Chapter 8 Chapter Summary and Review

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• Multi-Language Glossary
• Vocabulary practice

chapter 7

VOCABULARY

- **exponential function,** *p. 412*
- **exponential growth function,** *p. 412*
- **asymptote,** *p. 412*
- **exponential decay function,** *p. 419*
- **growth factor, decay factor,** *p. 426*
- **natural base *e*,** *p. 429*
- **common logarithm,** *p. 433*
- **logarithm of *y* with base *b*,** *p. 433*

VOCABULARY EXERCISES

1. How is the graph of an exponential growth function different from the graph of an exponential decay function?

2. Give a real-life example of a quantity that can be modeled as a function of time by an exponential decay function.

3. Which functions in this chapter have horizontal asymptotes? Which functions have vertical asymptotes?

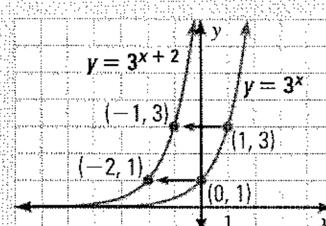
4. The logarithm of y with base b is referred to as the common logarithm under what condition for b?

8.1 EXPONENTIAL GROWTH

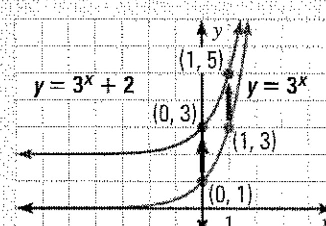
Examples on pp. 412–414

EXAMPLE An exponential growth function has the form $y = ab^x$ with $a > 0$ and $b > 1$. The graph of $y = 3^x$ is shown in blue below. It includes (0, 1) and (1, 3).

You can graph $y = 3^{x+2}$. Sketch the graph of $y = 3^x$. Then translate it 2 units to the left. The graph passes through $(-2, 1)$ and $(-1, 3)$. It has the x-axis as an asymptote. The domain is all real numbers, and the range is $y > 0$.



You can also graph $y = 3^x + 2$. Sketch the graph of $y = 3^x$. Then translate it 2 units up. The graph passes through (0, 3) and (1, 5). It has the line $y = 2$ as an asymptote. The domain is all real numbers, and the range is $y > 2$.



REVIEW HELP

Exercises	Examples
5	**1**, p. 412
6	**2**, p. 413
7, 8	**3**, p. 414

Graph the exponential function. Describe the horizontal asymptote. State the domain and range.

5. $y = 3^x$ **6.** $y = \frac{3}{2} \cdot 2^x$ **7.** $y = 3^{x+4}$ **8.** $y = 2^x - 4$

8.2 EXPONENTIAL DECAY

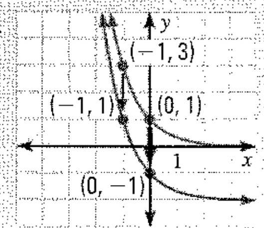
Examples on pp. 419–421

EXAMPLE An exponential decay function has the form $y = ab^x$ with $a > 0$ and $0 < b < 1$.

The graph of $y = \left(\frac{1}{3}\right)^x$ is shown in blue below. It passes through (0, 1) and (−1, 3).

You can graph $y = \left(\frac{1}{3}\right)^x - 2$. Sketch the graph of $y = \left(\frac{1}{3}\right)^x$.

Then translate it 2 units down. The graph passes through (0, −1) and (−1, 1). It has the line $y = -2$ as an asymptote. The domain is all real numbers, and the range is $y > -2$.



REVIEW HELP

Exercises	Examples
9	**1,** p. 419
10	**2,** p. 420
11	**3,** p. 420
12	**4,** p. 421

Graph the exponential function. Describe the horizontal asymptote. State the domain and range.

9. $y = \left(\frac{3}{5}\right)^x$ **10.** $y = 3\left(\frac{1}{5}\right)^x$ **11.** $y = \left(\frac{1}{5}\right)^{x+4}$ **12.** $y = \left(\frac{1}{5}\right)^x + 4$

8.3 MODELING WITH EXPONENTIAL FUNCTIONS

Examples on pp. 426–429

EXAMPLE You can write an exponential function of the form $y = ab^x$ whose graph passes through two points, such as (1, 21) and (2, 63).

