40. **Arc Length** A point moves at 65 meters per second on the circumference of a circle. How far does the point travel in 1 minute?
41. **Arc Length** A point is moving with an angular velocity of 3 radians per second on a circle of radius 6 meters. How far does the point travel in 10 seconds?
42. **Angular Velocity** Convert 30 revolutions per minute (rpm) to angular velocity in radians per second.
43. **Angular Velocity** Convert 120 revolutions per minute to angular velocity in radians per second.
44. **Linear Velocity** A point is rotating at 5 revolutions per minute on a circle of radius 6 inches. What is the linear velocity of the point?
45. **Arc Length** How far does the tip of a 10-centimeter minute hand on a clock travel in 2 hours?
46. **Arc Length** How far does the tip of an 8-centimeter hour hand on a clock travel in 1 day?

SECTION 4.6 | INVERSE TRIGONOMETRIC FUNCTIONS

In Chapter 2 we encountered situations in which we needed to find an angle given a value of one of the trigonometric functions. This is the reverse of what a trigonometric function is designed to do. When we try to use a function in the reverse direction, we are really using what is called the *inverse* of the function. We begin this section with a brief review of the inverse of a function. If this is a new topic for you, a more thorough presentation of functions and inverse functions is provided in Appendix A.

First, let us review the definition of a function and its inverse.

DEFINITION

A *function* is a rule or correspondence that pairs each element of the domain with exactly one element from the range. That is, a function is a set of ordered pairs in which no two different ordered pairs have the same first coordinate.

The *inverse* of a function is found by interchanging the coordinates in each ordered pair that is an element of the function.

With the inverse, the domain and range of the function have switched roles. To find the equation of the inverse of a function, we simply exchange x and y in the equation and then solve for y. For example, to find the inverse of the function $y = x^2 - 4$, we would proceed like this:

The inverse of	$y = x^2 - 4$	
is	$x = y^2 - 4$	Exchange x and y
or	$y^2 - 4 = x$	
	$y^2 = x + 4$	Add 4 to both sides
	$y = \pm\sqrt{x + 4}$	Take the square root of both sides

The inverse of the function $y = x^2 - 4$ is given by the equation $y = \pm\sqrt{x + 4}$.

The graph of $y = x^2 - 4$ is a parabola that crosses the x-axis at -2 and 2 and has its vertex at $(0, -4)$. To graph the inverse, we take each point on the graph of $y = x^2 - 4$, interchange the x- and y-coordinates, and then plot the resulting point. Figure 1 shows both graphs. Notice that the graph of the inverse is a reflection of the graph of the original function about the line $y = x$.

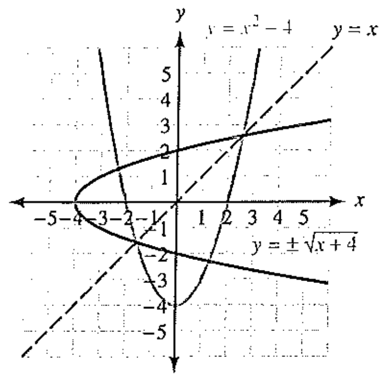


Figure 1

From Figure 1 we see that the inverse of $y = x^2 - 4$ is not a function because the graph of $y = \pm\sqrt{x + 4}$ does not pass the vertical line test. In order for the inverse to also be a function, the graph of the original function must pass the *horizontal* line test. Functions with this property are called *one-to-one* functions. If a function is one-to-one, then we know its inverse will be a function as well.

INVERSE FUNCTION NOTATION

If $y = f(x)$ is a one-to-one function, then the inverse of f is also a function and can be denoted by $y = f^{-1}(x)$.

Because the graphs of all six trigonometric functions do not pass the horizontal line test, the inverse relations for these functions will not be functions themselves. However, we will see that it is possible to define an inverse that is a function if we restrict the original trigonometric function to certain angles. In this section, we will limit our discussion of inverse trigonometric functions to the inverses of the three major functions: sine, cosine, and tangent. The other three inverse trigonometric functions can be handled with the use of reciprocal identities.

THE INVERSE SINE RELATION

To find the inverse of $y = \sin x$, we interchange x and y to obtain

$$x = \sin y$$

This is the equation of the inverse sine relation.

To graph $x = \sin y$, we simply reflect the graph of $y = \sin x$ about the line $y = x$, as shown in Figure 2.

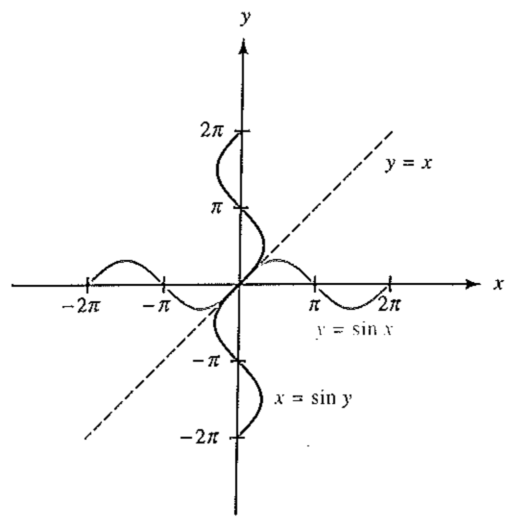


Figure 2

As you can see from the graph, $x = \sin y$ is a relation but not a function. For every value of x in the domain, there are many values of y. The graph of $x = \sin y$ fails the vertical line test.

