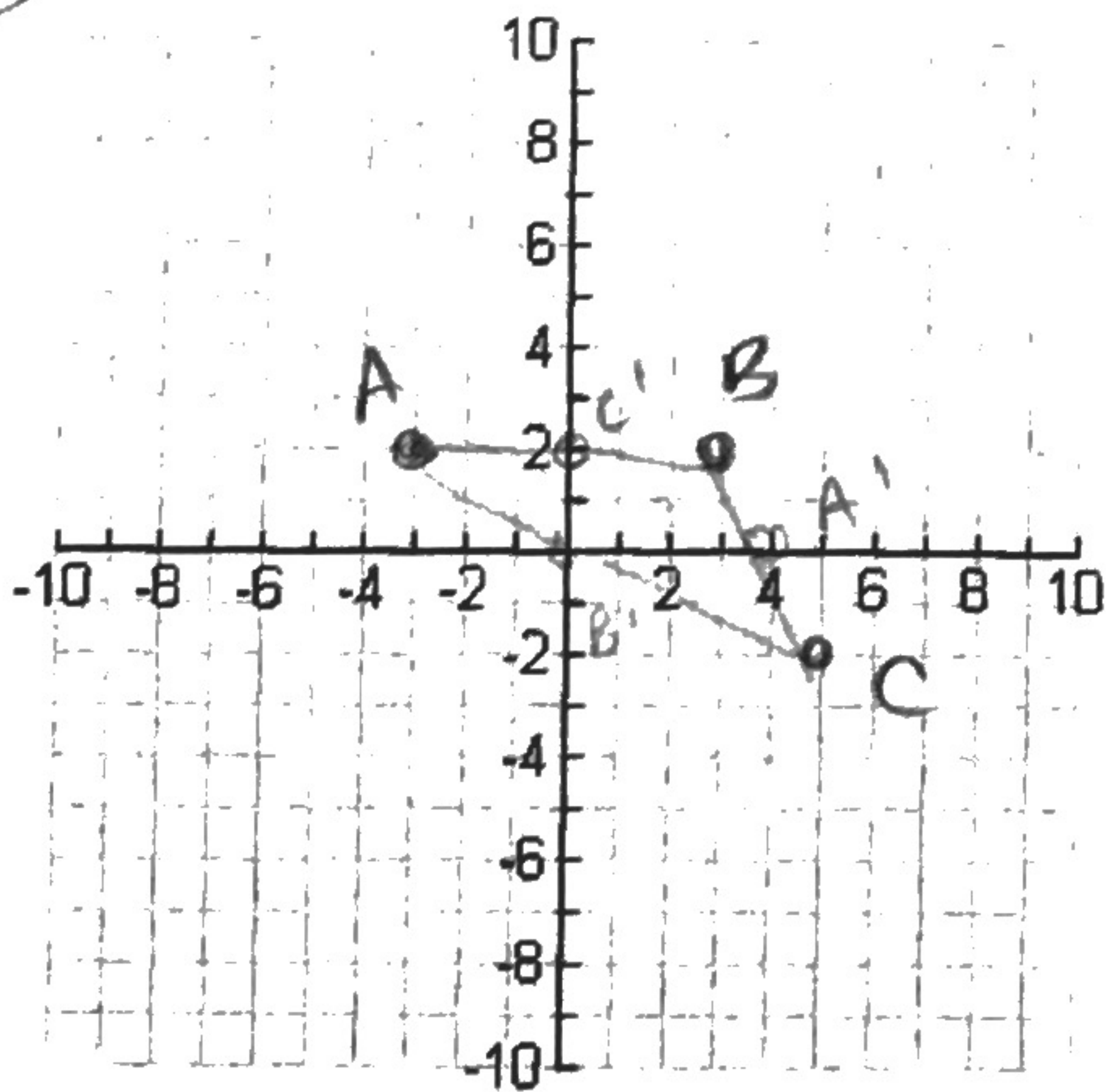
$$17^2 + 3^2 < 32^2$$

Scalene, obtuse

Ex 2 - Midsegment → needs 2 midpoints, connect them ↓

Together: A(-3, 2) B(3, 2) C(5, -2)

You try: Find the other 2 midsegments.



$$A'C' \quad y = -\frac{1}{2}(x-4) \text{ or } y = -\frac{1}{2}x + 2$$

$$B'C' \quad y = -2(x-1) \text{ or } y = -2x + 2$$

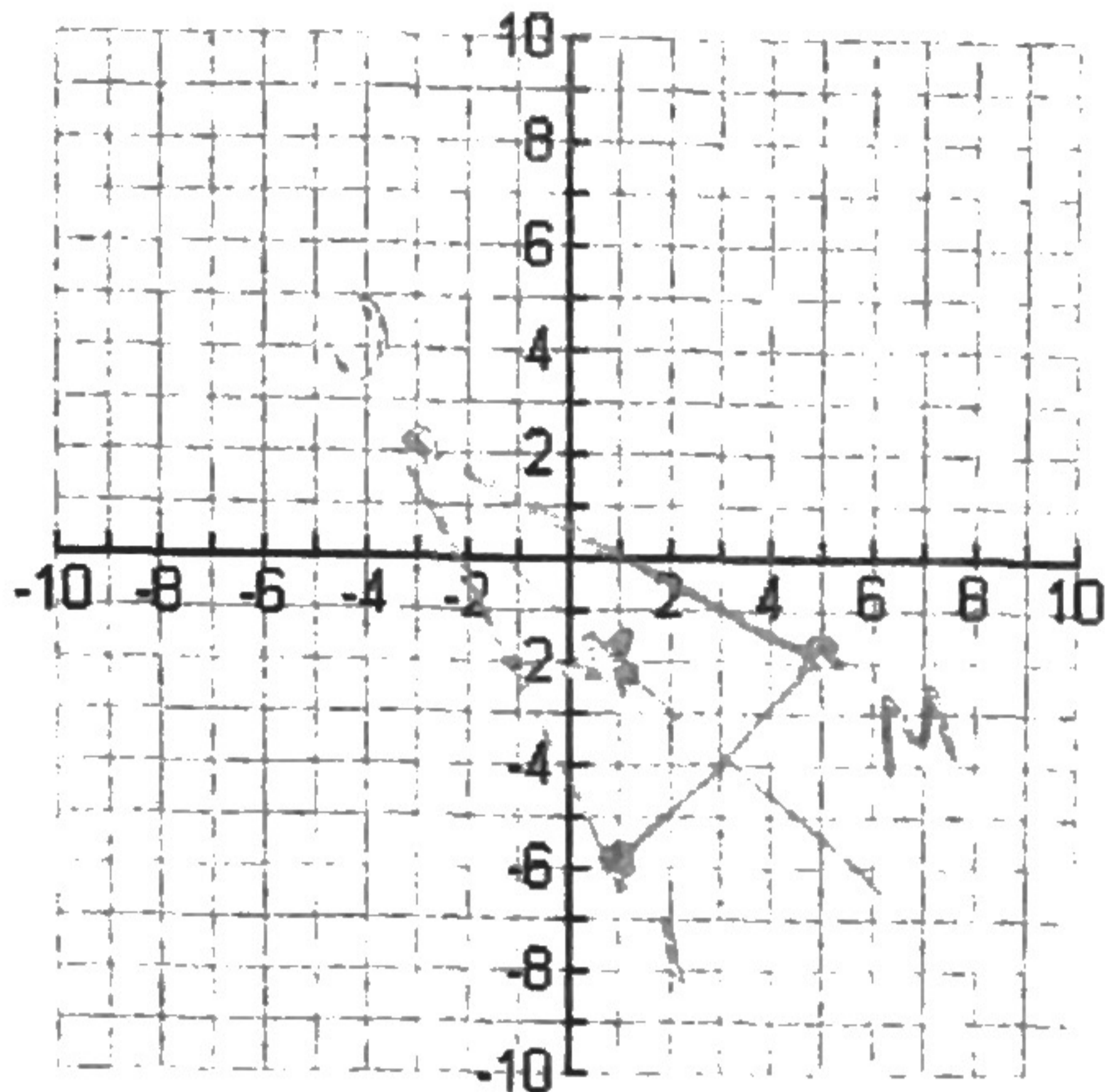
$$A'B' \quad y = 0$$

- AB midpoint C'(0, 2)
- BC midpoint A'(4, 0)
- CA midpoint B'(1, 0)

Ex. 3 - Medians

→ vertex to midpoint

Together:  $(-3, 2)$   $(1, -6)$   $(5, -2)$



You try: Find the other 2 medians.