STEP 1 Substitute the coordinates of the two points into $y = ab^x$ to obtain two equations in a and b.

$$21 = ab^1$$
$$63 = ab^2$$

STEP 2 Solve the first equation for a.

$$a = \frac{21}{b}$$

STEP 3 Substitute the expression for a into the second equation. Solve for b.

$$63 = \left(\frac{21}{b}\right)b^2$$
$$63 = 21b$$
$$3 = b$$

STEP 4 You know $b = 3$, so you can solve for a.

$$a = \frac{21}{b} = \frac{21}{3} = 7$$

So, $y = 7 \cdot 3^x$ is the exponential function whose graph passes through (1, 21) and (2, 63).

REVIEW HELP

Exercises	Example
13–16	**3,** p. 427

Write an exponential function of the form $y = ab^x$ whose graph passes through the given points.

13. (1, 16), (2, 128) **14.** (2, 20), (3, 40) **15.** (1, 2), (3, 18) **16.** (2, 10), (3, 20)

8.4 LOGARITHMS AND LOGARITHMIC FUNCTIONS

Examples on pp. 433–435

EXAMPLE You can evaluate logarithmic expressions and graph logarithmic functions.

To evaluate $\log_{1/3} 81$, ask yourself what power of $\frac{1}{3}$ gives you 81.

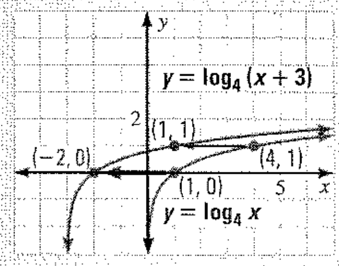
$\left(\frac{1}{3}\right)^? = 81$ — What power of $\frac{1}{3}$ gives 81?

$\left(\frac{1}{3}\right)^{-4} = 81$ — $3^4 = 81$ so use a negative exponent.

$\log_{1/3} 81 = -4$ — Definition of $\log_b y$

EXAMPLE To graph $y = \log_4 (x + 3)$, first sketch the graph of $y = \log_4 x$. Then translate the graph 3 units to the left.

The graph includes $(-2, 0)$ and $(1, 1)$. It has the line $x = -3$ as a vertical asymptote. The domain is $x > -3$, and the range is all real numbers.



REVIEW HELP

Exercises	Examples
17–20	**2**, p. 434
21–24	**4**, p. 435

Evaluate the expression.

17. $\log_3 243$ **18.** $\log_5 \frac{1}{25}$ **19.** $\log_{1/4} 256$ **20.** $\log_{49} 7$

Graph the function. Describe the vertical asymptote. State the domain and range.

21. $y = \log_3 x + 5$ **22.** $y = \log_3 x - 5$ **23.** $y = \log_3 (x - 5)$ **24.** $y = \log_3 (x + 5)$

8.5 PROPERTIES OF LOGARITHMS

Examples on pp. 442–444

EXAMPLE You can use product, quotient, and power properties of logarithms to expand and condense logarithmic expressions. Assume all variables are positive.

Expand $\log_4 \frac{6y}{x} = \log_4 6y - \log_4 x$ — Quotient property

$= \log_4 6 + \log_4 y - \log_4 x$ — Product property

Condense $\log_2 7 + \log_2 3 - 3\log_2 y = \log_2 7 + \log_2 3 - \log_2 y^3$ — Power property

$= \log_2 21 - \log_2 y^3$ — Product property

$= \log_2 \frac{21}{y^3}$ — Quotient property

REVIEW HELP

Exercises	Examples
25–28	**2**, p. 443
29–32	**3**, p. 443

Expand the expression. Assume all variables are positive.

25. $\log_3 3x$ **26.** $\log xy^2$ **27.** $\log_5 \frac{9x}{y}$ **28.** $\log_2 \frac{x^4}{3y}$

Condense the expression. Assume all variables are positive.

29. $\log 4 + \log 12$ **30.** $2 \log 6 - \log 9$ **31.** $\log_3 4 + 3 \log_3 x$ **32.** $2 \log_2 x + \log_2 y$

8.6 SOLVING EXPONENTIAL AND LOGARITHMIC EQUATIONS

Examples on pp. 448–451

EXAMPLE You can solve exponential equations by taking the common logarithm of each side.

$4^x = 11$	Write original equation.
$\log 4^x = \log 11$	Take common logarithm of each side.
$x \log 4 = \log 11$	Power property of logarithms
$x = \frac{\log 11}{\log 4}$	Divide each side by log 4.
$x \approx 1.73$	Use a calculator.