THE INVERSE SINE FUNCTION

In order that the function $y = \sin x$ have an inverse that is also a function, it is necessary to restrict the values that x can assume so that we may satisfy the horizontal line test. The interval we restrict it to is $-\pi/2 \leq x \leq \pi/2$. Figure 3 displays the graph of $y = \sin x$ with the restricted interval showing. Notice that this segment of the sine graph passes the horizontal line test, and it maintains the full range of the function $-1 \leq y \leq 1$. Figure 4 shows the graph of the inverse relation $x = \sin y$ with the restricted interval after the sine curve has been reflected about the line $y = x$.

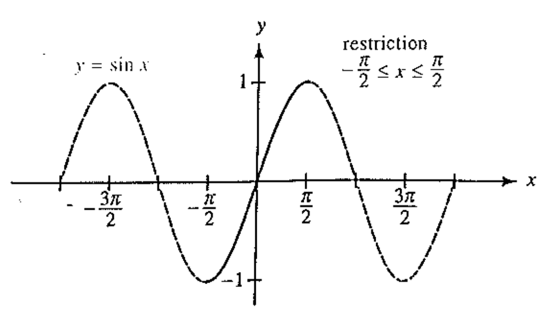


Figure 3

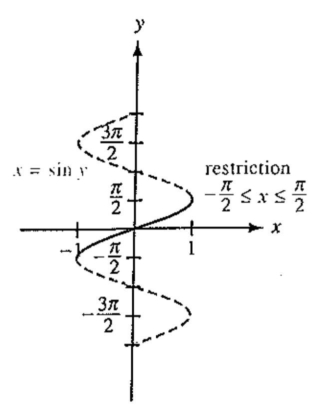
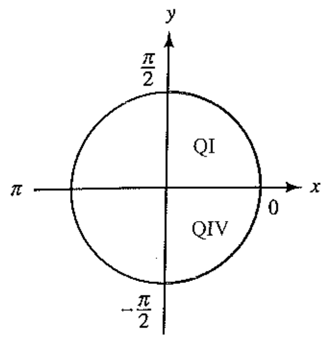


Figure 4

It is apparent from Figure 4 that if $x = \sin y$ is restricted to the interval $-\pi/2 \leq y \leq \pi/2$, then each value of x between -1 and 1 is associated with exactly one value of y, and we have a function rather than just a relation. The equation $x = \sin y$, together with the restriction $-\pi/2 \le y \le \pi/2$, forms the inverse sine function. To designate this function, we use the following notation.



The inverse sine function will return an angle between $-\pi/2$ and $\pi/2$, inclusive, corresponding to QIV or QI.

NOTATION

The notation used to indicate the inverse sine function is as follows:

Notation	Meaning
$y = \sin^{-1} x$ or $y = \arcsin x$	$x = \sin y$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

In words: y is the angle between $-\pi/2$ and $\pi/2$, inclusive, whose sine is x.

NOTE The notation $\sin^{-1} x$ is not to be interpreted as meaning the reciprocal of $\sin x$. That is,

$$\sin^{-1} x \neq \frac{1}{\sin x}$$

If we want the reciprocal of $\sin x$, we use $\csc x$ or $(\sin x)^{-1}$, but never $\sin^{-1} x$.

THE INVERSE COSINE FUNCTION

Just as we did for the sine function, we must restrict the values that x can assume with the cosine function in order to satisfy the horizontal line test. The interval we restrict it to is $0 \le x \le \pi$. Figure 5 shows the graph of $y = \cos x$ with the restricted interval. Figure 6 shows the graph of the inverse relation $x = \cos y$ with the restricted interval after the cosine curve has been reflected about the line $y = x$.