$J(-3, 2)$   $J'(3, -4)$   $y = -(x+3)+2$   
or  $-(x-3)-4$

$M(5, -2)$   $M'(-1, -2)$   $y = -2$

$I(1, -6)$   $I'(1, 0)$   $x = 1$

or... shortcut...

$\left( \frac{-3+1+5}{3}, \frac{2-6-2}{3} \right)$

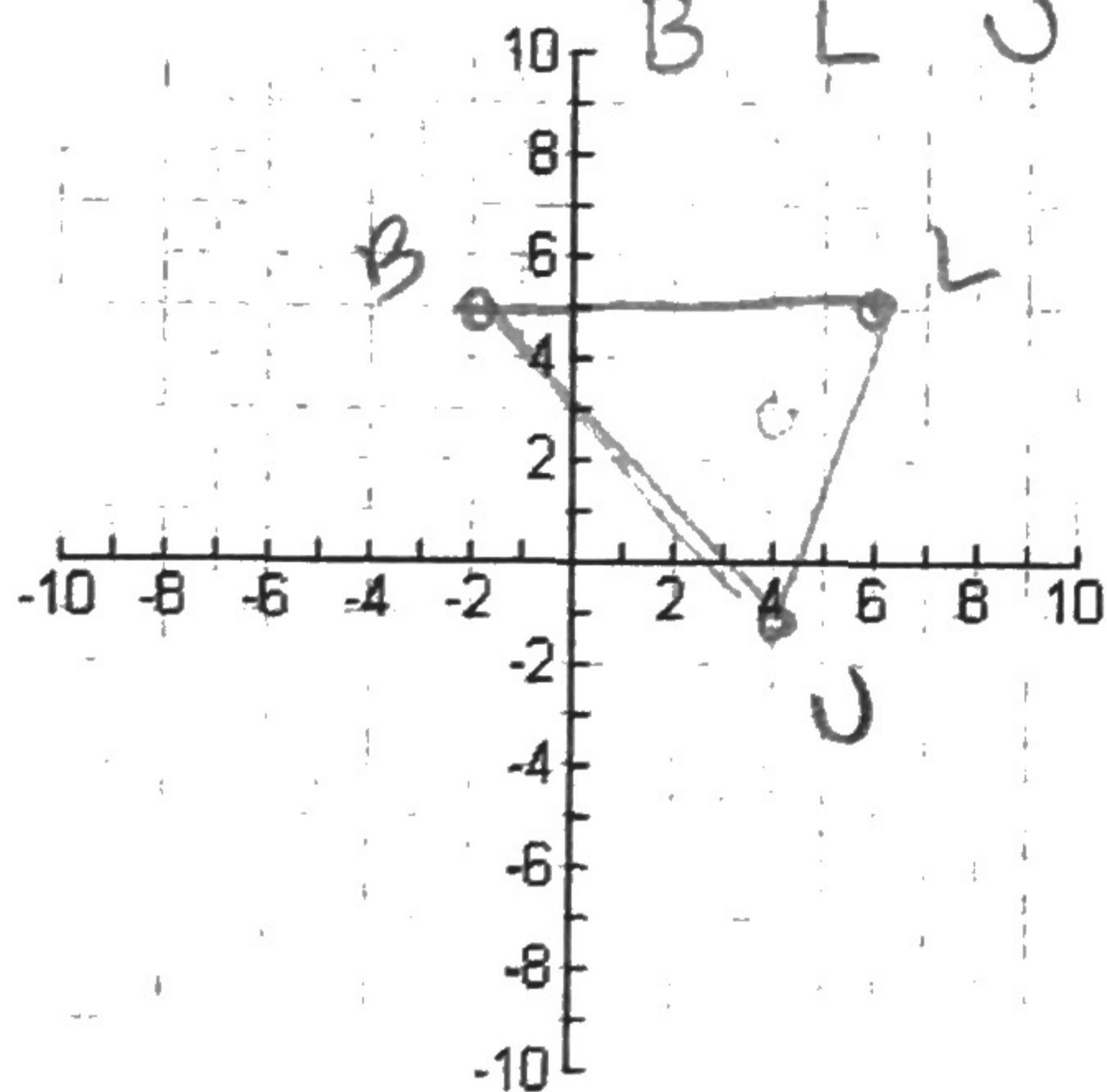
Where is the centroid?

$(1, -2)$

Ex 4 - Altitude

→ vertex and

Together:  $(-2, 5)$   $(6, 5)$   $(4, -1)$



You try: Find the other 2 altitudes.