CHECK Check the solution algebraically by substituting into the original equation. The solution checks because $4^{1.73} \approx 11$. ✓

REVIEW HELP

Exercises	Examples
33–35	**1**, p. 448
36–38	**2**, p. 449
39–41	**3**, p. 449

Use the equal powers property to solve the equation.

33. $7^{3x+5} = 7^{11}$ **34.** $6^3 = 6^{2x+5}$ **35.** $8^{4x-9} = 8^x$

Take the common logarithm of each side to solve the equation. Round your answer to the nearest thousandth.

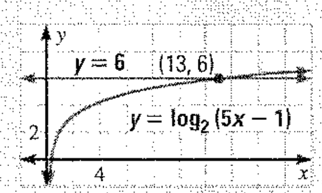
36. $7^x = 31$ **37.** $5^x = 80$ **38.** $2^x = 1000$

39. $10^{5x} = 45$ **40.** $10^{2x+1} = 14$ **41.** $3^{x-7} = 50$

EXAMPLE You can solve logarithmic equations by exponentiating each side of the equation.

$\log_2 (5x - 1) = 6$	Write original equation.
$2^{\log_2 (5x-1)} = 2^6$	Exponentiate each side.
$5x - 1 = 64$	$b^{\log_b x} = x$
$5x = 65$	Add 1 to each side.
$x = 13$	Divide each side by 5.

CHECK Check the solution graphically by graphing both sides of the equation. The two graphs intersect at $x = 13$. ✓



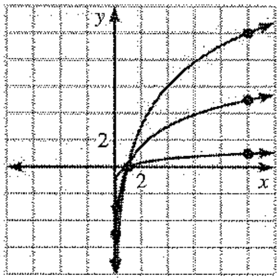
REVIEW HELP

Exercises	Examples
42, 43	**4**, p. 450
44, 45	**5**, p. 450

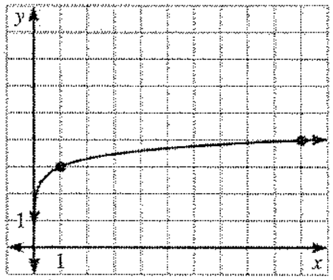
Solve the equation. Check for extraneous solutions.

42. $\log_6 (5x - 2) = \log_6 (2x + 7)$ **43.** $\log_4 (2x + 3) = \log_4 (x - 5)$

44. $\log_3 (4x + 1) = 4$ **45.** $\log_7 (2x - 3) = 2$

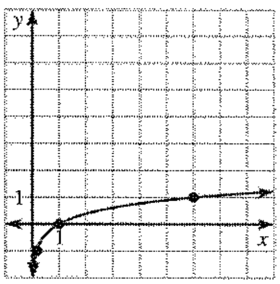
81.  All graphs include the point (1, 0) and have the same basic shape; the graphs are stretched vertically as k increases.

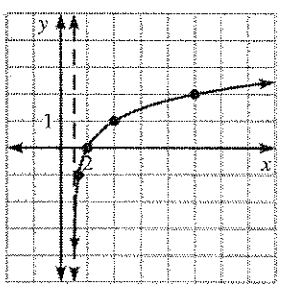
83. 1.11 85. 3.30 87. 1.79 89. 11.07 91. 3.3, 4.3, 5.3

93.  *Sample answer:* $A_1 = 10, A_2 = 10^{1/2}$; ratio: $10^{1/2} \approx 3.16$; no

95. −2 phi units 97. very coarse sand, fine gravel, coarse gravel, small cobble, small boulder, and large boulder

8.4 Technology Activity (p. 440)

1.  *Sample answer:* (1, 0), (6, 1), $x = 0$

3.  *Sample answer:* (2, 0), (4, 1), $x = 1$

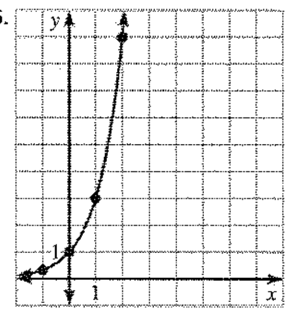
8.5 Guided Practice (p. 445) 1. 10 3. 1.699 5. 2.172 7. $\log 2 + \log x$ 9. $\log_5 3$ 11. $\log x - \log y$ 13. $2 \log_4 x - \log_4 3 - \log_4 y$ 15. $\log_8 21$ 17. $\log_4 56$ 19. $\log \frac{y^5}{x}$ 21. $\log_4 x^2 - \log_4 16y = 2 \log_4 x - \log_4 16 - \log_4 y = 2 \log_4 x - 2 - \log_4 y$