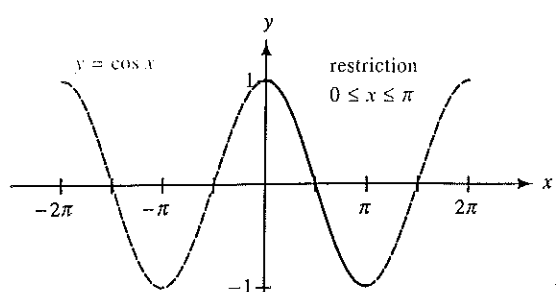


Figure 5

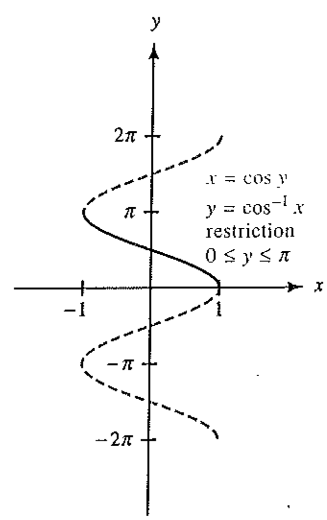
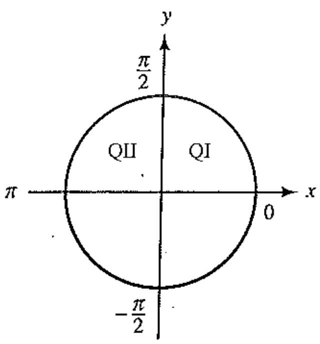


Figure 6

The equation $x = \cos y$, together with the restriction $0 \le y \le \pi$, forms the inverse cosine function. To designate this function we use the following notation.



NOTATION

The notation used to indicate the inverse cosine function is as follows:

Notation	Meaning
$y = \cos^{-1} x$ or $y = \arccos x$	$x = \cos y$ and $0 \le y \le \pi$
In words: y is the angle between 0 and π, inclusive, whose cosine is x.	

The inverse cosine function will return an angle between 0 and π, inclusive, corresponding to QI or QII.

THE INVERSE TANGENT FUNCTION

For the tangent function, we restrict the values that x can assume to the interval $-\pi/2 < x < \pi/2$. Figure 7 shows the graph of $y = \tan x$ with the restricted interval. Figure 8 shows the graph of the inverse relation $x = \tan y$ with the restricted interval after it has been reflected about the line $y = x$.

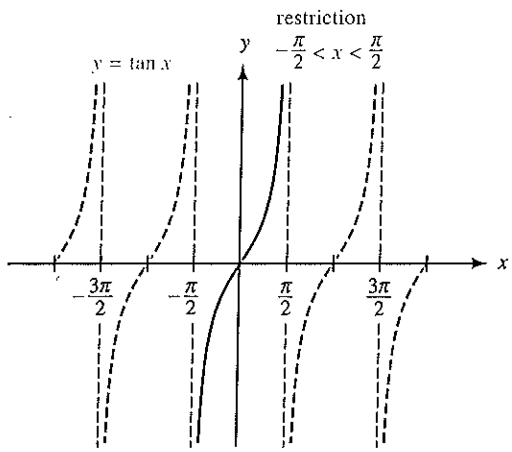


Figure 7

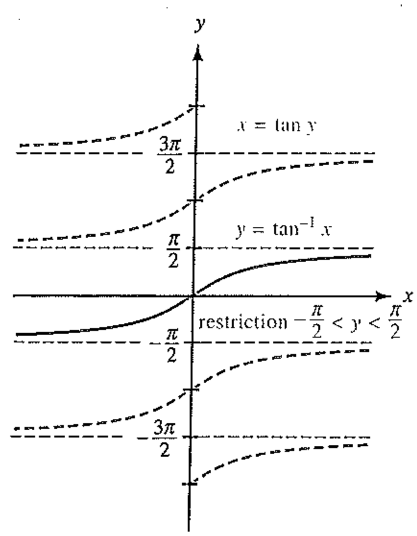
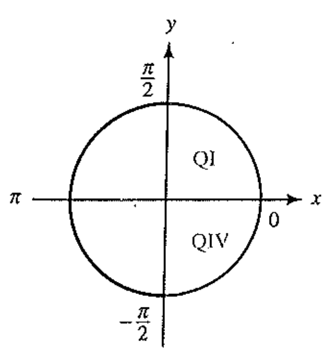


Figure 8

The equation $x = \tan y$, together with the restriction $-\pi/2 < y < \pi/2$, forms the inverse tangent function. To designate this function we use the following notation.



The inverse tangent function will return an angle between $-\pi/2$ and $\pi/2$, corresponding to QIV or QI.

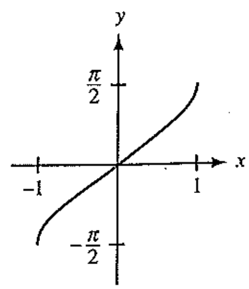
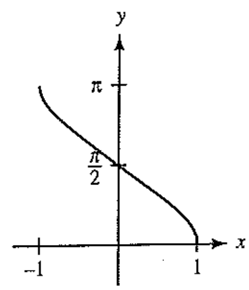
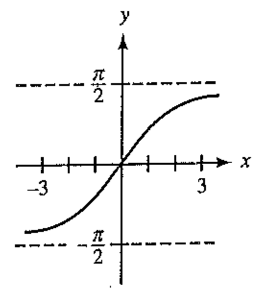
NOTATION

The notation used to indicate the inverse tangent function is as follows:

Notation	Meaning
$y = \tan^{-1} x$ or $y = \arctan x$	$x = \tan y$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$
In words: y is the angle between $-\pi/2$ and $\pi/2$ whose tangent is x.	