$B(-2, 5)$   $LU = \frac{-6}{-2} = 3$   $y = -\frac{1}{3}(x+2)+5$

$L(6, 5)$   $BU = \frac{-6}{6} = -1$   $y = 1(x-6)+5$

$U(4, -1)$

$x = 4$

$y = 1(4-6)+5 = 3$

or you could do

$-\frac{1}{3}(x+2)+5 = (x-6)+5$

$-(x+2) = 3(x-6)$

$-x-2 = 3x-18$

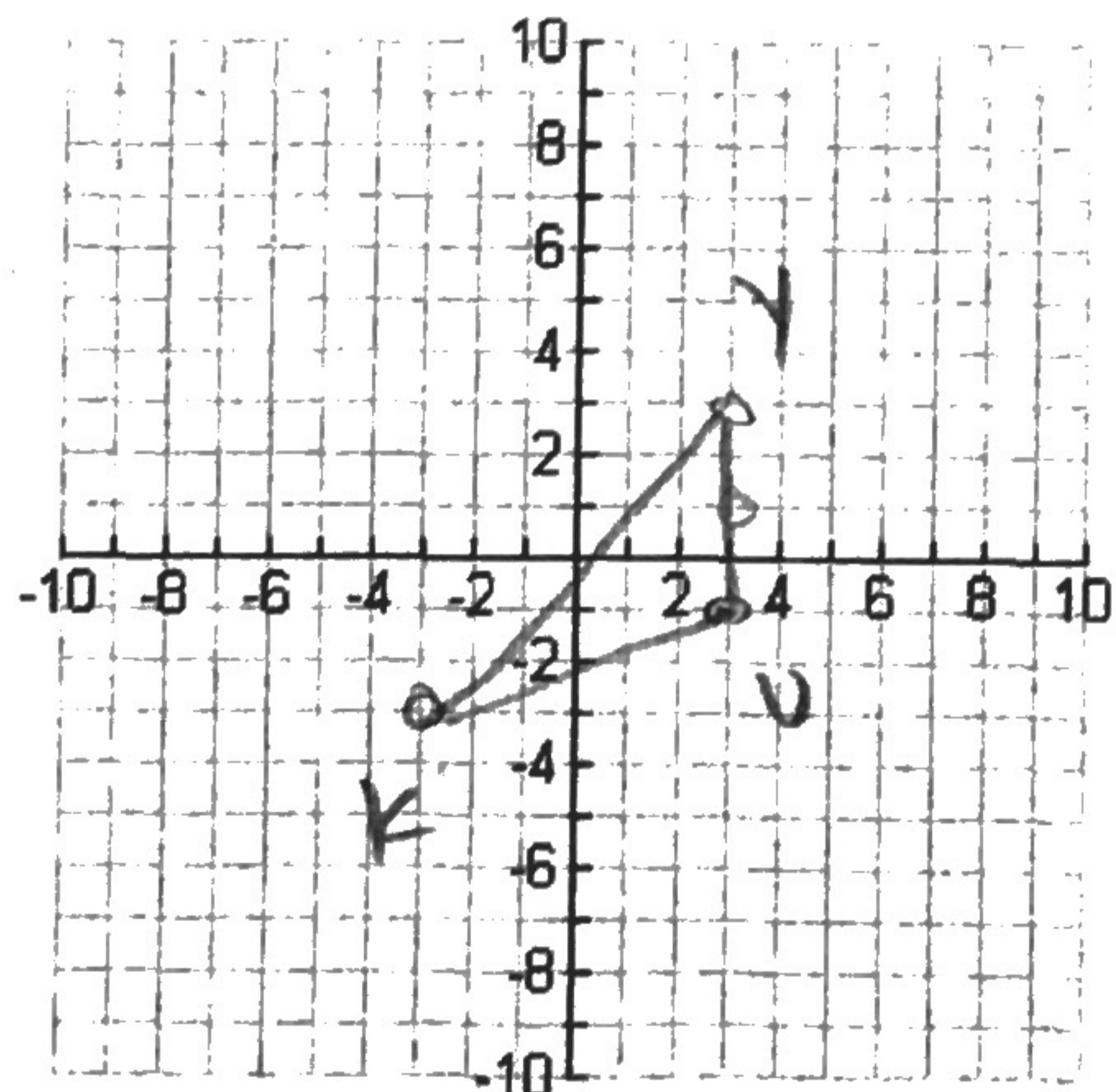
$16 = 4x$   $x = 4$

Where is the orthocenter?

$(4, 3)$

Ex. 5 - Perpendicular Bisectors

Together:  $(3, 3)$   $(3, -1)$   $(-3, -3)$



You try: Find the other 2 perpendicular bisectors.

$K'(3, 1)$   $y = 1$

$U'(0, 0)$   $KU = \frac{1}{1}$   $y = -x$

$K'(0, -2)$   $KU = \frac{-2}{-6} = \frac{1}{3}$   $y = -3(x)-2$

Where is the circumcenter?

$(1, -1)$

Name \_\_\_\_\_ Period \_\_\_\_\_

Special Segments on a Coordinate Plane

Classify the following triangles. Be sure to justify each classification.