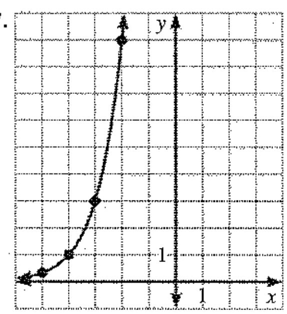
8.5 Practice and Applications (pp. 445–446) 23. A 25. C 27. 2.73 29. 0.504 31. −1.365 33. 1.869 35. $\log 5 + \log x$ 37. $\log_4 7 + 4 \log_4 x$ 39. $\log x + \log y$ 41. $\log_6 7 + \log_6 x + 4 \log_6 y$ 43. $\log_4 x - \log_4 5 - 2 \log_4 y$ 45. $3 \log_7 y - \log_7 20 - 5 \log_7 x$ 47. $\log_5 \frac{2}{3}$ 49. $\log_2 75$ 51. $\log_7 \frac{x^2}{2}$ 53. $\log_5 \frac{x^4}{32}$ 55. $\log_4 x^8y^6$ 57. $\log_2 xy^2$ 59. yes; the loudness is about 88.0 decibels. 61. 4.8 decibels louder 63. more acidic 65. 3, 4.5, 6.8

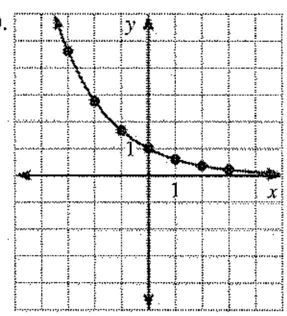
8.6 Guided Practice (p. 452) 1. exponential equation 3. $\frac{1}{2}$ 5. −2 7. $-\frac{1}{2}$ 9. 2.010 11. −1.041 13. $\frac{5}{2}$ 15. $\frac{8}{5}$ 17. 14 19. *Sample answer:* Both sides of the original equation should be in the exponent. So, it should have 3^4 instead of 4^3; $5x - 1 = 81$, and $x = \frac{82}{5}$.

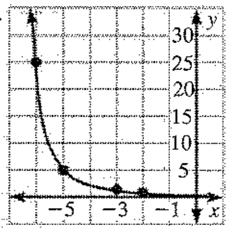
8.6 Practice and Applications (pp. 452–453) 21. 5 23. 2 25. $\frac{4}{3}$ 27. 1.771 29. 0.276 31. 0.715 33. −0.477 35. 2008 37. 7 39. 2 41. $\frac{7}{4}$ 43. $\frac{5}{2}$ 45. −1 47. 1 49. no solution 51. 20 53. −10 55. 111.5 yr 57. 6.97 yr

Chapter Review (pp. 455–458) 1. *Sample answer:* The graph of a growth function increases from left to right, and the graph of a decay function decreases from left to right. 3. Exponential functions have horizontal asymptotes; logarithmic functions have vertical asymptotes.

5.  $y = 0$; domain: all real numbers, range: $y > 0$

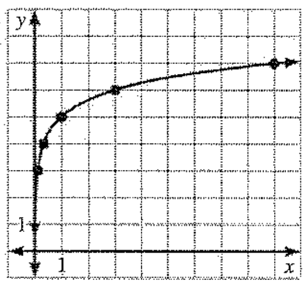
7.  $y = 0$; domain: all real numbers, range: $y > 0$

9.  $y = 0$; domain: all real numbers, range: $y > 0$

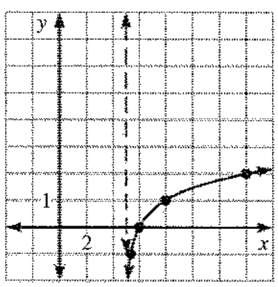
11. 

$y = 0$; domain: all real numbers, range: $y > 0$

13. $y = 2 \cdot 8^x$ **15.** $y = \frac{2}{3} \cdot 3^x$ **17.** 5 **19.** −4

21. 

$x = 0$; domain $x > 0$, range: all real numbers

23. 