To summarize, here are the definitions of the three inverse trigonometric functions we have presented, along with the domain, range, and graph for each.

INVERSE TRIGONOMETRIC FUNCTIONS

Inverse sine	Inverse cosine	Inverse tangent
$y = \sin^{-1} x = \arcsin x$	$y = \cos^{-1} x = \arccos x$	$y = \tan^{-1} x = \arctan x$
		
Domain: $-1 \le x \le 1$	Domain: $-1 \le x \le 1$	Domain: all real numbers
Range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$	Range: $0 \le y \le \pi$	Range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$

EXAMPLE 1 Evaluate in radians without using a calculator or tables.

a. $\sin^{-1} \frac{1}{2}$ **b.** $\arccos\left(-\frac{\sqrt{3}}{2}\right)$ **c.** $\tan^{-1}(-1)$

SOLUTION

a. The angle between $-\pi/2$ and $\pi/2$ whose sine is $\frac{1}{2}$ is $\pi/6$.

$$\sin^{-1} \frac{1}{2} = \frac{\pi}{6}$$

b. The angle between 0 and π with a cosine of $-\sqrt{3}/2$ is $5\pi/6$.

$$\arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$$

c. The angle between $-\pi/2$ and $\pi/2$ the tangent of which is -1 is $-\pi/4$.

$$\tan^{-1}(-1) = -\frac{\pi}{4}$$

NOTE In part c of Example 1, it would be incorrect to give the answer as $7\pi/4$. It is true that $\tan 7\pi/4 = -1$, but $7\pi/4$ is not between $-\pi/2$ and $\pi/2$. There is a difference.

USING TECHNOLOGY

GRAPHING THE INVERSE SINE FUNCTION

A graphing calculator can be used to graph and evaluate an inverse trigonometric function. To graph the inverse sine function, set your calculator to degree mode and enter the function

$$Y_1 = \sin^{-1} x$$

Set your window variables so that

$$-1.5 \le x \le 1.5;\ -120 \le y \le 120, \text{ scale} = 45$$

The graph of the inverse sine function is shown in Figure 9.

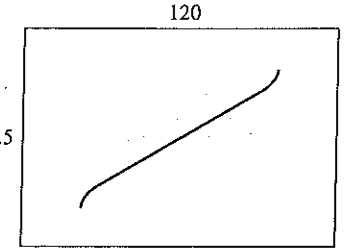


Figure 9

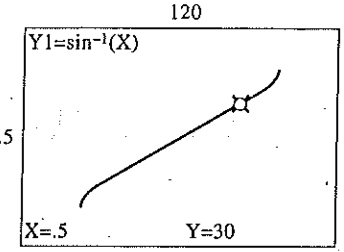


Figure 10

To find $\sin^{-1}\frac{1}{2}$ in Example 1a, use the appropriate command to evaluate the function for $x = 0.5$. As Figure 10 illustrates, the result is $y = 30°$. The angle between $-90°$ and $90°$ having a sine of 1/2 is 30°, or $\pi/6$ in radians.

EXAMPLE 2 Use a calculator to evaluate each expression to the nearest tenth of a degree.

a. arcsin (0.5075) **b.** arcsin (−0.5075)
c. $\cos^{-1}$ (0.6428) **d.** $\cos^{-1}$ (−0.6428)
e. arctan (4.474) **f.** arctan (−4.474)

SOLUTION The easiest method of evaluating these expressions is to use a calculator. Make sure the calculator is set to degree mode, and then enter the number and press the appropriate key. Scientific and graphing calculators are programmed so that the restrictions on the inverse trigonometric functions are automatic.

a. arcsin (0.5075) = 30.5°
b. arcsin (−0.5075) = −30.5° } Reference angle 30.5°
c. $\cos^{-1}$ (0.6428) = 50.0°
d. $\cos^{-1}$ (−0.6428) = 130.0° } Reference angle 50°
e. arctan (4.474) = 77.4°
f. arctan (−4.474) = −77.4° } Reference angle 77.4°

In Example 5 of Section 1.5, we simplified the expression $\sqrt{x^2+9}$ using the trigonometric substitution $x = 3 \tan \theta$ to eliminate the square root. We will now see how to simplify the result of that example further by removing the absolute value symbol.

EXAMPLE 3 Simplify $3 \left| \sec \theta \right|$ if $\theta = \tan^{-1} \dfrac{x}{3}$ for some real number x.