1. A(1, 3) B(3, -1) C(5, 3)

$$AB = \sqrt{4^2 + 2^2}$$

$$\sqrt{20} = 2\sqrt{5}$$

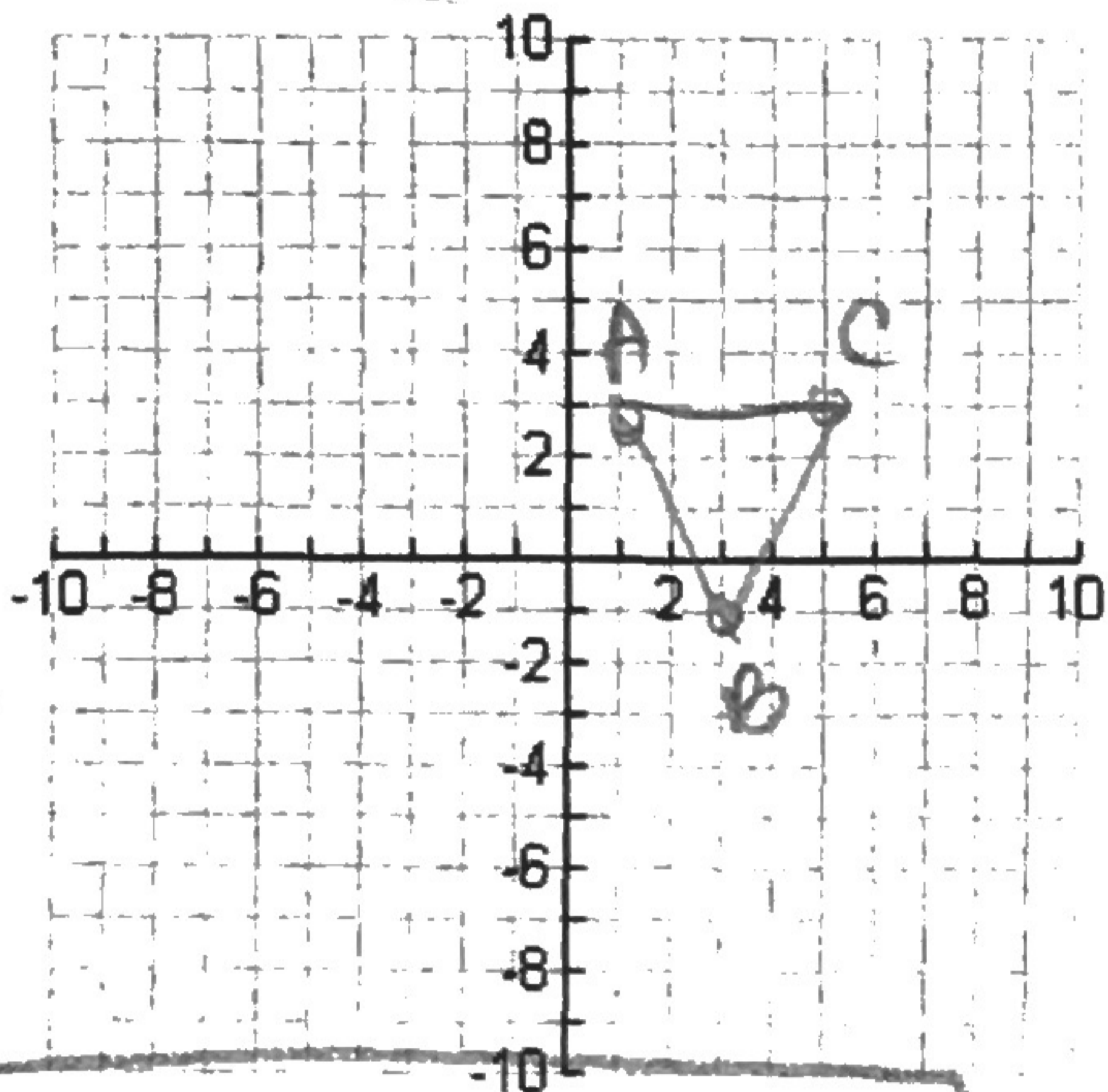
$$BC = \sqrt{4^2 + 2^2}$$

$$\sqrt{20} = 2\sqrt{5}$$

$$AC = \sqrt{16} = 4$$

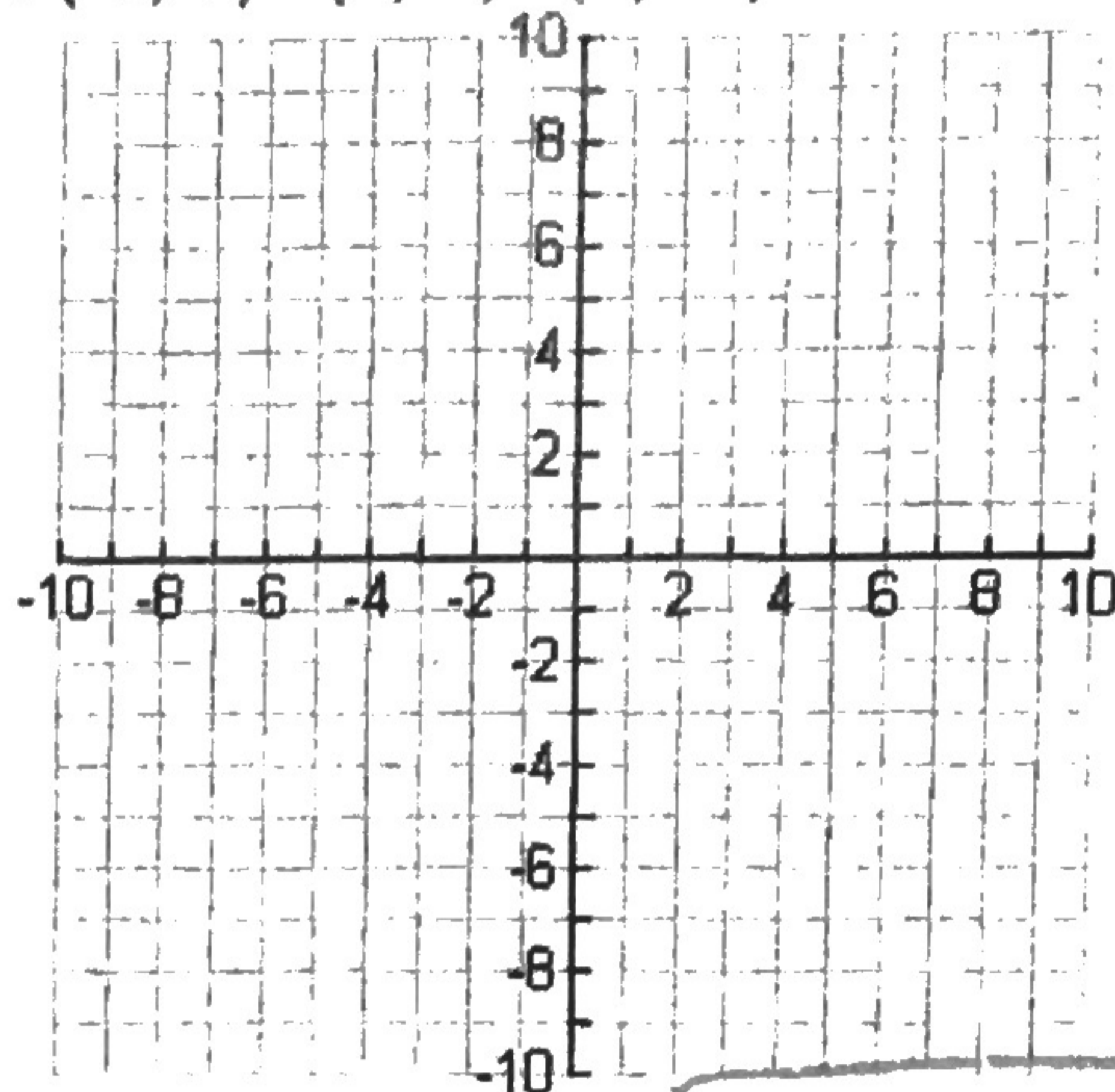
$$\sqrt{16} + \sqrt{20} > \sqrt{20}$$

$$36 > 20$$



**Isosceles, Acute**

2. D(-2, 3) E(4, 5) F(0, -3)



$$DE = \sqrt{2^2 + 6^2}$$

$$= 2\sqrt{10}$$

$$EF = \sqrt{8^2 + 4^2}$$

$$= 4\sqrt{5}$$

$$FD = \sqrt{6^2 + 2^2}$$

$$= 2\sqrt{10}$$

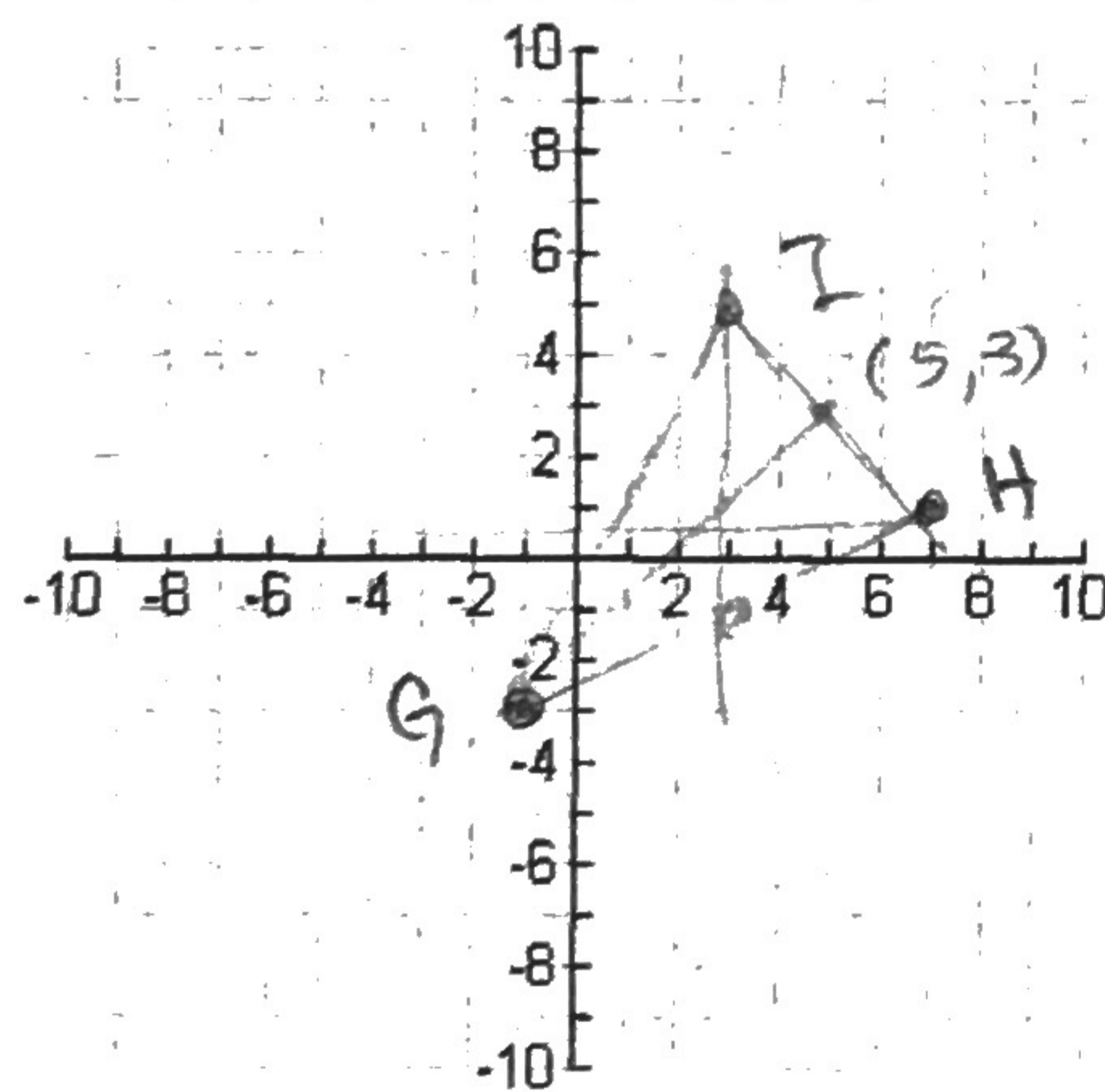
$$\sqrt{10} + \sqrt{10} = \sqrt{20}$$

**Isosc. Right**

→ Vertex + Midpoint

Using the points given, draw in each median. State the location of the centroid of each triangle.