$x = 5$; domain $x > 5$, range: all real numbers

25. $1 + \log_3 x$ **27.** $\log_5 9 + \log_5 x - \log_5 y$ **29.** log 48 **31.** $\log_3 4x^3$ **33.** 2 **35.** 3 **37.** 2.723 **39.** 0.331 **41.** 10.561 **43.** no solution **45.** 26

Chapter 9

9.1 Guided Practice (p. 469) **1.** constant **3.** inverse **5.** neither **7.** inverse **9.** inverse **11.** jointly **13.** not jointly **15.** jointly **17.** not jointly

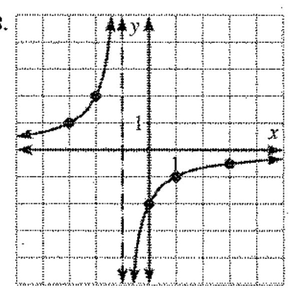
9.1 Practice and Applications (pp. 469–471)

19. inverse **21.** inverse **23.** direct **25.** direct **27.** $y = \frac{3}{x}$ **29.** $y = \frac{36}{x}$ **31.** $y = \frac{2}{x}$ **33.** $t = 9r$ **35.** about 1.8 h; distance **37.** direct **39.** inverse **41.** $z = 4xy$ **43.** $z = \frac{1}{3}xy$ **45.** $z = 10xy$ **47.** $y = \frac{1}{2}xz^2$ **49.** 2 **51.** about 66 cm^3

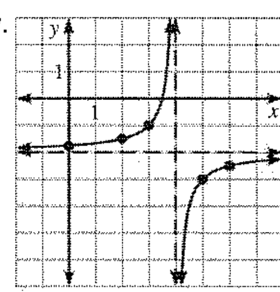
53. The load the beam can safely support is cut in half.

9.2 Guided Practice (p. 475) **1.** hyperbola **3.** The y-axis is the vertical asymptote and the line with equation $y = -4$ is the horizontal asymptote **5.** The line with equation $x = -2$ is the vertical asymptote and the line with equation $y = 2$ is the horizontal asymptote. **7.** The line with equation $x = 1$ is the vertical asymptote and the line with equation $y = -7$ is the horizontal asymptote.

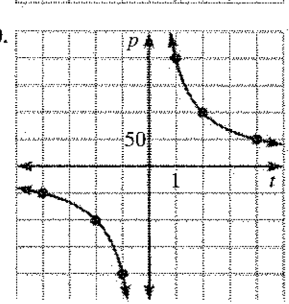
9.2 Practice and Applications (pp. 475–477) **9.** The y-axis is the vertical asymptote and x-axis is the horizontal asymptote; (1, 4), (−1, −4). **11.** The y-axis is the vertical asymptote and x-axis is the horizontal asymptote; (1, −7), (−1, 7). **13.** The y-axis is the vertical asymptote and x-axis is the horizontal asymptote; (1, 5), (−1, −5). **15.** The y-axis is the vertical asymptote and x-axis is the horizontal asymptote; (1, 11), (−1, −11). **17.** The y-axis is the vertical asymptote and the line with equation $y = -5$ is the horizontal asymptote; the domain is the set of nonzero real numbers and the range is all real numbers except −5. **19.** The line with equation $x = 1$ is the vertical asymptote and the x-axis is the horizontal asymptote; the domain is all real numbers except 1 and the range is all nonzero real numbers. **21.** The line with equation $x = -6$ is the vertical asymptote and the line with equation $y = -3$ is the horizontal asymptote; the domain is all real numbers except −6 and the range is all real numbers except −3. **23.** The line with equation $x = -5$ is the vertical asymptote and the line with equation $y = -1$ is the horizontal asymptote; the domain is all real numbers except −5 and the range is all real numbers except −1. **25.** The line with equation $x = 7$ is the vertical asymptote and the line with equation $y = 3$ is the horizontal asymptote; the domain is all real numbers except 7 and the range is all real numbers except 3. **27.** C **29.** The vertical asymptote should be the line with equation $x = 4$ and horizontal asymptote should be the line with equation $y = -2$.

33. 

The domain is the real numbers except −1 and the range is the nonzero real numbers.

37. 

The domain is the real numbers except 4 and the range is the real numbers except −2.

39. 

The p-axis is the vertical asymptote and the t-axis is the horizontal asymptote; that part of the graph found in quadrant I models the situation.

SELECTED ANSWERS