SOLUTION Since $\theta = \tan^{-1} \dfrac{x}{3}$, we know from the definition of the inverse tangent function that $-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$. For any angle θ within this interval, $\sec \theta$ will be a positive value. Therefore, $\left| \sec \theta \right| = \sec \theta$ and we can simplify the expression further as

$$3 \left| \sec \theta \right| = 3 \sec \theta$$

EXAMPLE 4 Evaluate each expression.

a. $\sin\left(\sin^{-1} \dfrac{1}{2}\right)$

b. $\sin^{-1}(\sin 135°)$

SOLUTION

a. From Example 1a we know that $\sin^{-1} \dfrac{1}{2} = \dfrac{\pi}{6}$. Therefore,

$$\sin\left(\sin^{-1} \frac{1}{2}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

b. Since $\sin 135° = \dfrac{1}{\sqrt{2}}$, $\sin^{-1}(\sin 135°) = \sin^{-1}\left(\dfrac{1}{\sqrt{2}}\right)$ will be the angle y, $-90° \le y \le 90°$, for which $\sin y = \dfrac{1}{\sqrt{2}}$. The angle satisfying this requirement is $y = 45°$. So,

$$\sin^{-1}(\sin 135°) = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$$

NOTE Notice in Example 4a that the result of 1/2 is the same as the value that appeared in the original expression. Because $y = \sin x$ and $y = \sin^{-1} x$ are inverse functions, the one function will "undo" the action performed by the other. (For more details on this property of inverse functions, see Appendix A.) The reason this same process did not occur in Example 4b is that the original angle 135° is not within the restricted domain of the sine function and is therefore outside the range of the inverse sine function. In a sense, the functions in Example 4b are not really inverses because we did not choose an input within the agreed-upon interval for the sine function.

EXAMPLE 5 Simplify $\tan^{-1}(\tan x)$ if $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

SOLUTION Because x is within the restricted domain for the tangent function, the two functions are inverses. Whatever value the tangent function assigns to x, the inverse tangent function will reverse this association and return the original value of x. So, by the property of inverse functions,

$$\tan^{-1}(\tan x) = x$$

EXAMPLE 6 Evaluate $\sin(\tan^{-1}\frac{3}{4})$ without using a calculator.

SOLUTION We begin by letting $\theta = \tan^{-1}(\frac{3}{4})$. (Remember, $\tan^{-1} x$ is the angle whose tangent is x.) Then we have

$$\text{If } \theta = \tan^{-1}\frac{3}{4}, \text{ then } \tan\theta = \frac{3}{4} \text{ and } 0^\circ < \theta < 90^\circ$$

We can draw a triangle in which one of the acute angles is θ (Figure 11). Since $\tan\theta = \frac{3}{4}$, we label the side opposite θ with 3 and the side adjacent to θ with 4. The hypotenuse is found by applying the Pythagorean Theorem.

From Figure 11 we find $\sin\theta$ using the ratio of the side opposite θ to the hypotenuse.

$$\sin\left(\tan^{-1}\frac{3}{4}\right) = \sin\theta = \frac{3}{5}$$

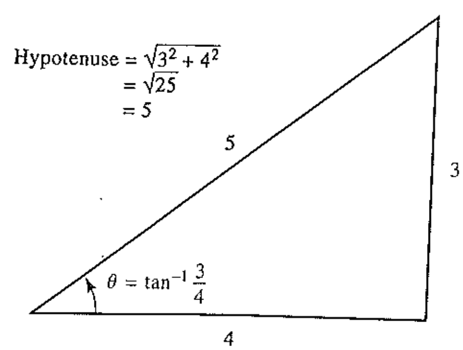


Figure 11

CALCULATOR NOTE If we were to do the same problem with the aid of a calculator, the sequence would look like this:

Scientific Calculator

3 [÷] 4 [=] [tan⁻¹] [sin]

Graphing Calculator

[sin] [(] [tan⁻¹] [(] 3 [÷] 4 [)] [)] [ENTER]

The display would read 0.6, which is $\frac{3}{5}$.

Although it is a lot easier to use a calculator on problems like the one in Example 6, solving it without a calculator will be of more use to you in the future.

EXAMPLE 7 Write the expression $\sin(\cos^{-1} x)$ as an equivalent expression in x only.

SOLUTION We let $\theta = \cos^{-1} x$. Then $\cos\theta = x = \dfrac{x}{1}$ and $0 \le \theta \le \pi$. We can visualize the problem by drawing θ in standard position with terminal side in either quadrant I or quadrant II (Figure 12). Let $P = (x, y)$ be a point on the terminal side of θ. By Definition I, $\cos\theta = \dfrac{x}{r}$, so r must be equal to 1. We can find y by applying the Pythagorean Theorem. Notice that y will be a positive value in either quadrant.

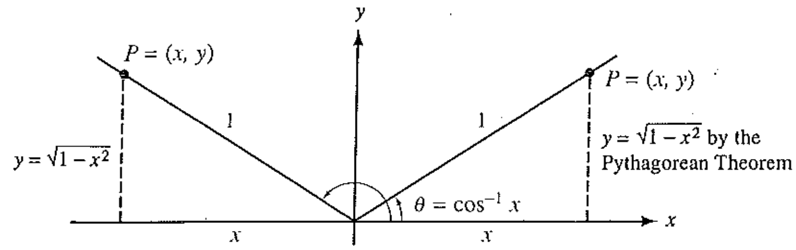


Figure 12

Since $\sin\theta = \dfrac{y}{r}$,

$$\sin(\cos^{-1} x) = \sin\theta = \frac{\sqrt{1 - x^2}}{1} = \sqrt{1 - x^2}$$

This result is valid whether x is positive (θ terminates in QI) or negative (θ terminates in QII).