3. G(-1, -3) H(7, 1) J(3, 5)



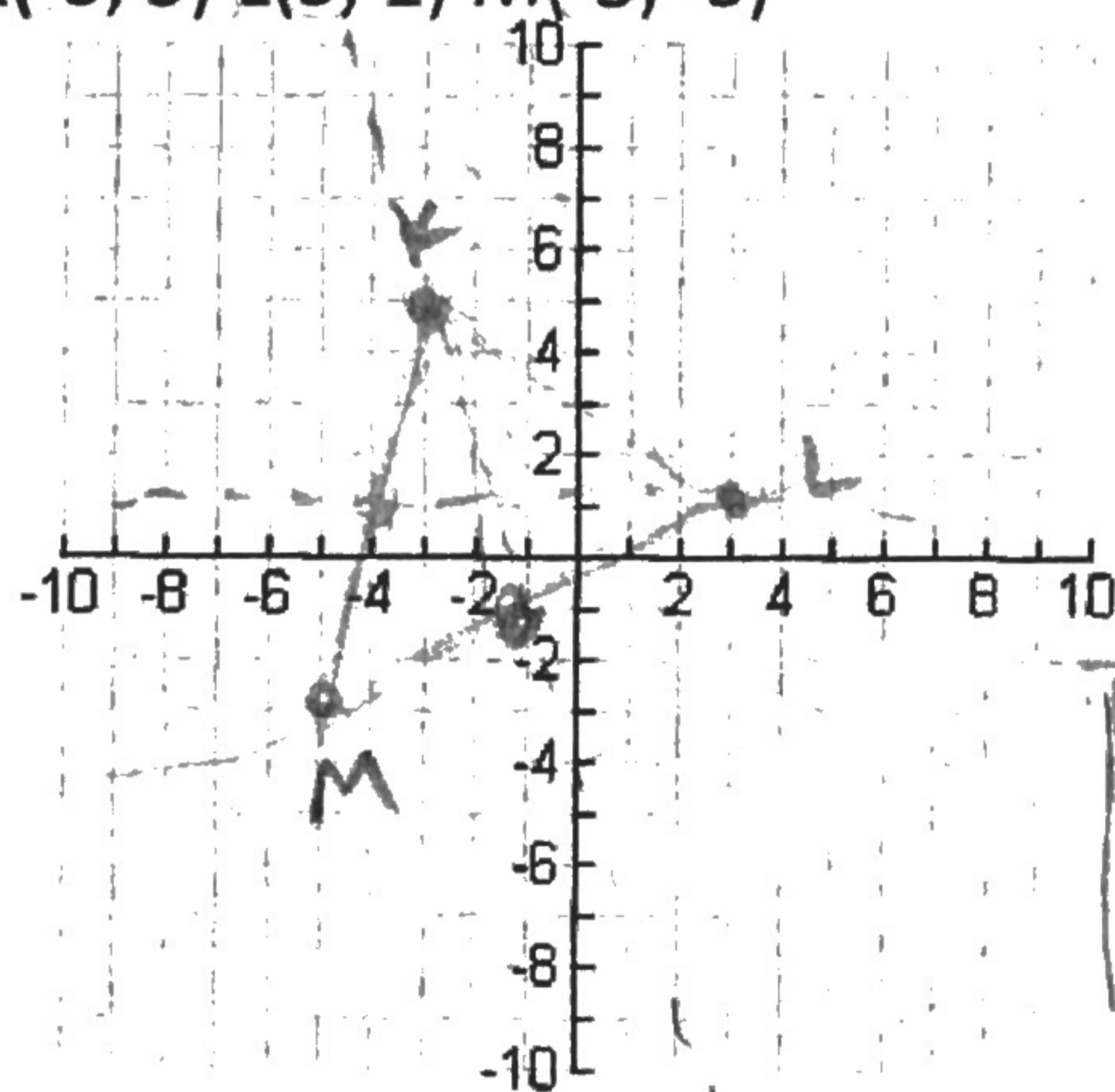
$$x = 3$$

$$y = 1$$

$$y = 1(x+1) - 3$$

**(3, 1)**

4. K(-3, 5) L(3, 1) M(-5, -3)



$$y = 1$$

$$y = -3(x+1) - 1$$

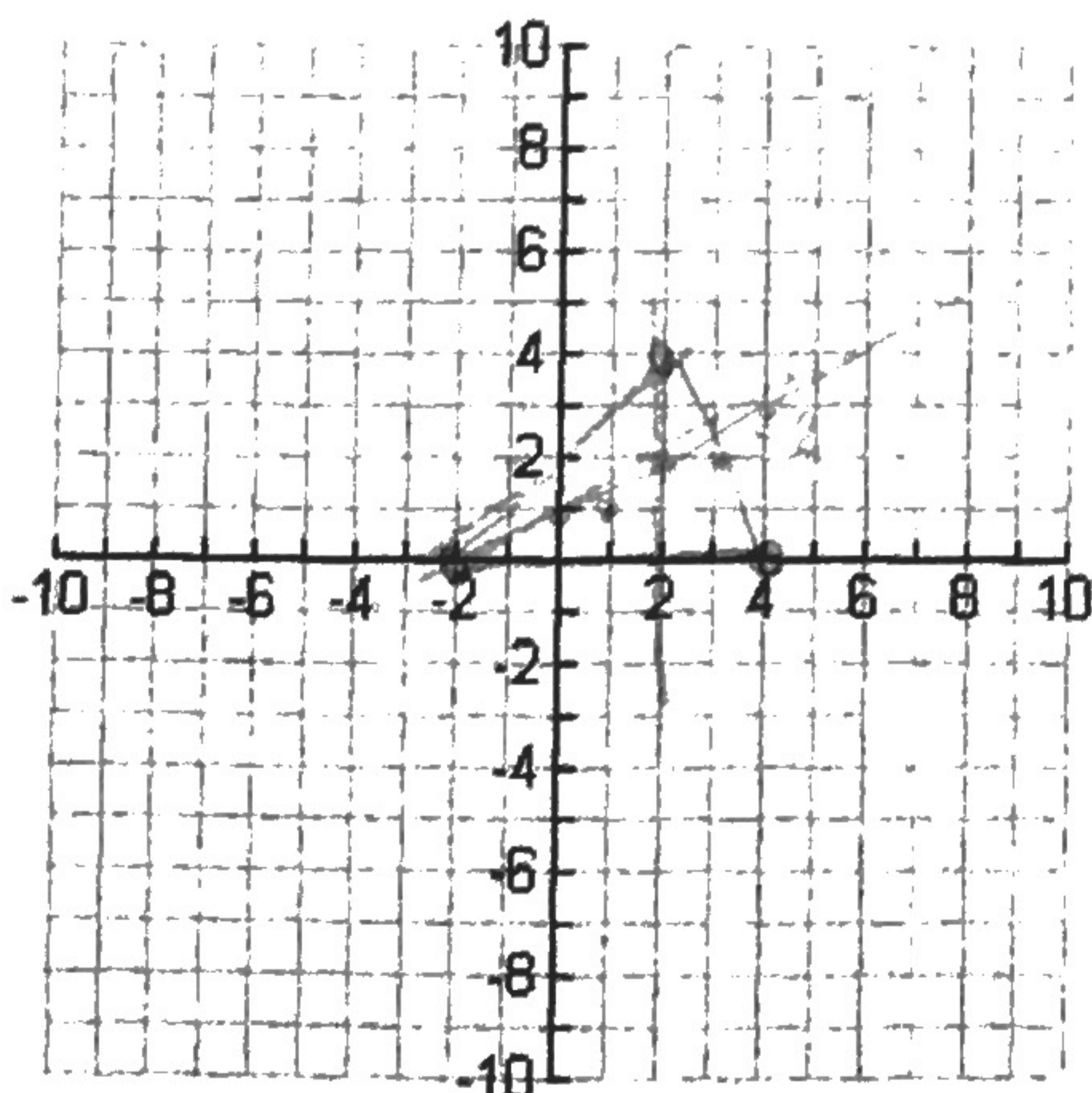
$$y = \frac{6}{5}x + 3$$

**(-5/3, 1)**

→ Vertex, ⊥

Using the points given, draw in each altitude. State the location of the orthocenter of each triangle.

5. (-2, 0) (4, 0) (2, 4)



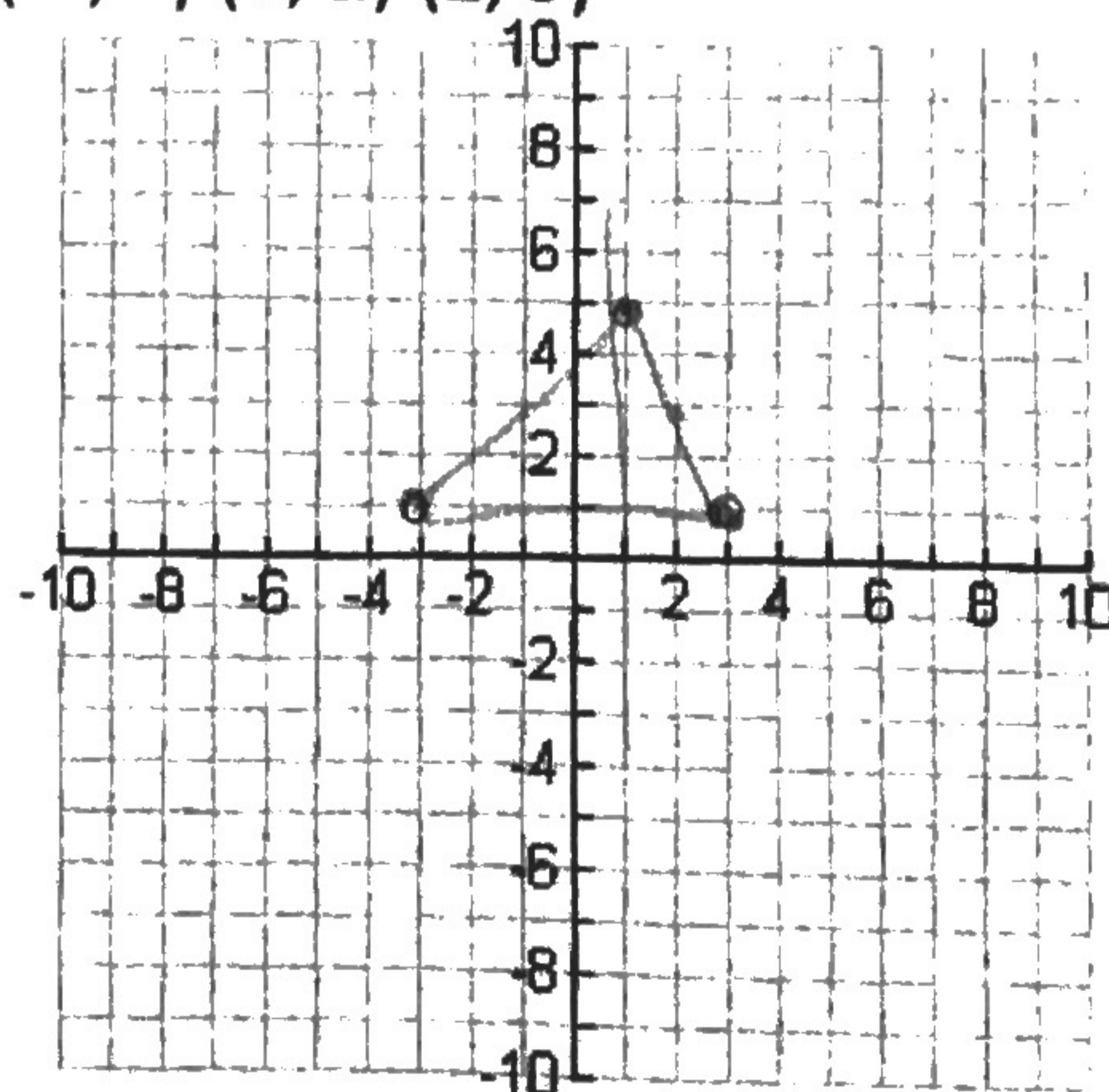
$$x = 2$$

$$y = -(x-4)$$

$$y = \frac{1}{2}(x+2)$$

$y = \frac{1}{2}(x+2)$   
 $x = 2$   
 $y = -(x-4)$   
**(2, 2)**

6. (-3, 1) (3, 1) (1, 5)