GETTING READY FOR CLASS

After reading through the preceding section, respond in your own words and in complete sentences.

a. Why must the graph of $y = \sin x$ be restricted to $-\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$ in order to have an inverse function?

b. What restriction is made on the values of x for $y = \cos x$ so that it will have an inverse that is a function?

c. What is the meaning of the notation $y = \tan^{-1} x$?

d. If $y = \arctan x$, then what restriction is placed on the value of y?

PROBLEM SET 4.6

1. Graph $y = \cos x$ between -2π and 2π, and then reflect the graph about the line $y = x$ to obtain the graph of $x = \cos y$.
2. Graph $y = \sin x$ between $-\pi/2$ and $\pi/2$, and then reflect the graph about the line $y = x$ to obtain the graph of $y = \sin^{-1} x$ between $-\pi/2$ and $\pi/2$.
3. Graph $y = \tan x$ for x between $-3\pi/2$ and $3\pi/2$, and then reflect the graph about the line $y = x$ to obtain the graph of $x = \tan y$.
4. Graph $y = \cot x$ for x between 0 and 2π, and then reflect the graph about the line $y = x$ to obtain the graph of $x = \cot y$.

Evaluate each expression without using a calculator, and write your answers in radians.

5. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

6. $\cos^{-1}\left(\frac{1}{2}\right)$

7. $\cos^{-1}(-1)$

8. $\cos^{-1}(0)$

9. $\tan^{-1}(1)$

10. $\tan^{-1}(0)$

11. $\arccos\left(-\frac{1}{\sqrt{2}}\right)$

12. $\arccos(1)$

13. $\sin^{-1}\left(-\frac{1}{2}\right)$

14. $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

15. $\arctan(\sqrt{3})$

16. $\arctan\left(\frac{1}{\sqrt{3}}\right)$

17. $\arcsin(0)$

18. $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$

19. $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

20. $\tan^{-1}(-\sqrt{3})$

21. $\cos^{-1}\left(-\frac{1}{2}\right)$

22. $\sin^{-1}(1)$

23. $\arccos\left(\frac{\sqrt{3}}{2}\right)$

24. $\arcsin(-1)$

Use a calculator to evaluate each expression to the nearest tenth of a degree.

25. $\sin^{-1}(0.1702)$

26. $\sin^{-1}(-0.1702)$

27. $\cos^{-1}(-0.8425)$

28. $\cos^{-1}(0.8425)$

29. $\tan^{-1}(0.3799)$

30. $\tan^{-1}(-0.3799)$

31. $\arcsin(0.9627)$

32. $\arccos(0.9627)$

33. $\cos^{-1}(-0.4664)$

34. $\sin^{-1}(-0.4664)$

35. $\arctan(-2.748)$

36. $\arctan(-0.3640)$

37. $\sin^{-1}(-0.7660)$

38. $\cos^{-1}(-0.7660)$

39. Use your graphing calculator to graph $y = \sin^{-1} x$ in degree mode. Use the graph with the appropriate command to evaluate each expression.

a. $\sin^{-1}\left(-\frac{1}{2}\right)$ b. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ c. $\arcsin\left(-\frac{1}{\sqrt{2}}\right)$

40. Use your graphing calculator to graph $y = \cos^{-1} x$ in degree mode. Use the graph with the appropriate command to evaluate each expression.

a. $\cos^{-1}\left(\frac{1}{2}\right)$ b. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ c. $\arccos\left(\frac{1}{\sqrt{2}}\right)$

41. Use your graphing calculator to graph $y = \tan^{-1} x$ in degree mode. Use the graph with the appropriate command to evaluate each expression.

a. $\tan^{-1}(-1)$ b. $\tan^{-1}(\sqrt{3})$ c. $\arctan\left(-\frac{1}{\sqrt{3}}\right)$

42. Simplify $4\,|\cos\theta|$ if $\theta = \sin^{-1}\frac{x}{4}$ for some real number x.

43. Simplify $2\,|\sin\theta|$ if $\theta = \cos^{-1}\frac{x}{2}$ for some real number x.

44. Simplify $5\,|\sec\theta|$ if $\theta = \tan^{-1}\dfrac{x}{5}$ for some real number x.

Evaluate without using a calculator.