$$x = 1$$

$$y = -(x-3) + 1$$

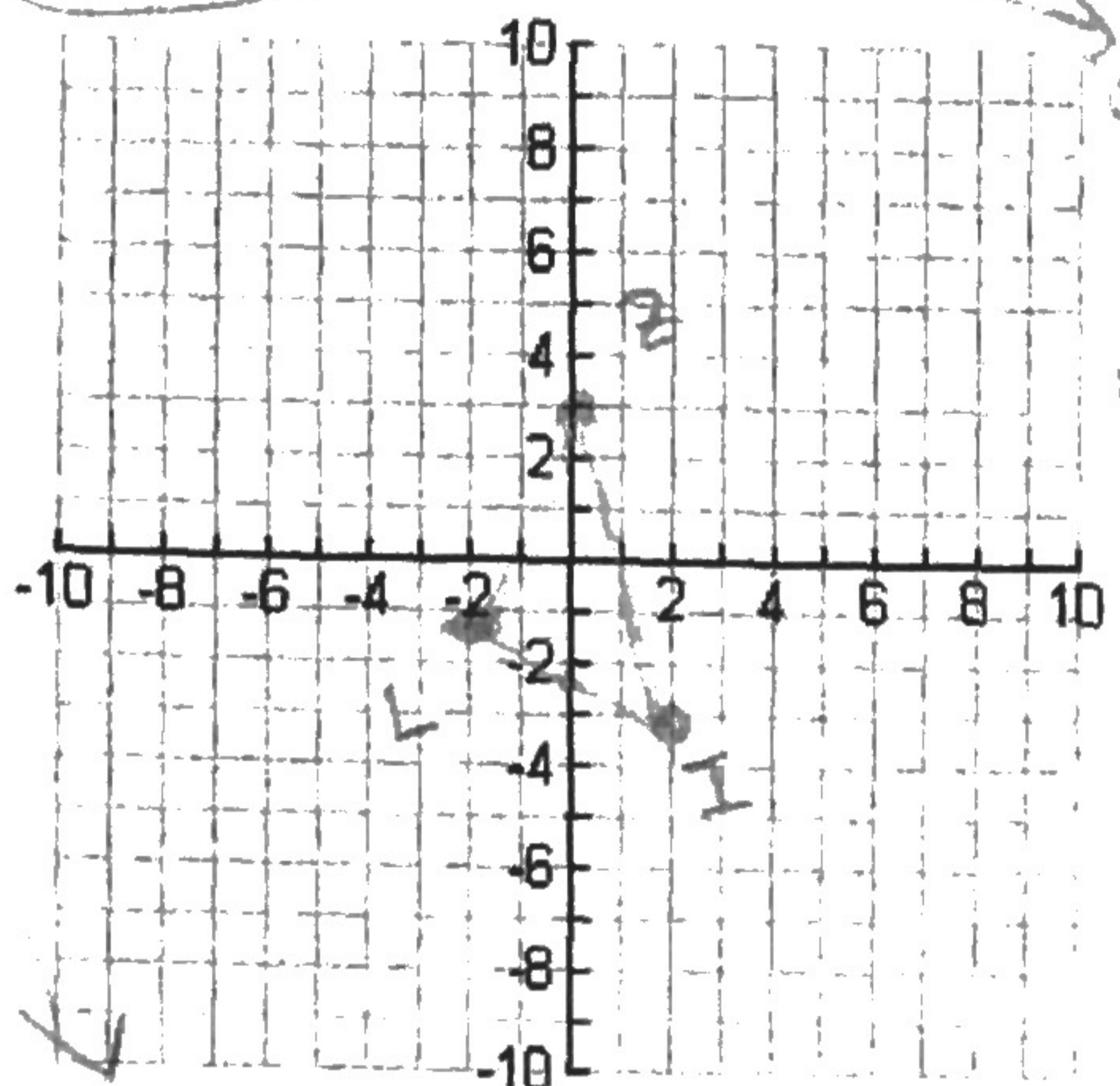
$$y = \frac{1}{2}(x+3) + 1$$

**(1, 3)**

→ midpoint, ↓

Using the points given, draw in each perpendicular bisector. State the location of the circumcenter for each triangle.

7. (-2, -1) (2, -3) (0, 3)



Since it's a right  $\Delta$ , we know the circumcenter will be on the midpoint of the hypotenuse #shortcuts!

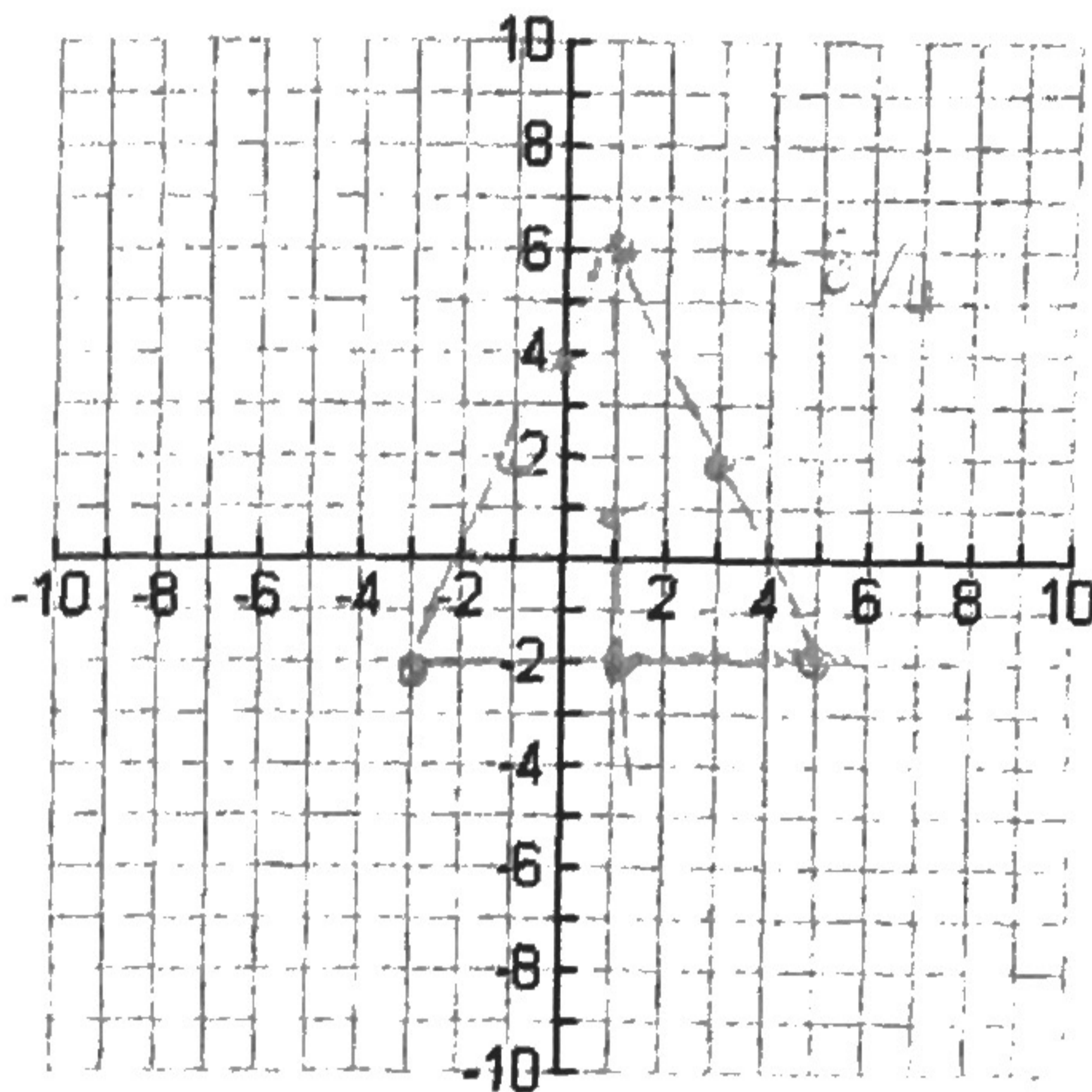
$$y = 2x - 2$$

$$y = \frac{1}{3}(x - 1)$$

$$y = -\frac{1}{2}(x + 1) + 1$$

(1, 0)

8. (-3, -2) (1, 6) (5, -2)