45. $\sin\left(\sin^{-1}\dfrac{3}{5}\right)$

46. $\cos\left(\cos^{-1}\dfrac{3}{5}\right)$

47. $\cos\left(\cos^{-1}\dfrac{1}{2}\right)$

48. $\sin\left(\sin^{-1}\dfrac{1}{\sqrt{2}}\right)$

49. $\tan\left(\tan^{-1}\dfrac{1}{2}\right)$

50. $\tan\left(\tan^{-1}\dfrac{3}{4}\right)$

51. $\sin^{-1}(\sin 225°)$

52. $\sin^{-1}(\sin 330°)$

53. $\sin^{-1}\left(\sin\dfrac{\pi}{3}\right)$

54. $\sin^{-1}\left(\sin\dfrac{\pi}{4}\right)$

55. $\cos^{-1}(\cos 120°)$

56. $\cos^{-1}(\cos 45°)$

57. $\cos^{-1}\left(\cos\dfrac{7\pi}{4}\right)$

58. $\cos^{-1}\left(\cos\dfrac{7\pi}{6}\right)$

59. $\tan^{-1}(\tan 45°)$

60. $\tan^{-1}(\tan 60°)$

61. $\tan^{-1}\left(\tan\dfrac{5\pi}{6}\right)$

62. $\tan^{-1}\left(\tan\dfrac{2\pi}{3}\right)$

Evaluate without using a calculator.

63. $\cos\left(\tan^{-1}\dfrac{3}{4}\right)$

64. $\csc\left(\tan^{-1}\dfrac{3}{4}\right)$

65. $\tan\left(\sin^{-1}\dfrac{3}{5}\right)$

66. $\tan\left(\cos^{-1}\dfrac{3}{5}\right)$

67. $\sec\left(\cos^{-1}\dfrac{1}{\sqrt{5}}\right)$

68. $\sin\left(\cos^{-1}\dfrac{1}{\sqrt{5}}\right)$

69. $\sin\left(\cos^{-1}\dfrac{1}{2}\right)$

70. $\cos\left(\sin^{-1}\dfrac{1}{2}\right)$

71. $\cot\left(\tan^{-1}\dfrac{1}{2}\right)$

72. $\cot\left(\tan^{-1}\dfrac{1}{3}\right)$

73. Simplify $\sin^{-1}(\sin x)$ if $-\pi/2 \le x \le \pi/2$.

74. Simplify $\cos^{-1}(\cos x)$ if $0 \le x \le \pi$.

For each expression below, write an equivalent expression that involves x only. (For Problems 81 through 84, assume x is positive.)

75. $\cos(\cos^{-1} x)$

76. $\sin(\sin^{-1} x)$

77. $\cos(\sin^{-1} x)$

78. $\tan(\cos^{-1} x)$

79. $\sin(\tan^{-1} x)$

80. $\cos(\tan^{-1} x)$

81. $\sin\left(\cos^{-1}\dfrac{1}{x}\right)$

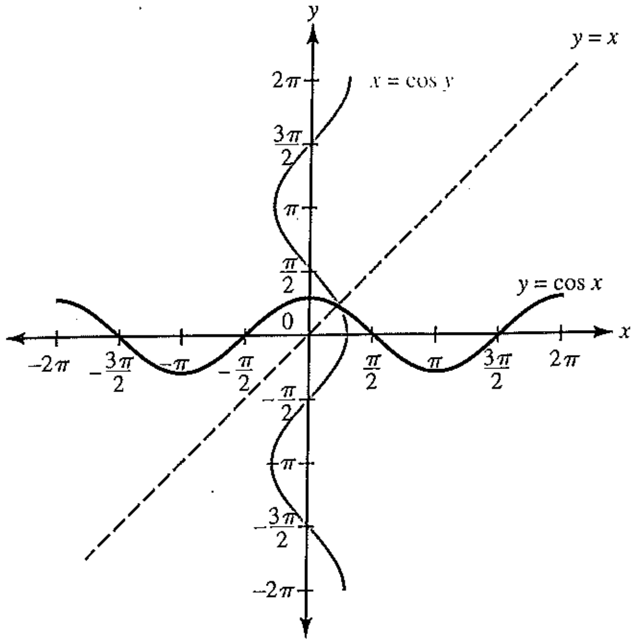
82. $\cos\left(\sin^{-1}\dfrac{1}{x}\right)$

83. $\sec\left(\cos^{-1}\dfrac{1}{x}\right)$

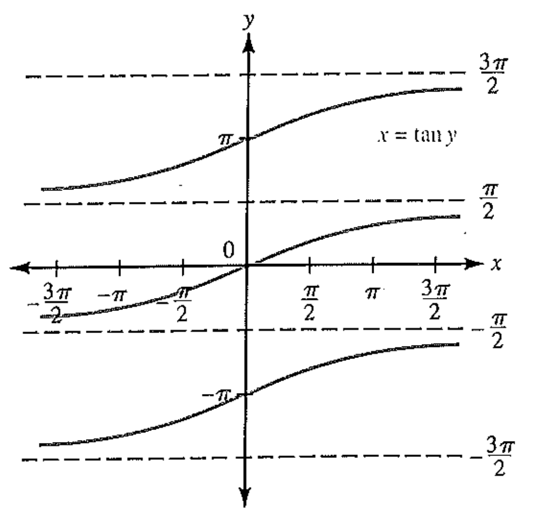
84. $\csc\left(\sin^{-1}\dfrac{1}{x}\right)$

PROBLEM SET 4.6

1.