$$x = 1$$

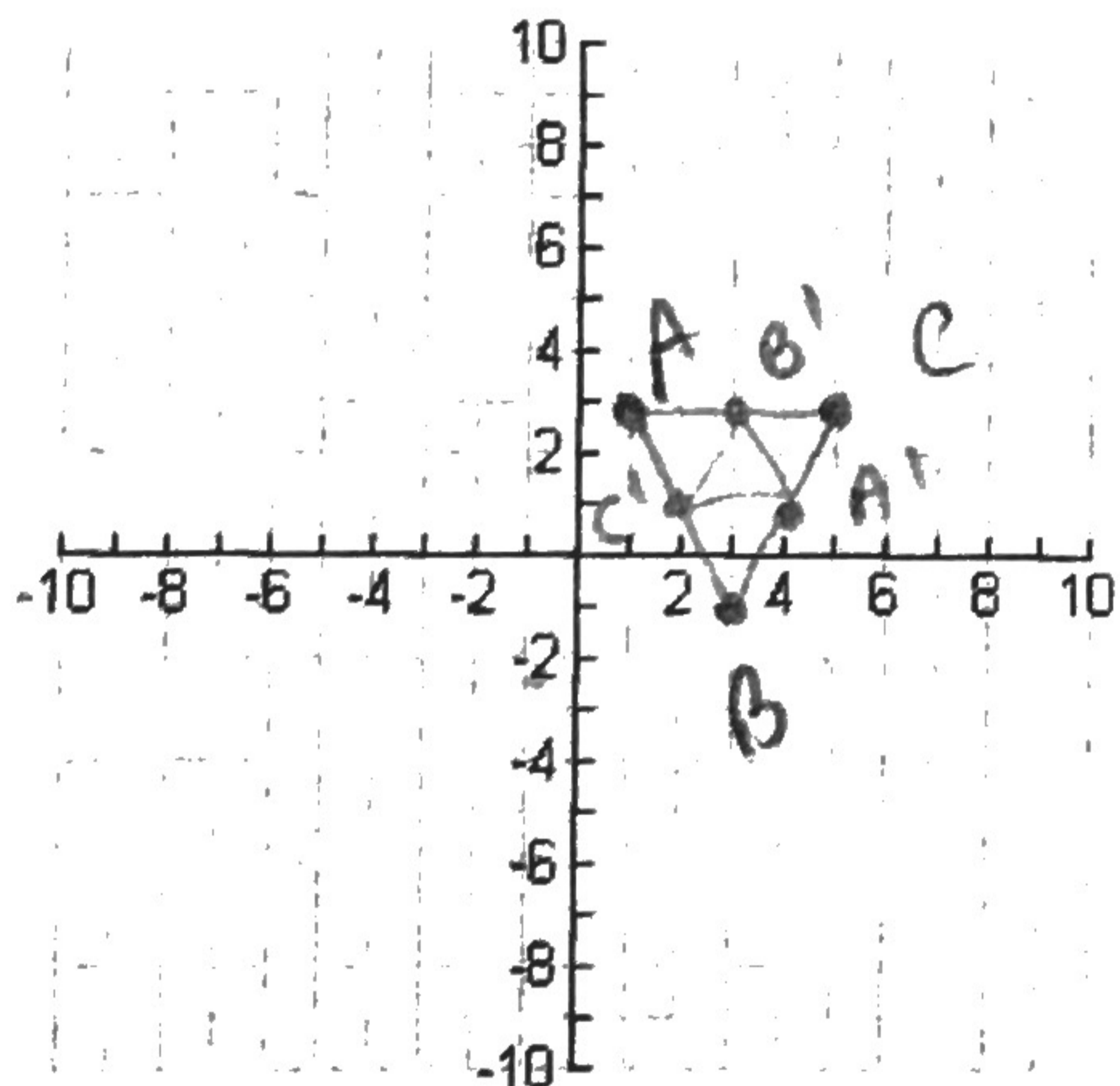
$$y = \frac{1}{2}(x - 3) + 2$$

$$y = -\frac{1}{2}(x + 1) + 2$$

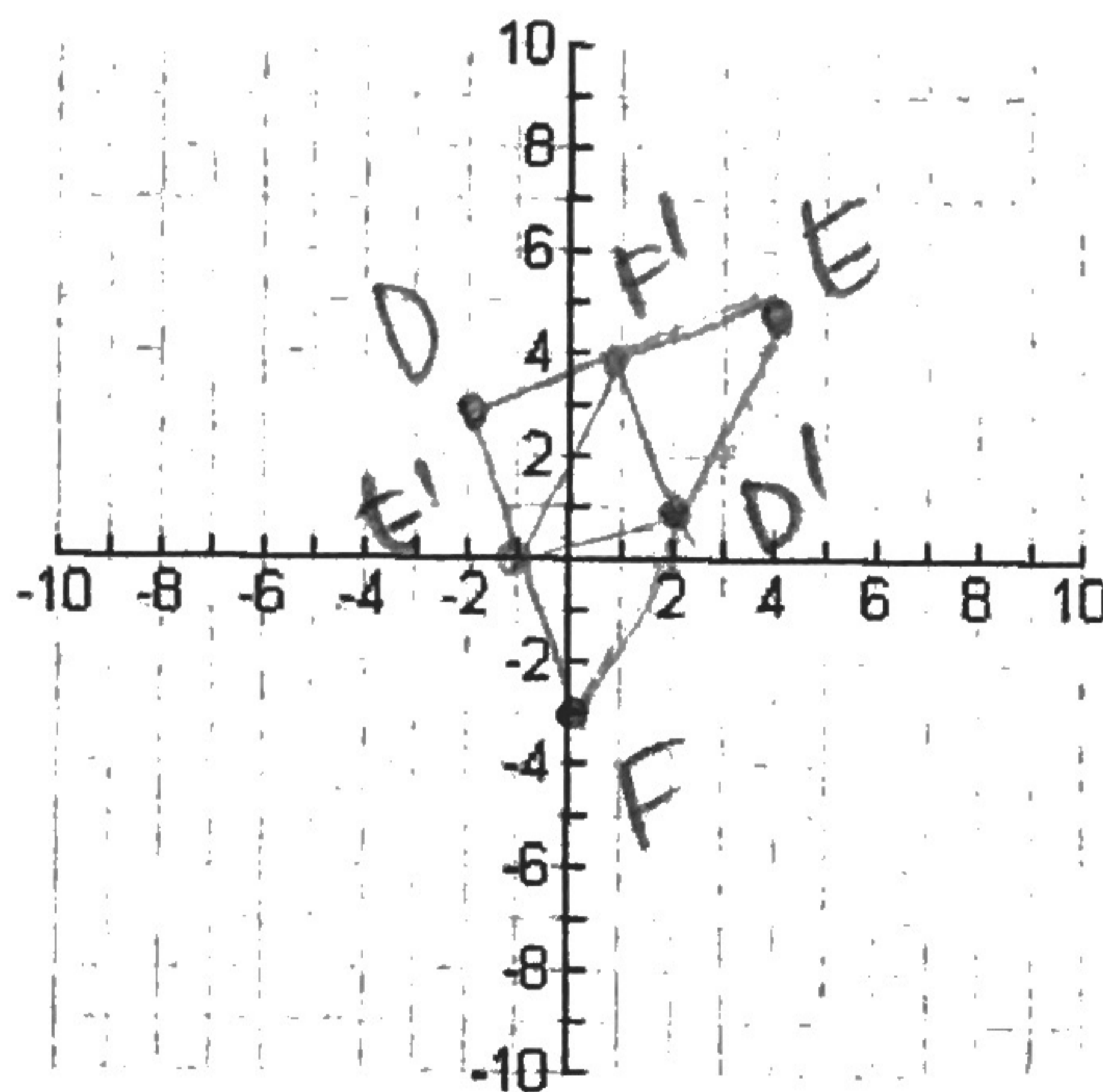
(1, 1)

Using the points given, draw each midsegment. Then show that the midsegments are parallel and  $\frac{1}{2}$  the length of the sides.

9. A(1, 3) B(3, -1) C(5, 3)



10. D(-2, 3) E(4, 5) F(0, -3)



$$AC = 4 \text{ slope} = 0$$

$$A'C' = 2, \text{ slope} = 0$$

$$BC = \sqrt{4^2 + 2^2} \text{ slope} = \frac{4}{2} = 2$$

$$= 2\sqrt{5}$$

$$B'C' = \sqrt{2^2 + 1^2} \text{ slope} = 2$$

$$= \sqrt{5}$$

$$AB = \sqrt{4^2 + 2^2} \text{ slope} = -2$$

$$= 2\sqrt{5}$$

$$A'B' = \sqrt{2^2 + 1^2} \text{ slope} = -2$$

$$= \sqrt{5}$$

$$DE = \sqrt{2^2 + 6^2} \text{ slope} = \frac{2}{6} = \frac{1}{3}$$

$$= 2\sqrt{10}$$

$$D'E' = \sqrt{1^2 + 3^2} \text{ slope} = \frac{1}{3}$$

$$= \sqrt{10}$$

$$FE = \sqrt{8^2 + 4^2} \text{ slope} = \frac{8}{4} = 2$$

$$= 4\sqrt{5}$$

$$F'E' = \sqrt{4^2 + 2^2} \text{ slope} = \frac{4}{2} = 2$$

$$= 2\sqrt{5}$$

$$DF = \sqrt{6^2 + 2^2} \text{ slope} = \frac{-6}{2} = -3$$

$$= 2\sqrt{10}$$

$$D'F' = \sqrt{3^2 + 1^2} \text{ slope} = -3$$

$$= \sqrt{10}$$