3. We have not included the graph of $y = \tan x$ with the graph of $x = \tan y$ because placing them both on the same coordinate system makes the diagram too complicated. It is best to graph $y = \tan x$ lightly in pencil and then reflect the graph about the line $y = x$ to get the graph of $x = \tan y$.

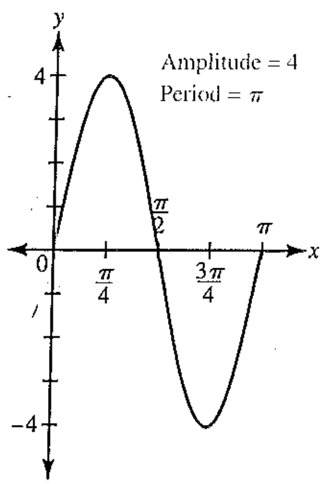


5. $\frac{\pi}{3}$ 7. π 9. $\frac{\pi}{4}$ 11. $\frac{3\pi}{4}$ 13. $-\frac{\pi}{6}$ 15. $\frac{\pi}{3}$ 17. 0 19. $-\frac{\pi}{6}$ 21. $\frac{2\pi}{3}$ 23. $\frac{\pi}{6}$
25. 9.8° 27. 147.4° 29. 20.8° 31. 74.3° 33. 117.8° 35. −70.0° 37. −50.0°
39. a. −30° b. 60° c. −45° 41. a. −45° b. 60° c. −30° 43. $2 \sin \theta$
45. $\frac{3}{5}$ 47. $\frac{1}{2}$ 49. $\frac{1}{2}$ 51. −45° 53. $\frac{\pi}{3}$ 55. 120° 57. $\frac{\pi}{4}$ 59. 45° 61. $-\frac{\pi}{6}$

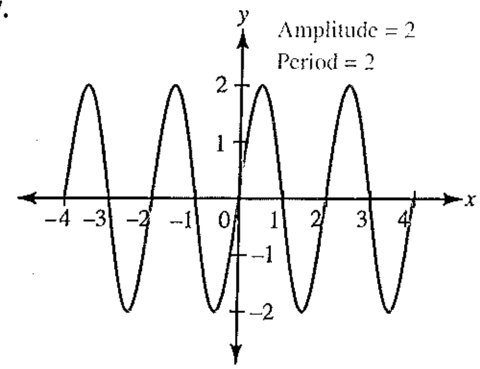
63. $\frac{4}{5}$ **65.** $\frac{3}{4}$ **67.** $\sqrt{5}$ **69.** $\frac{\sqrt{3}}{2}$ **71.** 2 **73.** x **75.** x **77.** $\sqrt{1-x^2}$ **79.** $\frac{x}{\sqrt{x^2+1}}$

81. $\frac{\sqrt{x^2-1}}{x}$ **83.** x

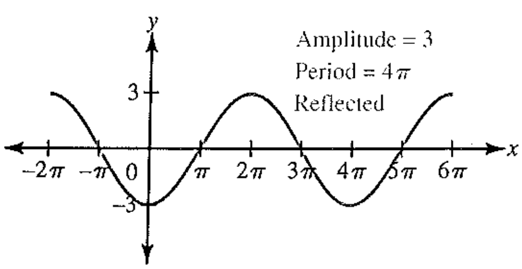
85.



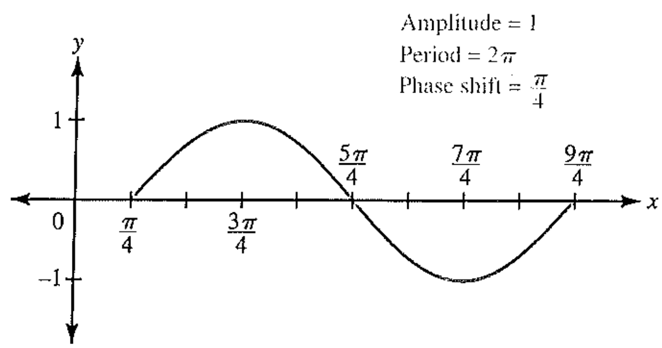
87.



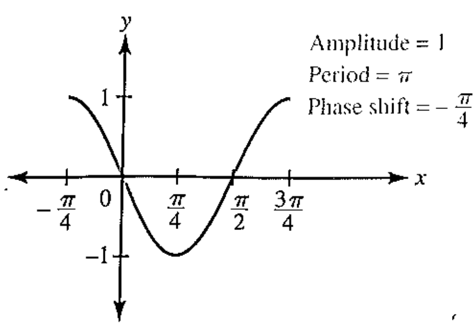
89.



91.



93.



95.